

# Should we draw, or should we work with numbers? Investigating proportional reasoning among 5th to 7th graders

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*Abstract.* Proportional reasoning is an essential component of our everyday life and our mathematics studies. The rate of development in this area varies between age groups. In order to find out the level of students in Grades 5–7, we developed an online test. We consider it important to emphasize and support the use of visual representations in this subject, and therefore the tasks of the test on the eDia (Csapó & Molnár, 2019) interface have three types of input and output. We distinguish between ratios represented visually in the form of discrete quantities, ratios represented visually in the form of continuous quantities and ratios represented by text or numbers. Our study aimed to explore the differences between task types. Results indicate a representation-dependent developmental shift: in Grades 5–6, students perform best on tasks involving visual discrete quantities, whereas in Grade 7, performance increases markedly on text-text tasks. This suggests that visual representations function as an early scaffold, while later instruction strengthens symbolic processing.

*Key words and phrases:* proportional reasoning, discrete and continuous quantities, visual representations.

*MSC Subject Classification:* Primary: 97C30; Secondary: 97D40, 97D60.

## Introduction

### The role and importance of visual representations

Visual representations are an important part of teaching mathematics: students are exposed to them from a very early age and learn concepts through



them. Visual representations play a crucial role in mathematics education from the beginning to the end of school. They help the development of problem-solving skills and the comprehension of abstract concepts. Research suggests that the use of visual representations leads to better understanding and better mathematical reasoning (Debrenti, 2015).

To increase the effectiveness of mathematics teaching, it is crucial to develop learners' ability to switch between different representations. Certain types of representations can be dominant in the teaching and learning of mathematics; however, the need to translate from one representation to another emerges often. The translation of representations is an important skill that students need to develop in order to become more proficient in learning mathematics. While the importance of visual representations in mathematics education has increased, further research is needed to explore different aspects of their implementation (Mainali, 2020).

A concept is mastered when the external representations become internal, when they are integrated into a so-called internal representational network (Ambrus, 2003). The external representation can be, for example, a number line, a graph, a picture, or a figure. An internal representation, on the other hand, is a mental image. Their interconnection is essential to the understanding of a concept.

The development of metacognitive knowledge elements can facilitate the linkage between external and internal representations (Csíkos et al., 2012).

In all areas of mathematics, the emergence and incorporation of visual representations are important, and thus, the role of visual representations can influence the possibilities for developing proportional reasoning. In our research, we investigated how proportional reasoning of 5th- to 7th-grade students is expressed when visual representations are involved. We sought to examine how the effectiveness of different representation types in proportional reasoning varies across grade levels. Rather than assuming that visual representations are universally beneficial, we investigate whether their role changes during development. In line with the title question, we do not treat drawing versus working with numbers as an either-or choice; instead, we examine how the relative effectiveness of visual and symbolic representations changes across Grades 5–7, and whether this change is related to students' computational fluency with rational numbers.

## The development of proportional reasoning

Proportional reasoning is a critical cognitive skill that is central to mathematical thinking and underpins advanced mathematical concepts (Ben-Chaim et al., 2012; McLaughlin, 2003). It involves comparing ratios and recognizing equations between them, and is considered a key indicator of formal reasoning (McLaughlin, 2003). Proportional reasoning is complex and pervasive in the mathematics curriculum, extending to other subjects such as science and history (Dole, 2010). Its development represents a significant milestone in students' cognitive development, and is essential for understanding a wide range of mathematical topics, including functions, graphs, and algebraic equations. Research suggests that engaging students in exploring and analysing diverse situations related to proportions can increase students' interest and understanding of the concept of proportionality (Dole, 2010). The development of proportional reasoning is a gradual process, supported by increasingly elaborated multiplicative thinking and the ability to compare two quantities in relative (multiplicative) rather than absolute (additive) terms (Lamon, 2005; Dole, 2010). The essence of proportional reasoning is the understanding of the multiplicative structures inherent in proportional situations (Harel et al., 1992; Dole 2010).

There is a lack of consensus in the research literature concerning several aspects of proportional reasoning, e.g., the age at which it develops and the different components or areas it might have.

Piaget's experiments led him to the conclusion that proportional reasoning can be acquired at the stage of formal thinking, around the age of 12 (Inhelder & Piaget, 1958). Lovell confirmed Piaget's findings in 1971. Piaget's experiment was later assessed as too difficult because of the knowledge of physics required for the experiments. They agreed that proportional reasoning was an indicator of formal thinking. Karplus tested more than 700 secondary school children on a task requiring them to change the length scale, use the concept of proportion, and construct an enlarged version of a figure using paperclips. It has been found that the development of proportional reasoning is not uniform across pupils, and they show different abilities in understanding proportions (Karplus & Peterson, 1970). In an experiment of Noelling (1980), children were asked to compare orange juice mixtures of different compositions. In the experiment, children were given tasks where two containers of orange juice had different concentrations, and had to judge how the taste changed when mixed. In the research, Noelling explored how children at different ages are able to understand and manage the concept of proportionality. They found that proportional reasoning develops gradually and

does not appear in children's thinking all at once. Whereas in earlier years, simple quantitative relations were recognised, conscious proportional reasoning develops around the age of 12 (Noelting, 1980).

While a complete understanding of proportional reasoning can be obtained by the age of 12, the development of proportional reasoning starts much earlier. For example, most preschool children can make a judgement about whether a caricature or a scale drawing looks right or wrong. Other common experiences, such as the realization that more money buys more candy, that objects appear different because of distance or distortion, that some everyday objects maintain certain scale relationships to one another, all play a role in the development of a sense of proportion (Lamon, 1995).

Research shows that proportional reasoning emerges in early childhood, with children as young as 5 showing rudimentary skills (Nippold & Sullivan, 1987). By the age of 7, children may be using part-whole relationships to compare proportions (Sophian & Wood, 1997).

Goswami (1989) has shown that children are able to think proportionally earlier than previous theories supposed. He pointed out that children's ability to think in proportions depends on the formulation of the task. In the experiment, pictures were shown of different missing value tasks. The majority of 6-7-year-olds successfully solved proportional analogy tasks for simple geometric shapes (Goswami, 1989). In a later study, Goswami conducted an experiment with simple proportions, using known foods, pizza (as a continuous quantity) and chocolate (as a discrete quantity) as the units of measurement. Preschool children recognised that half of a pizza was equivalent to half of a box of chocolate, but often also recognised that six-eighths of a pizza was equivalent to three-quarters of a box of chocolate. It remains questionable whether this performance would have been maintained if larger denominators had been used (Singer-Freeman & Goswami, 2001).

A study by Boyer et al. (2008) shows that children aged 6-9 years can name equal proportions, but accuracy decreases as the ratio increases. The use of different models and representations is common in early studies, which brings the investigation of different types of quantities to the foreground of research. Studies revealed that children show different developmental progress for discrete (countable by number of pieces) and continuous (measurable) quantities. Different results were found when examining the two types of quantity. We consider that it is important to review and consider the studies of these two types of quantities. The use of both types of quantities may be helpful when introducing ratios to

10-14-year-olds, and the results may also prove useful in other areas, including the teaching of fractions.

### Research results concerning discrete and continuous quantities

In research with toddlers, preschoolers, and primary-school-aged children, the two types of quantities have been found to be crucial, although there is no consistent result on which tasks children are more successful at, i.e., those concerning discrete quantities or those related to continuous quantities.

Several studies have shown that children process continuous quantities more efficiently than discrete quantities (Abreu-Mendoza et al., 2021; Begolli et al., 2020; Boyer & Levine, 2015; Boyer et al., 2008; Hurst & Cordes, 2018; Jeong et al., 2007). In fact, children seem to be effective with continuous visualization from a very early age, long before they are familiar with proportional symbols (Boyer et al., 2008; Jeong et al., 2007). Children's better performance may be related to the different strategies they use for different types of quantities (Jeong et al., 2007). When they need counting strategies, they make errors more easily, while for continuous quantities, they use more intuitive decision-making, magnitude estimation, and comparison. For continuous quantities, children's accuracy also tends to decrease with increasing scales, and until the age of 12, they have difficulty thinking discretely (Boyer et al., 2008). Performance in ratio tasks for continuous variables seems to be related to analogical reasoning skills (Hurst et al., 2018).

Other authors have found that problems with discrete quantities are easier to solve than problems with continuous quantities (Resnick & Singer, 1993, Vanluydt et al., 2020).

Among the studies, different types of tasks were characterized: the investigations differed in whether they included a missing value problem or a comparison problem. In a comparison problem, the goal is to select a ratio equal to a given ratio from the given options, whereas in a missing value problem, the goal is to produce a ratio equal to a ratio, three of the four data in the ratio pairs are known, and a fourth needs to be found. These task types also seem to influence whether children are more successful on discrete or continuous quantity tasks.

The other circumstance that may also be an influencing factor is the one-to-many or more-to-many relationships in each task. There was also a difference in each study as to whether children were asked to solve the tasks using manipulatives or on paper or computer.

The above-mentioned studies were conducted primarily with young children from kindergarten to fourth grade.

The aim of our research was to find out how the type of quantities (discrete or continuous) affect the ability to solve tasks among 10-14-year-old students. Based on the above-cited literature, it seems that the development of proportional reasoning skills starts around the age of 5, and we believe that many helpful steps could be taken before proportional reasoning is explicitly taught. For these possible aids, we wanted to explore pupils' attitudes to different types of quantity at this age range. For this purpose, we created an online test focusing on discrete and continuous quantities as well as on tasks given with numbers or text.

Our research questions:

- RQ1: Is the newly developed test sufficiently reliable across all grades?
- RQ2: How does students' performance differ across representation types (discrete visual, continuous visual, text/number) in Grades 5–7?
- RQ3: Do grade-level differences emerge in the performance gap between visual and symbolic representations?
- RQ4: Does the success of operations with rational numbers affect the success of proportional tasks?

## Method

### Participants

Participants of the experiment were pupils of Grades 5, 6 and 7 from three schools – two in Budapest and one in the countryside. The total number of participants was 245, out of which 83 pupils were in Grade 5; 81 in Grade 6; and 81 in Grade 7. The measurement was carried out at the end of the school year, when the 5th and 6th graders had not yet learned about ratios in maths lessons, as opposed to the 7th graders having learnt about them.

### Structure of the test, types of tasks

We used the eDia interface (Csapó & Molnár, 2019) to create our measurement tool and to conduct the measurement. We aimed to create a diverse set of tasks for the online diagnostic measurement, and we have based the structure of the tasks on the previous types used in the eDia interface (Csapó et al., 2015).

We conducted two measurements using an earlier version of the test: first a paper-based version, followed by an online version (Bereczki, 2023). After evaluating the results of the online test, we adjusted and added some introductory tasks on rational numbers. Previously, open-ended tasks were also included in the test, we converted them into closed tasks, and modified those tasks that could have been omitted in the analysis according to itemwise reliability or showed extreme results in the measurement.

The measurement tool was designed according to the different types of quantities, it includes three different types of tasks in terms of quantities: (1) discrete, (2) continuous, and (3) given by text or number. For discrete tasks, both originally discrete and discretised tasks were included. We considered the quantity in the question as an input factor and the quantity used in the response as an output factor. Accordingly, we worked with 9 different types of tasks (Figure 1). The instrument contains 4 versions of each type of task, for a total of 36 items. The types of each task are abbreviated below as shown in Table 1.

			INPUT /STIMULUS		
			VISUAL		TEXT OR NUMBER
			DISCRETE	CONTINUOUS	
OUTPUT / RESPONSE	VISUAL	DISCRETE	DD	CD	TD
		CONTINUOUS	DC	CC	TC
	TEXT OR NUMBER		DT	CT	TT

Table 1. Structure of the tasks of the measurement tool

We worked with different proportions within the different types of tasks. The ratios were classified into four different types, which were (a) 1:2 ratio, (b) 2:3 ratio, (c) 1: $x$  ratio where  $x > 2$ , and (d) both members of the ratio pair greater

than 2, e.g., 3:5. Both comparison problems and missing value problems were included.

Some examples of problems:

- (1) Observe the ratio of the areas of the LEGO pieces. Choose the piece whose area has the same ratio to the shown blue piece as the small yellow piece to the big one. Click on the button.



Figure 1. Task of type discrete-discrete (DD)

- (2) We ordered pizza with two toppings: broccoli and mushroom. The ratio between them was two parts with broccoli to four parts with mushroom. Which pizza did we choose? Click on the button.



Két féle feltéttel rendeltünk pizzát. Brokkolisat  és gombásat  .  
Két rész brokkolis és négy rész gombás volt az arány.  
Melyik pizzát választottuk?  
Kattints a megfelelő gombra!



Figure 2. Task of type continuous-continuous (CC)

- (3) Choose the picture in which the ratio of the red and the green gummy bears is 5:10. Click on the button under the right image.

Válaszd ki azt a képet, amelyiken a piros és zöld gumicukrok aránya 5:10.  
Kattints a megfelelő kép alatti gombra!



Figure 3. Task of type text-discrete (TD)

- (4) What is the ratio between the small and the large bar of chocolate? Click on the button.

Mennyi a kicsi és a nagy sportszelet hosszának aránya?  
Kattints a megfelelő gombra!



Figure 4. Task of type continuous-text (CT)

- (5) We prepare a jug of beverage, pouring syrup and water into the jug, by using smaller glasses. On the left, you can see how many glasses of syrup and water we poured into the jug. Choose the one out of the jugs on the right that best matches the beverage we created. Click on the appropriate button.

Egy kancsó szörpöt készítünk, pohárral belemérjük a kancsóba a szörpöt és a vizet.  
 A bal oldalon láthatod, hogy hány pohárral töltöttünk a kancsóba. A jobb oldali kancsók  
 közül válaszd ki azt az ábrát, amelyik legjobban szemlélteti az összeöntött  
 folyadékokat?  
 Kattints a megfelelő gombra!

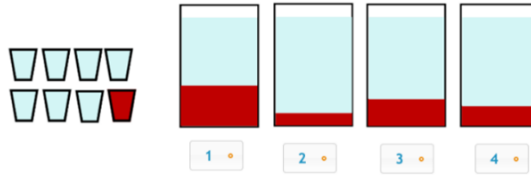


Figure 5. Task of type discrete-continuous (DC)

- (6) Grandma made a cake and covered it with chocolate. She used white chocolate and milk chocolate according to the ratio shown above. Which is grandma's cake? Click on the appropriate button.



Figure 6. Task of type discrete-continuous (DC)

### Measurement process

During the measurement, students had 45 minutes to complete the tasks. They were not allowed to use any aids (e.g., a ruler or calculator), and had to work independently.

The supervising teachers gave the pupils the necessary instructions but did not help them to complete the tasks. The pupils were familiar with similar types of measurement. For each task, they could click on the correct answer to give their solution.

## Results

In this section, we first report instrument reliability, then grade-level differences, followed by representation-type differences (discrete, continuous, text/number), and finally the relation between proportional reasoning and computational fluency.

Overall, the reliability of the entire test was found to be good across the whole sample, with a Cronbach's alpha of 0.857. There were no strong differences between grades. Cronbach's alpha was 0.860 for Grade 5 ( $N = 83$ ); 0.812 for Grade 6 ( $N = 81$ ); and 0.850 for Grade 7 ( $N = 81$ ).

The subtests were also tested separately for reliability. For the calculation skills test on rational numbers, the Cronbach's alpha was 0.808 for the 5th graders, 0.764 for the 6th graders, and 0.793 for the 7th graders, and 0.801 for the total sample. For the proportion tasks, Cronbach's alpha was 0.728 for 5th graders, 0.725 for 6th graders, and 0.781 for 7th graders.

	5 <sup>th</sup> grade		6 <sup>th</sup> grade		7 <sup>th</sup> grade		All students	
	$N = 83$		$N = 81$		$N = 81$		$N = 245$	
	MEAN	S.D.	MEAN	S.D.	MEAN	S.D.	MEAN	S.D.
Entire test	0,59	0,14	0,64	0,12	0,70	0,13	0,64	0,14
Calculation skills	0,76	0,17	0,82	0,14	0,85	0,14	0,81	0,15
Proportion tasks	0,47	0,15	0,50	0,14	0,59	0,15	0,52	0,15

*Table 2.* Mean and standard deviation of students' scores on the test

When comparing the grades pairwise, we found that their overall test scores differed significantly by grade.

In the calculation skill test, there is a significant difference between Grades 5 and 6, and between Grades 5 and 7 (based on the results of the Levene test for homogeneity,  $p < .001$  Dunnette T3 post-hoc comparison gives  $p = .049$  and  $p < .001$ ), but no difference between Grades 6 and 7 (based on the results of the Levene test for homogeneity,  $p < .001$  Dunnette T3 post-hoc comparison gives  $p = 0.393$ ).

In the proportion tasks part of the test, there is no significant difference between grades 5 and 6 (based on the results of the Levene test for homogeneity,  $p < .001$  Dunnette T3 post-hoc comparison gives  $p = .382$ ), but both grades (Grades 5 and 6) performed significantly lower than Grade 7 (based on the results of the Levene test for homogeneity,  $p < .001$  Dunnette T3 post-hoc comparison

gives  $p < .001$  and  $p = .001$ ). The distribution of results by task type is shown in Table 3 and Figure 7.

As shown in Table 2, mean scores increase across grades both for the entire test and for the proportional reasoning subtest. While Grade 6 already shows higher averages than Grade 5 in calculation skills, the most pronounced increase in proportional reasoning performance appears between Grades 6 and 7. The standard deviations are comparable across grades, suggesting similar variability within cohorts.

Table 3 presents the mean performance for each input-output representation type, providing an overview of how students performed across the different visual and symbolic task formats.

		INPUT (Stimulus)					
		Mean	S.D.	Mean	S.D.	Mean	S.D.
		DISCRETE		CONTINUOUS		TEXT FIGURE	
OUTPUT (Response)	DISCRETE	62,50%	30,33%	58,00%	27,23%	54,00%	27,20%
	CONTINUOUS	62,25%	24,13%	48,50%	25,05%	48,25%	22,80%
	TEXT/FIGURE	43,50%	25,58%	40,75%	27,20%	51,00%	32,40%

Table 3. Average and standard deviation of students' scores on each type of task

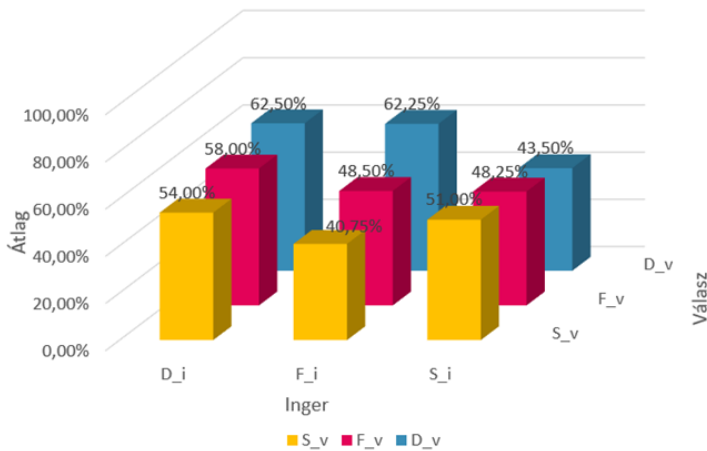


Figure 7. Students' performance on each type of task

To examine the differences between task types, repeated measures analysis of variance was applied. Mauchly's Test of Sphericity indicated that the assumption of sphericity was violated ( $p < .001$ ), therefore the Huynh-Feldt correction was applied. The analysis showed a significant difference between the task variables ( $F(7.764; 24.448) = 26.875, p < 0.001$ ).

Pairwise comparisons revealed systematic differences between representation types. Overall, tasks involving discrete visual representations tended to yield higher performance than tasks requiring symbolic (text or numerical) responses. Detailed pairwise comparisons are reported in Table 4.

	DD	DC	DT	CD	CC	CT	TD	TC
DC								
DT	DD>DT							
CD			DT>CD					
CC	DD>CC	DC>CC	DT>CC	CD>CC				
CT	DD>CT	DC>CT	DT>CT	CD>CT	CC>CT			
TD	DD>TD	DC>TD	DT>TD	CD>TD				
TC	DD>TC	DC>TC		CD>TC		CT>TC		
TT	DD>TT	DC>TT		CD>TT		CT>TT	TD>TT	

Table 4. Significant differences between the means of the variables ( $p < .05$ )

A secondary analysis examined the effect of ratio scale. Students performed significantly worse on ratios of the form 1:x (e.g., 1:3, 1:4, 1:7) than on the other ratio types ( $p < .001$ , Cohen's  $d = .38 - .49$ ).

To explore the relationship between proportional reasoning and students' computational fluency with rational numbers, correlations were calculated between the calculation skills test and the different representation-based task variables.

We examined the correlation between each quantity type and the numeracy test (which involved operations with rational numbers). The scores on the calculation skills test were grouped into four groups along quartiles, and the scores of the four groups were examined separately for each task type. Table 5 shows the results. Correlation values indicating a significant relationship between numeracy and each task variable are highlighted in grey ( $p < .05$ ). There appears to be no significant correlation between task variables and the calculation skills test for students performing in the two middle quartiles, while for students performing poorly or very well on the calculation skills test, there is a significant correlation

when tasks are given in discrete quantities or text/numbers, and no significant correlation when the task input is a continuous quantity.

	Proportional reasoning sum	DD	DC	DT	CD	CC	CT	TD	TC	TT
DD	,676**									
DC	,642**	,386**								
DT	,611**	,327**	,247**							
CD	,456**	,288**	,191**	,245**						
CC	,458**	,283**	,261**	,260**	0,084					
CT	,410**	,130*	,149*	,212**	,188**	0,120				
TD	,613**	,334**	,297**	,325**	,135*	0,089	,209**			
TC	,503**	,245**	,283**	,148*	0,082	0,120	0,039	,248**		
TT	,730**	,378**	,435**	,362**	,210**	,175**	,204**	,487**	,357**	
Calc.Skill_1quart.	-,426**	-,329**	-,321**	-,301**	-0,098	-,172**	-,132*	-,298**	-,158*	-,336**
C.S. 2quart.	-0,123	-,153*	-0,063	-0,035	-0,118	0,051	0,060	-,127*	-0,089	-0,122
C.S. 3quart.	0,076	0,078	0,067	-0,029	0,003	-0,037	0,060	,172**	-0,027	0,092
C.S. 4quart.	,443**	,377**	,299**	,334**	,193**	,144*	0,020	,249**	,247**	,346**

Table 5. Correlations between variables and the numeracy test ( $p < .05$ )

Comparisons were made by grade on the results achieved on each task variable. By comparing the performance of students at different grades on each quantity type, we found that there was a significant difference between Grades 5 and 6 for text-discrete ( $p = .02$ ) and text-text ( $p = .03$ ) task types. Grades 6 and 7 show a significant difference for several variables, for discrete-text ( $p = .02$ ), continuous-discrete ( $p = .04$ ), continuous-continuous ( $p = .005$ ), continuous-text ( $p = .002$ ) and text-text ( $p < .001$ ) task types. There is no significant difference between Year 5 and Year 7 performance only for the continuous-continuous and text-continuous types, and for all other types. In Figure 8, significant differences are shown by arrows.

The performance of Grades 5 and 6 appears to be close, with Grade 7 differing more significantly from the two lower grades. The largest difference in averages is for the text-to-text tasks, the only task variable where all three grades differ significantly. On this task type, the performance of the seventh graders shows a significant outperformance compared to the other grades. The mean (and standard deviation) of the grades on each task variable is shown in Table 6.

Table 6 shows the mean performance for each representation type by grade. Overall, the performance of Grades 5 and 6 appears relatively similar, whereas Grade 7 shows higher averages across most representation types, particularly for tasks requiring symbolic (text-text) responses.

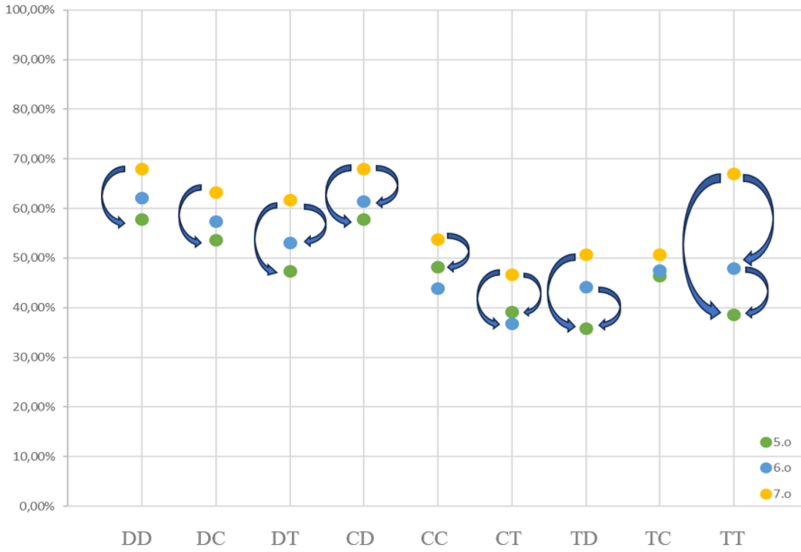


Figure 8. Significant differences between grades in the individual task variables

	DD	DC	DT	CD	CC	CT	TD	TC	TT
5.o.	57,83% (31,22%)	53,61% (28,46%)	47,29% (25,61%)	57,83% (25,58%)	48,19% (26,13%)	39,16% (24,13%)	35,84% (22,49%)	46,39% (26,23%)	38,55% (29,55%)
6.o.	62,04% (29,61%)	57,41% (26,05%)	53,09% (28,88%)	61,42% (23,40%)	43,83% (24,54%)	36,73% (18,15%)	44,14% (27,18%)	47,53% (25,50%)	47,84% (31,67%)
7.o.	67,90% (29,64%)	63,27% (26,54%)	61,73% (25,34%)	67,90% (22,44%)	53,70% (23,75%)	46,60% (24,61%)	50,62% (24,99%)	50,62% (29,84%)	66,98% (29,53%)

Table 6. Means (standard deviations) of each grade on each task variable

In addition to the differences between grades, we also examined how the performance of each task variable develops within each grade, between which variables there is a significant difference within a grade. The comparisons of these patterns provide a further opportunity by observing the structure of the performance of the individual grades, and further examining the possible rearrangements and the differences between them.

To examine the differences between representation types within Grade 5, repeated-measures ANOVA was conducted. The analysis indicated significant differences among task types ( $F(8; 8.819) = 9.835, p < .001$ ). Pairwise comparisons showed that tasks involving discrete visual representations (DD, DC and

CD) yielded higher performance than tasks requiring symbolic responses (CT, TD and TT). Detailed pairwise comparisons are presented in Table 7.

Within each grade, repeated-measures ANOVA indicated significant differences among representation types. The detailed pairwise comparisons are summarized in the following tables.

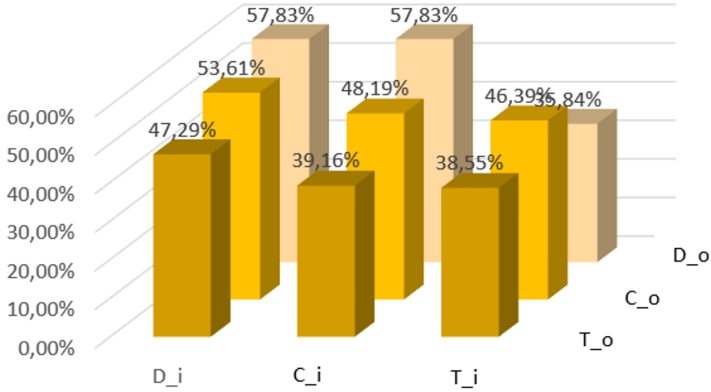


Figure 9. Results of 5th graders on each task variable

5.	DD	DC	DT	CD	CC	CT	TD	TC
DC								
DT								
CD								
CC								
CT	DD>CT	DC>CT		CD>CT				
TD	DD>TD	DC>TD		CD>TD				
TC								
TT	DD>TT	DC>TT		CD>TT				

Table 7. Significant differences between the means of the variables ( $p < .05$ ) in Grade 5

To examine the differences between representation types within Grade 6, repeated-measures ANOVA was conducted. The analysis indicated significant differences among task types ( $F(7.309, 10.590) = 10.921, p < .001$ ). The mean of the

discrete-discrete variable is significantly higher than the continuous-continuous, continuous-text, text-discrete, text-continuous, text-to-text variables. The average of the discrete-continuous variable is significantly higher than the averages of the continuous-continuous, continuous-text and text-discrete variables. The averages achieved on discrete-text tasks are higher only in the case of continuous-text tasks, and do not differ significantly from other task types. The averages of the continuous-discrete variables are significantly better than four variables: continuous-continuous, continuous-text, text-discrete, and text-continuous. Detailed pairwise comparisons are presented in Table 8.

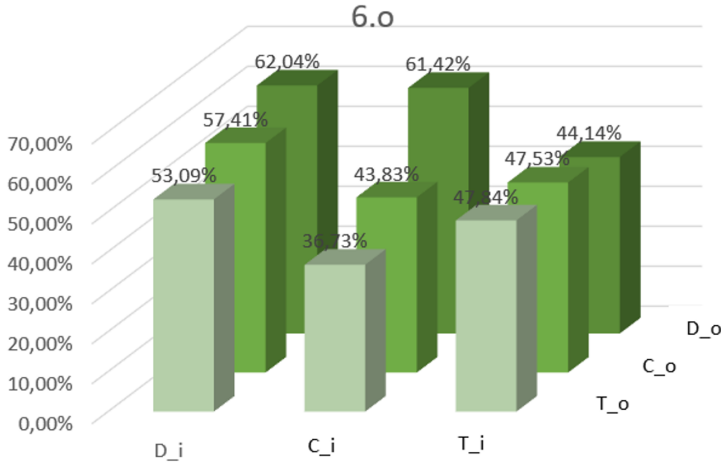


Figure 10. Results of 6th graders on each task variable

6.	DD	DC	DT	CD	CC	CT	TD	TC
DC								
DT								
CD								
CC	DD>CC	DC>CC		CD>CC				
CT	DD>CT	DC>CT	DT>CT	CD>CT				
TD	DD>TD	DC>TD		CD>TD				
TC	DD>TC			CD>TC				
TT	DD>TT							

Table 8. Significant differences between the means of the variables ( $p < .05$ ) in Grade 6

To examine the differences between representation types within Grade 7, repeated-measures ANOVA was conducted. The analysis indicated significant differences among task types ( $F(8, 9.229) = 11.173, p < .001$ ). Pairwise comparisons showed that tasks involving discrete visual representations (DD and CD) yielded higher performance than several task types requiring continuous or symbolic responses (such as CC, CT, TD and TC). Compared to the previous classes, we found that in Grade 7 the text-to-text shows a variable high average, significantly higher than the continuous-continuous, continuous-text, text-discrete or text-continuous quantities. Detailed pairwise comparisons are presented in Table 9.

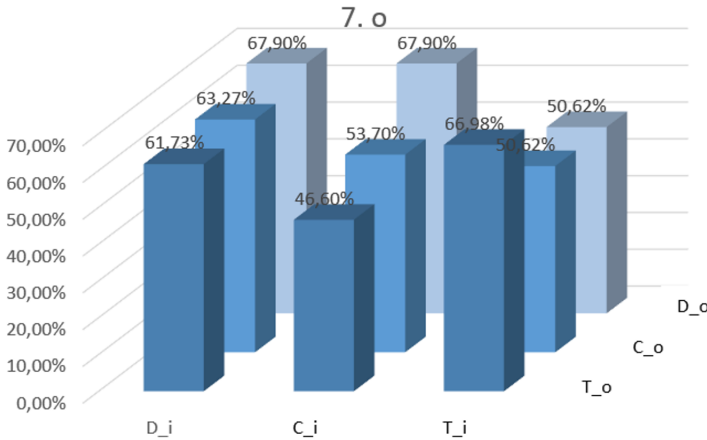


Figure 11. Results of 7th graders on each task variable

6.	DD	DC	DT	CD	CC	CT	TD	TC
DC								
DT								
CD								
CC	DD>CC	DC>CC		CD>CC				
CT	DD>CT	DC>CT	DT>CT	CD>CT				
TD	DD>TD	DC>TD		CD>TD				
TC	DD>TC			CD>TC				
TT	DD>TT							

Table 9. Significant differences between the means of the variables ( $p < .05$ ) in Grade 7

Comparing the three grades, it seems that compared to the more even performance of the fifth grade, the knowledge level of the sixth grade shows a rearrangement on certain variables, and in the results of the seventh graders, text-to-text tasks play a prominent role. We compared the correlations between the different types of quantities in all three grades to see which variables have a significant relationship between the pairs of variables in all three grades.

To compare the significant correlations between the different types of quantities by grades, we used Fisher's z-test. The z-test showed a significant difference between grades in one case, in the case of the TD-TT quantity type, between Grades 5 and 6 ( $p = .048$ ) and 5 and 7 ( $p = .003$ ). There was no difference between the correlations calculated on this variable between Grades 6 and 7. The relationship between the two types of tasks seems to show a development trend with the growth of the grade. The difference is not significant, but in the case of all three grades, there is a correlation between DD-DT, DD-TD, DD-TT, DC-TC, DC-TT correlation pairs. The correlations between each type of quantity are shown in Table 10.

The correlations between representation types differed somewhat across grades. Fewer significant relationships were observed in Grades 5 and 6, whereas Grade 7 showed a larger number of significant correlations between task types. This pattern suggests that the structure of students' proportional reasoning becomes more interconnected in the higher grade.

This result suggests that the knowledge structure of students is rearranged around Grade 6, and their knowledge becomes more compact by Grade 7. The continuous-continuous quantity does not show a correlation with any other type of quantity in the 6th grade; it occupies a relatively isolated place in the knowledge structure.

Overall, the results reveal clear differences across representation types and grade levels. Students in Grades 5 and 6 tended to perform better on tasks involving discrete visual representations, whereas symbolic formats were more challenging. In Grade 7, performance on text-text tasks increased substantially compared to the lower grades, indicating a change in the relative success of visual and symbolic representations.

		5.	6.	7.
DD	DC	,448**	0,187	,469**
DD	DT	,294**	,376**	,247*
DD	CD	,342**	,273*	0,182
DD	CC	,363**	0,114	,337**
DD	CT	0,053	,257*	0,074
DD	TD	,323**	,273*	,354**
DD	TC	,240*	0,112	,350**
DD	TT	,404**	,328**	,336**
DC	DT	0,139	,364**	0,161
DC	CD	0,201	0,167	0,134
DC	CC	,429**	0,060	,231*
DC	CT	0,147	-0,021	,225*
DC	TD	,283**	0,206	,341**
DC	TC	,222*	,357**	,266*
DC	TT	,385**	,455**	,407**
DT	CD	,324**	0,132	0,189
DT	CC	,255*	0,182	,316**
DT	CT	0,162	,273*	0,152
DT	TD	0,091	,431**	,297**
DT	TC	-0,015	0,212	0,207
DT	TT	0,059	,477**	,367**
CD	CC	0,147	0,002	0,035
CD	CT	0,213	0,177	0,111
CD	TD	,235*	-0,065	0,147
CD	TC	0,054	0,022	0,135
CD	TT	0,201	0,107	0,172
CC	CT	0,162	0,059	0,049
CC	TD	-0,005	0,074	0,154
CC	TC	-0,043	0,050	,327**
CC	TT	,239*	-0,027	,232*
CT	TD	0,107	0,078	,359**
CT	TC	0,070	-0,038	0,035
CT	TT	0,091	-0,050	,403**
TD	TC	0,209	0,216	,293**
TD	TT	,246*	,475**	,600**
TC	TT	0,123	,496**	,449**

Table 10. Correlations between pairs of variables, \* and \*\* significant differences ( $p < .05$ )

## Discussion

In this study, we presented the results of an online survey among 5th, 6th and 7th graders. The aim of our study was to develop a measurement tool that can be used to map the characteristics of the proportional reasoning of the upper school age group, and to examine several types of tasks across several grades. We examined how the grades and the types of quantity representation in the tasks are related, and what role the types of representation play in solving the tasks.

The results of this study revealed clear differences in students' proportional reasoning across representation types and grade levels. Students in Grades 5 and 6 performed more successfully on tasks involving discrete visual representations, while symbolic representations were more challenging. In Grade 7, however, performance on text-text tasks increased substantially compared to the lower grades, indicating that older students were more able to solve proportional problems presented in symbolic form.

In previous research, visual representations were used to examine primarily younger age groups and tasks containing only discrete and continuous type representations. In our research, in addition to the use of these variables, the types of representations given by text or numerical data were also included. We wanted to get an overview of how successfully students solve different tasks, so that we can use them to support better understanding and the development of proportional reasoning in the future.

The measuring device is a test with good reliability, we found differences between grades and overall, in terms of individual task characteristics. On the numeracy test, the performance of fifth graders is lower than that of 6th and 7th graders, there is no difference between the latter two grades. On the proportionality test, there is no significant difference between Grades 5 and 6, while the performance of Grade 7 is significantly higher than that of the other two grades. We also found differences between the types of representation, especially those types of representations that contain discrete quantities.

The differences in the performance of the different grades can be explained by the structure of the curriculum in the case of both the numeracy test and the proportionality test. The development of the concept of numbers in rational numbers is still very rudimentary in Grade 5, the deepening of the topic is given a greater role in the sixth grade, so it may be that the results of the fifth grade differ significantly from those of the higher grades. The teaching of proportions

and proportional division in Grade 7 is in the curriculum, so this may explain the better performance of Grade 7 on the proportionality test compared to lower grades.

Previous studies have reported differences between tasks involving discrete and continuous quantities. However, their results are not fully consistent. In one study (Boyer et al., 2008), children performed better on tasks involving continuous quantities, whereas in another study (Vanluydt et al., 2020), better performance was found for discrete quantities. One possible explanation for this difference may be the type of proportional problem used in the tasks (comparison vs. complement problems). These findings suggest that not only the type of quantity representation but also the structure of the task may influence students' performance.

In our study, the results show a clearer pattern. Students in this age group performed significantly better on tasks involving discrete visual representations (DD), while tasks requiring textual or numerical responses were generally more challenging. Unlike the studies mentioned above, our instrument also included tasks based on textual and numerical information, which allowed us to compare visual and symbolic representations within the same framework. This broader set of task types provides a more comprehensive picture of how different representations influence proportional reasoning.

Our results therefore suggest that, for this age group, discrete visual representations provide a particularly strong support for solving proportional reasoning tasks. This may be related to the fact that discrete representations allow students to rely on exact visual structures and numerical correspondences. At the same time, as students' numerical skills develop, they increasingly attempt to apply strategies developed for discrete quantities to other types of representations as well.

Previous research has also indicated that students' performance on visual representations is related to their performance on tasks presented in numerical or textual form (Matthews et al., 2016), and that discrete or discretized representations may show a stronger relationship with symbolic tasks than continuous quantities (Begolli et al., 2020). Our findings are consistent with this interpretation, as students with stronger numeracy skills tended to perform better across several representation types. A key contribution of this study is that it examines proportional reasoning tasks involving visual, textual and numerical representations within the same framework.

Examining the correlation of numeracy skills with other task variables, we can see that having good numeracy skills is an advantage, and the lack of them is a disadvantage for all task variables, unless the tasks are with continuous quantities.

Examining the differences between the different grades, we can see that in the case of 5th graders, the performances achieved on the DD, DC and CD task types stand out from the rest. There is no difference between these types in this age group, but all three types have a higher problem solving compared to the continuous-text, text-discrete and text-to-text variables. This is because their previous experience is mainly limited to these quantities, and tasks given with text/ data based on numbers can be difficult for them. When comparing Grades 5 and 6, we did not find much difference between the two grades, we only experienced significantly higher performance in Grade 6 on two task types, the text-discrete and the text-to-text task type. Although the average of the tasks containing continuous quantity representations (CC) is higher for fifth graders than for sixth graders, their performance does not differ significantly.

There is a rearrangement in the performance of 6th graders: they get closer to rational numbers at this age, and their numeracy skills continue to strengthen, and the transformation of the knowledge structure at this age is also because they already encounter proportional tasks in this grade. The knowledge structure becomes more compact by the 7th grade. The 7th grade age is the limit of the appearance of the formal level of operations, at this age they are already able to perform operations with abstract concepts.

In Grade 7, students mainly encounter tasks given with text and data based on numbers, so it is not surprising that at this age, unlike before, the performance of students in these types of tasks is strengthened, so we can say that we measured outstanding performance on text-to-text tasks compared to before, far ahead of continuous quantity representation tasks. At this age, there is no longer a significant difference between performance on DD variables and performance on text-to-text variables, as we have previously experienced in all grades. Text-to-text tasks are the only type of tasks where there is a significant difference between all three classes, this is the type of variable that shows the greatest development at this age.

In summary, we see that the performance on proportional tasks in this age group depends on the type of variable of the input and output. In the case of visual-type tasks, especially in the case of discrete quantities, the younger age group (5th and 6th) is more successful than in the case of textual and numerical tasks. Later, as a result of school teaching and the development of students'

thinking, the distribution of performance according to the types of tasks changes. We believe that the development and reinforcement of the experience gained on visual tasks and the concept of proportion can help the later problem solving on the text or numerical tasks, but the steps of the developmental transition need to be confirmed by further research. The visual experiences of the students can help to bring out their existing knowledge, easier understanding and deeper knowledge towards abstract quantities.

According to Möhring et al. (2016), children's perceptions of proportions based on the visualization of proportions correlated with their general fraction knowledge. Thus, our assumption is that if students encounter a lot of tasks that require proportional thinking, which they receive in a visual form, it can help not only to understand their calculations on proportions, but also to better develop the concept of fractions. Further experiments are needed to support this.

During teaching, we recommend that the teaching of proportional reasoning be based on visual representations. Based primarily on the students' existing experiences, the experiences and analogies gained on discrete and continuous quantities can help the development and strengthening of the concept of ratio. We recommend linking these to textual or numerical problems and gradually transitioning to text-to-text tasks, while maintaining the use of representations as an aid in solving these types of tasks. We recommend that visual representations come to the fore in all three grades.

Overall, our results suggest that visual representations play a particularly important role in the earlier stages of learning proportional reasoning, while symbolic representations become increasingly effective as students' mathematical thinking develops. Thus, the question of whether students should draw or work with numbers may not represent an either-or choice: visual representations appear to support the early development of proportional reasoning, while symbolic representations become increasingly effective at later stages.

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