24/1 (2026), 103–125
DOI: 10.5485/TMCS.2026.16199
tmcs@science.unideb.hu
https://ojs.lib.unideb.hu/tmcs

Teaching Mathematics and Computer Science

Error analysis in teaching combinatorics: the development of prospective teachers' confidence and problem-solving skills

ZOLTÁN PAULOVICS, CSABA CSAPODI and ZOLTÁN LÓRÁNT NAGY

Abstract. This study investigates the pedagogical potential of error analysis in the teaching of combinatorics within mathematics teacher education. Building on previous research that highlights the role of incorrectly worked sample solutions in cognitive, metacognitive, and affective learning processes, we conducted a mixed-methods study with prospective mathematics teachers at Eötvös Loránd University. Quantitative results from Likert-scale questionnaires ($n = 26$) indicate that regular analysis of incorrectly worked solutions substantially enhanced participants' self-confidence, strengthened their problem-solving skills, and positively shaped their attitudes toward future teaching practice. Complementary qualitative data, analyzed through grounded theory, revealed five interrelated categories – self-reflection and confidence, discernment, deeper understanding, methodological surplus, and combinatorial surplus – that together explain the mechanisms through which error analysis supports professional growth. The findings suggest that systematic analysis of conceptual errors not only improves problem-solving competence but also fosters self-confidence, self-reflection, and teaching-related attitudes. By comparing our emergent model of error-analysis thinking with Schoenfeld's problem-solving framework, we argue that "discernment" constitutes a distinctive and central dimension of error-based learning. The study contributes both theoretically, by refining models of mathematical problem solving, and practically, by offering concrete recommendations for integrating error analysis into mathematics teacher education curricula.

Key words and phrases: error analysis, combinatorics education, prospective mathematics teachers, incorrectly worked sample solutions, problem-solving competence, confidence, teacher attitudes, Schoenfeld's framework, grounded theory.

MSC Subject Classification: 97C30, 97K20, 97D40, 97C70, 97C99.

Introduction

Solving combinatorial problems poses particular challenges for prospective mathematics teachers: beyond producing a correct final answer, it is essential for future instruction that the individual reasoning units should be presented in a logical, structured and transparent manner. To foster prospective teachers' problem-solving competence and self-confidence, increasing emphasis has been placed on exploiting errors as a learning tool: rather than relying exclusively on analyses of correct worked sample solutions, didactic research also investigates how the deliberate examination of worked solutions containing errors contributes to student development (Chi et al., 1989; Große & Renkl, 2007; Paulovics et al., 2023; Renkl, 1997; Rushton, 2018).

Traditional mathematics instruction often privileges error-free, "elegant" solutions: teachers predominantly demonstrate the correct route, while students seldom encounter dead ends or reconstruct principled mistakes. In contrast, error analysis – a complex process that integrates cognitive, metacognitive and affective components – can foster not only deeper content knowledge but also a transformation in students' relationships to learning (Van Gog et al., 2019; Große & Renkl, 2007; Paulovics et al., 2023). Given the prevalence of characteristic errors in combinatorics, classroom processing of erroneous lines of reasoning is especially warranted, because prospective mathematics teachers frequently exhibit limited skills in detecting and refuting errors (Paulovics & Csapodi, 2025).

Although numerous studies have examined the role of errors as learning stimuli (Curry, 2004; VanLehn, 1999), comparatively little attention has been given to the ways in which future teachers can incorporate this strategy into their own teaching practice. Moreover, our understanding remains limited regarding how regular analysis of incorrectly worked sample solutions influences prospective teachers' problem-solving competence, confidence, and pedagogical attitudes – particularly in combinatorics, where tasks often demand creative reasoning rather than mechanical algorithmic application.

Context of the study

The experiment reported by Paulovics et al. (2023) involved a compulsory course attended by second-year mathematics teacher education students at Eötvös Loránd University (Hungary). The course aimed to review and consolidate combinatorics topics that are prominent in the upper-secondary curriculum, to revisit certain graph-theoretic notions to a lesser extent, and to develop the problem-solving skills required for these domains.

During the semester, students had opportunities for individual problem solving; each session included instructor-led joint analyses of correctly worked sample solutions and of incorrectly worked sample solutions, and, occasionally, collaborative problem solving and group analyses were also employed. The incorrectly worked sample solutions shared a common structure: a fictional character (Winnie-the-Pooh) presented a counting solution and students were asked to determine whether his reasoning was correct. It is important to emphasize that the errors examined were conceptual rather than procedural. Following Eizenberg and Zaslavsky (2004), we refer to a conceptual error (or conceptually incorrect worked sample solution) when the line of reasoning contains a logical or principled flaw. Procedural errors (and procedurally incorrect worked sample solutions), by contrast, arise during the execution of a solution (i.e., calculation slips or omissions that lead to an incorrect final result). Error analysis, in this study, denotes the process of locating and explicating the faulty piece of reasoning within a given solution.

Paulovics et al. (2023) report that regular error analysis – particularly of solutions containing conceptual errors – can be an effective instrument in shaping prospective teachers' combinatorial competencies and attitudes. Since the effectiveness of teaching and learning combinatorial problem solving depends strongly on how future teachers relate to incorrect solutions and on their proficiency in error analysis, a deeper understanding of these processes and their didactic application can contribute to improved prospective teacher education.

Aim of the study

The present study aims to elucidate how regular analysis of erroneous solutions affects mathematics teacher-education students' combinatorial problem-solving skills, self-confidence, and teacher attitudes. In particular, we investigate how deliberate exposure to errors contributes to students' cognitive strategy development, strengthens metacognitive self-regulation, and fosters positive affective orientations toward learning.

Theoretical background

This section outlines the literature underpinning the study, with particular attention to the role of error analysis in problem solving across cognitive, metacognitive and affective dimensions, and to Schoenfeld's problem-solving framework (Schoenfeld, 1985, 2016), which structures these dimensions.

Incorrectly worked sample solutions

A correctly worked sample solution can be decomposed into three principal components: the task statement, the detailed steps of the solution, and the final result. Analyses of worked examples reduce learners' extraneous search for irrelevant details and can free cognitive resources for deeper processing. However, mere exposure to correct examples does not guarantee deep understanding: some learners remain passive and do not engage in generative self-explanation of the steps (Adams et al., 2014; Chi et al., 1989; Van Gog et al., 2019; Große & Renkl, 2007; Renkl, 1997). Thus, instructional design must actively promote self-explanation to foster structural knowledge.

Incorrectly worked examples differ from correct ones by means of at least one step contains an incorrect element. In error analysis, learners first identify the error, then interpret it and, if appropriate, correct it; this process not only improves accuracy but also deepens conceptual understanding (Rushton, 2018). Pairing examples and non-examples – or presenting correct and incorrect solutions in parallel – strengthens critical analytic skills and clarifies conceptual boundaries (Skemp, 1987; VanLehn, 1999). The CASCADE model (VanLehn, 1999) suggests that noticing early-stage errors or uncertainties and then reflecting on them (for example, via self-explanations) is crucial for deeper acquisition, a claim consistent with related theoretical positions (Siegler, 2002; Curry, 2004).

The effect of error analysis on attitudes to combinatorics and problem-solving skills

Paulovics et al. (2023) examined how regular analysis of incorrect solutions influenced prospective teachers' attitudes toward combinatorics and their problem-solving competence. Their combinatorics course for second-year prospective teachers was designed as an experimental intervention study with a two-group pre-posttest design, and the data were analyzed statistically using SPSS (Creswell, 2014). Paired t-tests revealed significant differences across the studied domains:

error analysis substantially increased positive attitudes toward combinatorics and improved end-of-semester test means relative to a control group.

The results demonstrated that error analysis yielded multi-level benefits: conceptual accuracy improved, misconceptions and errors decreased, and students became more active and reflective in interpreting errors. Moreover, the instructor gained deeper insight into students' problem-solving processes, which in turn enhanced pedagogical confidence and the intentionality of instructional choices. These findings provided a promising foundation for further investigation – one that the present study seeks to pursue.

Schoenfeld's model

Given the centrality of Schoenfeld's problem-solving framework (Schoenfeld, 1985, 2016) for interpreting our results, we provide a detailed account here to enable later comparison.

1. Resources

The resources component refers to the body of knowledge and tools available to a problem solver. This includes declarative knowledge (theorems, definitions), procedural knowledge (rules, algorithms), as well as intuitive schemas, routines, prior solution patterns, and meta-knowledge about when and how to apply them. Crucially, Schoenfeld emphasizes that possession of resources is insufficient; what matters is the ability to activate and use them effectively. Students often fail not because they lack knowledge, but because they do not mobilize it effectively and flexibly. Pedagogically, this underscores the need not only to transmit new knowledge but also to develop students' capacity to access and apply existing knowledge meaningfully.

2. Heuristics

Heuristics denote non-algorithmic strategies that guide problem solving when routine procedures fail. Unlike procedural rules, heuristics do not guarantee solutions but rather structure the search process – for example, drawing diagrams, rephrasing problems, working backwards, or testing special cases, as famously systematized by Pólya. Their effectiveness depends on deliberate, reflective practice across varied tasks. Instruction should therefore move beyond demonstration, requiring students to apply, analyze, and evaluate heuristics actively. Properly integrated, heuristics function as cognitive "maps", directing learners' exploration of problems.

3. Control

Control represents the metacognitive dimension of problem solving: the capacity for self-regulation through planning, monitoring, and evaluation. Planning involves selecting strategies and goals; monitoring tracks progress and identifies dead ends; evaluation assesses strategy effectiveness and prompts revision. These processes operate cyclically, transforming problem solving into a sequence of conscious decisions rather than mechanical execution. Successful solvers routinely exercise control by abandoning unproductive strategies, reflecting on their thinking, and learning from mistakes. By contrast, weak control often results in abandoned attempts or repetitive, ineffective procedures – even among students with adequate knowledge and heuristics. Importantly, control extends beyond the solution phase to task interpretation and post-solution reflection, making it a cornerstone of both problem solving and long-term learning. Instruction must therefore cultivate control through fostering strategic awareness, reflective thinking, and learner autonomy.

4. Beliefs

Finally, beliefs capture the affective and epistemological orientations that shape students' engagement with mathematics. These include views on the nature of mathematics, perceptions of personal ability, and assumptions about success and failure. Deeply ingrained, often unconscious, such beliefs regulate persistence, strategy choice, and attitudes toward difficulty. For instance, students convinced that problems should be solved quickly may abandon tasks prematurely when immediate success is lacking. Beliefs thus serve as active regulators of problem-solving behavior, influencing what strategies are considered legitimate or worthwhile. Schoenfeld argues that effective pedagogy must explicitly address and reshape students' beliefs, fostering dispositions that value perseverance, reflection, and interpretive engagement with mathematical tasks.

In sum, Schoenfeld's model – through its four interdependent dimensions of resources, heuristics, control, and beliefs – offers a comprehensive framework for understanding mathematical problem solving and for designing instructional practices that support not only cognitive development but also metacognitive growth and affective resilience.

Research questions

This study aims to build on the findings of Paulovics et al. (2023) by examining why frequent and systematic analysis of incorrectly worked sample solutions within a course led to significant improvements in prospective teachers' attitudes toward combinatorics and their problem-solving skills, as well as what factors account for the instructor's observed increase in pedagogical confidence during the semester.

More specifically, the guiding research question is: *In what ways did the implemented intervention influence prospective teachers' confidence in combinatorics, their problem-solving competence, and their teaching-related attitudes, and how can these effects be synthesized into an explanatory model of its impact?*

Research method

Participants were drawn from the experimental group of the Paulovics et al. (2023) study: second-year prospective mathematics teachers at Eötvös Loránd University. The sample comprised 26 students (aged 19–23; 22 female, 4 male). Following Taherdoost (2022), we developed a two-part questionnaire (approx. 30 minutes), which students completed during the final session of the semester.

The first part employed Likert-type scaling and included six closed-ended items. Respondents rated their agreement with each statement on a 5-point scale, where 1 = "strongly disagree" (not at all true of me), 2 = "disagree" (not very true of me), 3 = "neutral", 4 = "agree" (largely true of me), and 5 = "strongly agree" (completely true of me).

The six statements (presented in randomized order) were as follows:

(1) 'Because we analyzed incorrect solutions, I am less afraid that I will have to correct incorrect student solutions when I become a practicing teacher.'

(2) 'Repeated exposure to incorrect solutions has made me less confident in solving combinatorics problems.'

(3) 'Analyzing incorrect solutions has directly contributed to my solving counting or other combinatorics problems correctly more often.'

(4) 'Because of analyzing incorrect solutions, I performed better on my midterm exam.'

(5) 'I am glad that during the semester we frequently engaged in analysis of incorrect solutions.'

(6) 'If I teach in the future, I will often bring incorrect student solutions to class.'

Items (1) and (2) addressed confidence, Items (3) and (4) targeted problem-solving competence, and Items (5) and (6) concerned teaching-related attitudes.

The qualitative strand was informed by grounded theory (Strauss & Corbin, 1990), a methodology that enables researchers to explore phenomena from multiple angles and develop explanatory frameworks that remain closely tied to empirical data (Corbin & Strauss, 2014; Willig, 2013). Grounded theory thus facilitated in-depth interpretation of participants' perspectives in relation to existing literature and the researchers' pedagogical experience.

The second part of the questionnaire comprised two open-ended items that required extended written responses:

- 'In what ways did the analysis of incorrect solutions contribute to your confidence in your own problem solving?'
- 'To what extent did the analysis of incorrect solutions support the development of your combinatorial problem-solving skills?'

To assess the confidence-enhancing, skill-improving, and attitudinal effects of error analysis, we employed a mixed-methods approach: quantitative analysis of the Likert-scale data and qualitative coding of the open responses (Creswell, 2014).

Results

We report quantitative and qualitative findings below.

Quantitative data

Responses to the six Likert items are summarized in Table 1.

Confidence. For Item (1), 20 out of 26 respondents selected 4 or 5, indicating that a substantial majority agreed that familiarity with error analysis represented pedagogically valuable experience they expected to apply as practicing teachers. Only one respondent rated this item as 1 or 2. Similarly, Item (2), after reverse coding, yielded 18 confidence-positive responses, suggesting that repeated exposure to error analysis did not undermine but rather enhanced students' confidence in solving combinatorial tasks. Taken together, the two confidence items yielded 38 positive responses out of 52 (73%), showing that regular error analysis had a generally positive effect on participants' confidence.

Problem-solving competence. For Item (3), 16 respondents partially or fully agreed that analyzing incorrect solutions directly contributed to solving combinatorial tasks correctly; only 2 judged that such exercises had no effect on their end-of-semester problem-solving ability. Item (4) produced a more mixed pattern: 13 positive (4 or 5) responses contrasted with 6 negative (1 or 2) responses, suggesting that about half the students perceived an improvement in their midterm performance attributable to error analysis. Across both competence items, 29 responses indicated agreement and 8 disagreement, implying that students more often perceived benefits for their problem solving than not.

Teacher attitudes. Item (5) attracted exceptionally strong endorsement: 17 respondents gave the maximum rating of 5, and only one rated below 3, indicating widespread satisfaction with frequent engagement in error analysis. For Item (6), 14 out of 26 indicated they would themselves use incorrect solutions in their future classrooms. Aggregated agreement across the two attitude items was high (36 positive responses) and contrasted with only 5 rejections.

Values	**1**	**2**	**3**	**4**	**5**
Statement 1	0	1	5	7	13
Statement 2	9	9	3	5	0
Statement 3	1	1	8	9	7
Statement 4	3	3	7	9	4
Statement 5	0	1	3	5	17
Statement 6	2	2	8	8	6

Table 1. The distribution of quantitative data

Qualitative data

Following grounded theory procedures, we applied open, axial and selective coding (Strauss & Corbin, 1990). Below, we summarize the outcomes of each category.

Open coding. Open coding generated descriptive labels for early observations and emergent themes (Corbin & Strauss, 2014; Willig, 2013). Across the open responses, we identified 58 distinct idea units, which were consolidated into 18 initial codes. For example, Participant 3's response to Question 2 – "I handle my own mistakes much more confidently now" – was coded as "(self-)confident handling of errors." The emergence and subsequent reduction of new codes signaled theoretical saturation: the final three responses yielded only one previously unseen code (Guest et al., 2006).

Axial coding. Axial coding examined relationships among the open codes, yielding five higher-order categories. These categories (presented below) are illustrated with extensive participant quotations, and their frequencies are summarized in Table 2.

Self-reflection and confidence. A key experience for participants was the development of their ability to reflect on their own problem-solving processes – especially in analyzing seemingly incorrect or demonstrably flawed approaches and in initiating correction. This reflective practice formed the basis for increased self-confidence. As one student explained, "I found it easier to recognize the wrong solutions of my colleagues and my own, so I became more confident when I didn't feel my solution was wrong." Self-reflection thus supported not only the recognition of errors but also a more accurate evaluation of one's own performance, thereby reducing uncertainty.

The growth of self-reflection was reinforced by the "constant questioning" of solutions. Over time, this critical stance fostered greater confidence, as students realized that they had "mastered the basics better." With a more secure theoretical foundation, mistakes became less discouraging, and students were able to continue working productively even when encountering errors.

Several participants emphasized this shift, noting, for example: "I don't get so discouraged when I find out that my solution is wrong." Such statements indicate that many students had previously struggled to cope with unexpected setbacks in their problem solving, but through practice they developed resilience, self-checking skills, and a more proactive approach to error correction. As one student summarized: "I am more confident in finding errors in my existing thought process."

Discernment (error handling, error identification, and learning lessons). Discernment is an internal disposition in which individuals draw on prior knowledge, experience, and cognitive, metacognitive, and affective resources to judge the relevance – and thus the correctness – of an impulse (e.g., a proposed solution step) and its effect on the subsequent problem-solving process. Within error analysis, this process relies particularly on four skills: identifying erroneous reasoning, dealing with one's own mistakes, recognizing recurring error types, and applying lessons learned from previous errors.

Participants emphasized that error recognition developed into a skill during the course: "I started to see the types of errors" and "I learned to correct them." This newly acquired competence not only accelerated error detection but also reduced error frequency: "In similar tasks, these incorrect solutions no longer occur to me, or I quickly realize that they are wrong"; "I know better where to pay more attention, where there are greater opportunities for error"; and "I pay attention to possible points of error even while solving the problem, not just afterwards." Analyzing typical errors therefore enhanced both task-specific problem solving and long-term strategic thinking, increasing the likelihood of avoiding flawed reasoning.

The ability to recognize and filter out faulty lines of thought is also critical from a pedagogical perspective: future teachers must be able to detect student errors quickly and provide effective guidance. As one participant noted, they became "more sensitive to the pitfalls of combinatorial tasks, to what can go wrong", a shift that further strengthened their problem-solving capacity.

Deeper understanding (underlying mathematical structures, models, and their applications). This category proved central to the research: participants' feedback indicated that gaining a more thorough grasp of combinatorial models and their applications contributed substantially to both their problem-solving skills and their self-confidence. Regular analysis of incorrectly worked sample solutions not only clarified technical details but also created opportunities to recognize and correct fundamental misconceptions. As one participant explained, "I had to spend more time on a task, and I gained a better understanding of what we were doing and why." This process encouraged students to attend more closely to the logical structure of solutions and their interconnections.

Through regular analysis and reflection, the essential stages of problem solving became clearer: "the points where mistakes can be made during problem solving became clearly identifiable" and "the cornerstones became clearer." Such clarity fostered more structured thinking patterns, which participants described as "encouraging more systematic thinking." The analysis also revealed that many errors stemmed from deeper conceptual difficulties, helping students approach more complex problems with greater insight. As one noted: "I often didn't understand before how my incorrect solutions stemmed from errors in my approach" and "it made it much clearer how the task texts were linked to a type of solution."

Detailed examination of incorrect examples also strengthened students' self-reflection skills. Several participants reported: "I can better and more deeply justify why a wrong solution is wrong" and "it helped me understand why one or two wrong solutions that I often made were actually wrong." Such deeper understanding not only supported the successful handling of immediate problems but also provided long-term learning benefits.

In sum, deeper understanding contributed not only to improved task performance but also to a fundamental reshaping of students' problem-solving approaches and their methodological stance toward mathematics. This category is especially significant for teacher training, as it highlighted the value of error analysis both for applying combinatorial models effectively and for developing reflective pedagogical practices.

Approach to task solving – methodological surplus. From a research perspective, this category highlights a key dimension of participants' development: a more methodologically sound and precise approach to problem solving. Regular analysis of poorly developed examples underscored that the success of task-solving depends on the thoroughness and logical coherence of the applied methods. As several participants put it, "I think through solutions more carefully", showing that reflection and deliberate consideration played a central role in the course.

The added value of this approach was reinforced by greater attention to precision. Participants repeatedly emphasized that they were "solving combinatorial problems much more precisely" and striving to "check every little detail." These remarks suggest that analyzing incorrect examples brought not only error identification and correction into focus, but also meticulous self-checking and attention to detail. Such practices supported the successful solution of combinatorial tasks while simultaneously strengthening self-reflection skills.

Methodological precision further promoted more careful and meaningful reading of problem texts, a basic prerequisite for correctly interpreting combinatorial tasks. As one participant explained, "I read the texts better and with greater understanding", while another added, "I check my solutions more thoroughly." This feedback shows that methodological development extended beyond individual steps to encompass the entire problem-processing procedure. Ultimately, this category reflected a shift in learning attitudes, as participants stated: "I think through my own solutions more often and more thoroughly" and "I did not make the same mistakes."

Approach to problem solving – combinatorial surplus. From a research perspective, this category highlights the expansion and deepening of participants' combinatorial knowledge. Regular analysis of incorrectly worked examples helped prospective teachers approach problems in a more flexible and versatile way.

As one participant reflected, "We always discussed different lines of thought, and my own knowledge became more diverse."

Methodological diversity played a central role in this process. Participants emphasized that they "always had to try multiple solutions" during course assignments. These experiences not only strengthened their problem-solving abilities but also fostered a deeper understanding of logical relationships through the comparison of different approaches. In this way, the course contributed to the enrichment and systematization of problem-solving strategies.

Viewing combinatorial problems from multiple perspectives also broadened participants' capacities. One student noted, "I can solve a problem from multiple perspectives" and "I can understand another line of thought from multiple perspectives." This flexibility is especially important for teachers, as recognizing and integrating diverse approaches into classroom practice is an essential pedagogical competence.

Knowledge expansion was evident not only in the variety of solution techniques but also in the refinement of skills: "It greatly complemented and refined my skills." This process emphasized the methodological richness inherent in combinatorial problems, which the course made accessible to students. Overall, this category indicates that analyzing incorrect solutions enhanced participants' confidence while making their combinatorial knowledge deeper, more structured, and more nuanced – developments which are likely to support their future teaching.

Category	**Self-reflection and confidence**	**Discernment**	**Deeper understanding**	**Methodological surplus**	**Combinatorial surplus**
Total number of observed codes	11	20	9	10	8

Table 2. Number of observed codes in each category

Selective coding. The final stage of grounded theory data analysis is selective coding (Strauss & Corbin, 1990). This stage aims to integrate categories into a coherent framework, validate relationships between concepts, and refine categories that require further development. To explore these relationships, we constructed a complete weighted graph with five vertices, each representing one category. Weighted graphs are a common technique for illustrating connections between students' learning goals. Edge weights were calculated by identifying which categories co-occurred in each participant's argument: for every pair of categories mentioned, the weight of the connecting edge was increased by one.

Applying this procedure across all participants yielded the graph shown in Figure 1. Edge weights are reported in Table 4, and the frequency of references to each category is presented in Table 3. For example, the weight of edge C52 is eight, meaning that participants linked Categories 2 and 5 on eight occasions.

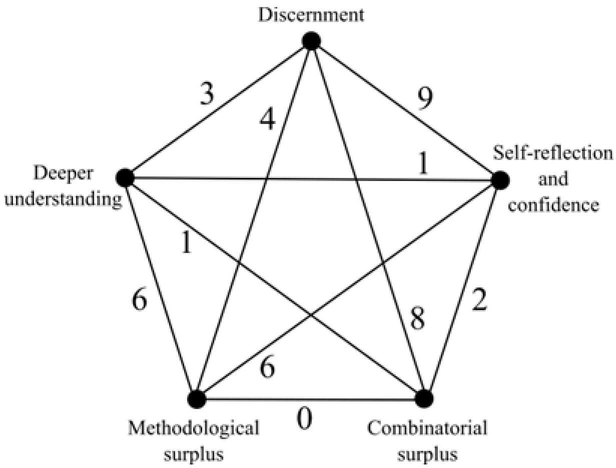

Figure 1. The complete weighted graph

Self-reflection and confidence	Discernment	Deeper understanding	Methodological surplus	Combinatorial surplus
21	34	16	19	13

Table 3. Number of references to certain categories

Four edges clearly have greater weight than the others: the weight of edges C12, C25, C14, and C34 is six, eight, or nine; furthermore, three edges have a weight greater than one: C23, C24, and C15. The weight of the other edges is zero or one. We illustrate these four stronger and three moderately strong connections with a diagram (Figure 2). To validate the model, we returned to the participants' responses, as suggested by Strauss and Corbin (1990), to compare the developed theory with the original data.

		Self-reflection and confidence	Discernment	Deeper understanding	Methodological surplus	Combinatorial surplus
		1	2	3	4	5
Self-reflection and confidence	1	3				
Discernment	2	9	10			
Deeper understanding	3	1	3	5		
Methodological surplus	4	6	4	6	3	
Combinatorial surplus	5	2	8	1	0	2

Table 4. Weights of the edges in the complete graph

The number of references to the category "Discernment" (34; Table 3) is strikingly high compared with the other categories. Moreover, Discernment is connected to each of the other categories with at least three weighted edges – representing the three strongest links in the network. Both the participants' definitions and their responses confirm the central role of this category. Discernment integrates cognitive, metacognitive, and affective elements, as illustrated in statements such as: "I recognize type errors more easily and quickly", "I have become more sensitive to the pitfalls of combinatorial tasks", "I pay attention to possible points of error even while solving", and "I can now solve tasks more critically and thoughtfully."

The category "Attitude toward problem solving – combinatorial surplus" reflects mainly the cognitive aspects of error analysis. Apart from its strong association with "Discernment", it was linked to only a few other categories. Typical responses included: "It helped me distinguish between different types of combinatorial tasks and solutions" and "We always discussed different lines of thought and my own knowledge became more diverse." These confirm that participants recognized cognitive gains through error analysis.

The category "Approach to task solving – methodological surplus" represents the metacognitive aspects of error analysis. It showed the strongest connections with the categories "Deeper understanding" and "Self-reflection, self-confidence", which primarily reflect affective dimensions of learning.

Finally, two categories were identified that are closely related to metacognition: “Deeper understanding” and “Attitude towards task solving – methodological surplus”. While student responses suggest that “Deeper understanding” is primarily metacognitive, it also incorporates several cognitive elements. By contrast, the link between “Approach to task solving – combinatorial surplus” and “Deeper understanding” was weak, with a weight of only 1.

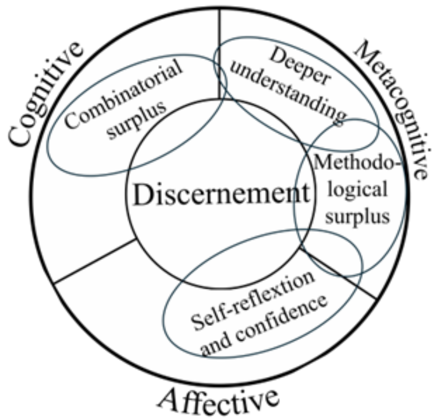

Figure 2. The model of error analysis thinking

Discussion and conclusion

Prospective mathematics teachers in our study evaluated the impact of error analysis on their professional development through both Likert-scale ratings and open-ended reflections.

The results of the study show that analyzing incorrectly worked sample solutions had a particularly positive effect on the confidence of teacher training students. Based on the quantitative data, the vast majority of participants agreed that the practice of error analysis provides useful pedagogical experience in the long term and did not reduce their confidence in solving combinatorial problems. This is a particularly important finding, as previous research (e.g., Schoenfeld, 1985; Törner, 2002) has emphasized that attitudes toward mistakes strongly influence participants’ confidence and problem-solving efficiency. Our results confirm the opposite: 73% of students reported an increase in confidence, which supports the relevance of incorporating error analysis into teacher training practice.

A more nuanced picture emerged with regard to problem-solving skills. Although the majority of prospective teachers felt that working with incorrect solutions contributed to improving their performance, the results here were less clear-cut than in the case of confidence. For example, one statement received a more balanced distribution of responses, which may indicate that prospective teachers internalized the experiences gained from error analysis to varying degrees. This is consistent with Pólya's (1945) heuristic approach, which views the development of problem solving as a gradual process, as well as with Schoenfeld's (1985, 2016) emphasis on the control component, according to which effective self-regulation and metacognitive functioning play a key role in enabling learners to apply strategies flexibly.

The results regarding teacher attitudes were clearly positive. The majority of participants welcomed the use of incorrect solutions and a significant proportion indicated their intention to apply this method in their future teaching practice. This is in line with the interpretation of errors as a pedagogical resource (Borasi, 1996; Radatz, 1980), which can promote reflective learning. The openness of prospective teachers to using mistakes as a teaching tool may contribute to the creation of a more constructive, learner-centered learning environment in the long term. Overall, the quantitative results confirm that didactic practices based on the analysis of incorrect solutions strengthens confidence, supports the development of problem-solving skills, and shapes participants' attitudes in a positive direction.

In analyzing the qualitative results of our research, we focus on the following dimensions: the five categories obtained by applying the grounded theory method (Self-reflection and confidence, Discernment, Deeper understanding, Attitude towards task solving – methodological surplus, Attitude towards task solving – combinatorial surplus) are situated within a complex space defined by cognitive, metacognitive, and affective dimensions. Based on the responses, we were able to create a tentative model of the mechanisms through which regular error analysis positively influences teacher training students' confidence in combinatorial problems, problem-solving skills, and teaching attitude. The resulting model differs from previous models, thus providing a good opportunity to observe and understand new aspects of the positive effects of error analysis. It is encouraging that, in addition to its uniqueness, many already known and important factors have also been identified in our model through comparison with other models.

It is natural to compare our model with Schoenfeld's problem-solving model, which is considered fundamental in the field of problem solving. In order to understand the process of problem solving or error analysis – and thus the development of mathematical problem-solving competence – all four main components of Schoenfeld's problem-solving model – resources, heuristics, control, and beliefs – must be considered together. According to the theory, successful problem solving depends on the integrated functioning of resources, heuristics, control processes, and beliefs, all of which must be learned and taught (Schoenfeld, 1985, 2016). In the following, we will explain the similarities and differences between the models in more detail.

Comparison of models

In our model, the central position of the concept of "discernment" suggests that the essence of student development lies in experiencing, learning about, and mastering the process of discernment, since this allows for more effective and successful error analysis through increasingly refined perception of the structural features characteristic of a given task (in a specific situation). As a general model of the problem-solving process, the Schoenfeld model does not, of course, explicitly include this important component of error analysis, the concept of discernment. This (first and very important added) contribution of our model shows, on the one hand, that narrowing down the general process of problem solving to a specific area (error analysis) necessarily introduces special concepts and innovations into the model, and, on the other hand, that highlighting and presenting this key concept can also have a fruitful impact on the interpretation of the general model.

From the definition of the concept of "discernment" – based on the participants' responses – and the many references to it, we can conclude that error analysis as a complex process (since it is not problem solving in the usual sense, whether easy or difficult), it is closely related to the combined treatment of cognitive, metacognitive, and affective components: that is, while in certain problem-solving situations, a particular area – such as the cognitive – may play a more prominent role, this is less true of error analysis.

It is important to clarify the relationship between the concepts of control and discernment in Schoenfeld's model by presenting the subtle differences between them. While control is a "conscious metacognitive activity" that spans the entire problem-solving process and through which every decision is checked, discernment is much more of an attitude that is less an active seeker of cognitive impulses and more an admiring observer and evaluator of them in terms of their purpose.

The quantitative data showed positive changes in confidence, problem-solving skills, and teacher attitudes. Although none of these can be clearly assigned to the category of discernment, each plays a role in strengthening this area. The increase in prospective teachers' confidence naturally helps to initiate the process of error detection, which, based on their responses, was an important component of success.

The category "Approach to problem solving – combinatorial surplus" is essentially located in the cognitive domain, but in contrast to Schoenfeld's category "Resources", a more flexible and versatile approach belonging to the domain of metacognition also appeared here. The development of deeper, more structured, and more nuanced combinatorial thinking can be observed not only at the level of combinatorial knowledge, but also in several components of the category "Heuristics", which basically relates to the metacognitive area, for example, in the more conscious use of problem-solving strategies (even if they specifically relate to the area of error detection and analysis).

In addition to the close relationship between the categories of "Deeper understanding" and "Attitude towards problem solving – methodological surplus" – which are fundamentally located in the metacognitive domain – it is also worth focusing on their differences in comparison with the Schoenfeld model. While a deeper understanding of the mathematical structure behind problems involves the use and strengthening of mathematical knowledge and facts, i.e., relevant resources, the methodological surplus highlighted by the participants is more about self-reflection and thus points to the problem solver's inner convictions.

Participants emphasized that they had made significant progress in reflecting on their own problem-solving processes, which may have contributed to their increased self-confidence. The category of "Self-reflection and confidence" can therefore be classified as fundamentally affective, but it also contains a number of metacognitive components, and Schoenfeld's "Beliefs" elements appeared only rarely in the participants' responses. This also means that the participants' responses reflected only to a small extent on this category – this suggests a characteristic of the error analysis process, but may also indicate the incompleteness of our model. Determining this would require further experimentation with a larger sample size than the number of participants in the study.

Possible use of the model in teacher training

Below, we analyze in detail what development opportunities the research results suggest for mathematics teacher training. We also briefly reflect on the ideas, as we have already tested several of them in practice since the research was conducted.

Incorporating regular analysis of incorrectly worked sample solutions into the curriculum

Since 73% of students became more confident as a result of error analysis, and their test results also improved significantly (Paulovics et al., 2023), it seems worthwhile to incorporate weekly error analysis tasks into mathematics teacher training, where students specifically examine and correct incorrectly worked sample solutions. High attitude values show that prospective teachers welcome joint error analysis. To prevent the task from becoming monotonous, it is worth implementing the error analysis block in a variety of ways during the exercises: in addition to joint thinking guided by the instructor, students can also be encouraged to think independently or in small groups (2-3 people) and discuss the examples.

Simulation of error analysis in class

After proper preparation, we can also simulate error analysis with the prospective teachers: the "teacher" is given a task to think about briefly, and the "student" prepares not only the task but also an incorrect solution. The teacher then has to understand and correct the student's solution "live". In our experience, this is also highly instructive for fellow prospective teachers, as they, being outside observers, can observe details that would otherwise remain hidden from them in the excitement of participation. Of course, this experience can also be very valuable for active participants, especially if we take the time to analyze what has been said, and allow them to reflect on their thoughts and words.

Constructive error culture

Since no student indicated that frequent error analysis would reduce their self-confidence, it should be emphasized that discussing errors in training – and later in teaching – is not an accusation, but a learning opportunity. In our experience, many prospective teachers find it difficult to admit to their mistakes in front of the whole group, so it is not worth forcing everyone to respond to the comments of others at the beginning – once a constructive and safe atmosphere has been established, most will open up on their own. One of the cornerstones of a culture of constructive error is authenticity: as an instructor, one may occasionally make mistakes during the exercise – the way you react to your own mistakes will have the greatest impact on your students.

References

Adams, D. M., McLaren, B. M., Durkin, K., Mayer, R. E., Rittle-Johnson, B., Isotani, S., & van Velsen, M. (2014). Using erroneous examples to improve mathematics learning with a web-based tutoring system. *Computers in Human Behavior*, *36*, 401–411. https://doi.org/10.1016/j.chb.2014.03.053

Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*. Ablex Publishing.

Chi, M. T. H., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanations: How students study and use examples in learning to solve problems. *Cognitive Science*, *13*(2), 145–182. https://doi.org/10.1207/s15516709cog1302_1

Corbin, J. M., & Strauss, A. (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative Sociology*, *13*, 3–21.

Corbin, J., & Strauss, A. (2014). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (4th ed.). Sage Publications.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Sage.

Curry, L. A. (2004). The effects of self-explanations of correct and incorrect solutions on algebra problem-solving performance. In K. Forbus, D. Gentner, & T. Regier (Eds.), *Proceedings of the Annual Meeting of the Cognitive Science Society* (Vol. 26, p. 1548). Lawrence Erlbaum.

Eizenberg, M. M., & Zaslavsky, O. (2004). Students' verification strategies for combinatorial problems. *Mathematical Thinking and Learning*, *6*(1), 15–36.

Große, C. S., & Renkl, A. (2007). Finding and fixing errors in worked examples: Can this foster learning outcomes? *Learning and Instruction*, *17*(6), 612–634. https://doi.org/10.1016/j.learninstruc.2007.09.008

Guest, G., Bunce, A., & Johnson, L. (2006). How many interviews are enough? An experiment with data saturation and variability. *Field Methods*, *18*(1), 59–82.

Paulovics, Z., Csapodi, Cs., & Nagy, Z. L. (2023). The impact of incorrect solutions on the attitude and problem-solving abilities of prospective mathematics teachers towards combinatorics. In *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (No. 12). Alfréd Rényi Institute of Mathematics.

Paulovics, Z., & Csapodi, Cs. (2025). Towards understanding the error analysis thinking of prospective mathematics teachers. *International Electronic Journal of Mathematics Education*, *20*(4), Art. ID em0848. https://doi.org/10.29333/iejme/16678

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method.* Princeton University Press.

Radatz, H. (1980). Students' errors in the mathematical learning process: A survey. *For the Learning of Mathematics*, *1*(1), 16–20.

Renkl, A. (1997). Learning from worked-out examples: A study on individual differences. *Cognitive Science*, *21*(1), 1–29. https://doi.org/10.1207/s15516709cog2101_1

Rushton, S. J. (2018). Teaching and learning mathematics through error analysis. *Fields Mathematics Education Journal*, *3*, Art ID 4. https://doi.org/10.1186/s40928-018-0009-y

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Academic Press.

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, *196*(2), 1–38.

Siegler, R. S. (2002). Microgenetic studies of self-explanation. In N. Granott, & J. Parziale (Eds.), *Microdevelopment. Transition Processes in Development and Learning* (pp. 31–58). Cambridge University Press. http://dx.doi.org/10.1017/cbo9780511489709.002

Skemp, R. R. (1987). The Psychology of Learning Mathematics. Lawrence Erlbaum.

Strauss, A., & Corbin, J. M. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* Sage.

Taherdoost, H. (2022). Designing a questionnaire for a research paper: A comprehensive guide to design and develop an effective questionnaire. *Asian Journal of Managerial Science*, *11*(1), 8–16.

Törner, G. (2002). Mathematical beliefs – A search for a common ground: Some theoretical considerations on structuring beliefs, some research questions, and some phenomenological observations. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 73–94). Kluwer Academic Publishers.

Van Gog, T., Rummel, N., & Renkl, A. (2019). Learning how to solve problems by studying examples. In J. Dunlosky, & K. A. Rawson (Eds.), *The Cambridge* *handbook of cognition and education* (pp. 183–208). Cambridge University Press.

VanLehn, K. (1999). Rule-learning events in the acquisition of a complex skill: An evaluation of cascade. *Journal of the Learning Sciences*, *8*(1), 71–125. https://doi.org/10.1207/s15327809jls0801_3

Willig, C. (2013). *Introducing qualitative research in psychology* (3rd ed.). Open University Press.

ZOLTÁN PAULOVICS
EÖTVÖS LORÁND UNIVERSITY, HUNGARY
AND
ALFRÉD RÉNYI INSTITUTE OF MATHEMATICS, HUNGARY

CSABA CSAPODI
EÖTVÖS LORÁND UNIVERSITY, HUNGARY
AND
ALFRÉD RÉNYI INSTITUTE OF MATHEMATICS, HUNGARY

ZOLTÁN LÓRÁNT NAGY
EÖTVÖS LORÁND UNIVERSITY, HUNGARY

(Received October, 2025)