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Teaching Mathematics and Computer Science

Game Theory for Managers and Mechanical Manager Students

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Abstract. In this article we describe the second part of a case study, in which 48 Mechanical Management students were involved. The participants of the case study were MSc level students at Szent István University, Gödöllő.

In the case study we looked for methods by which we can support the most important components of competence motivation and the development of mathematical and other key competences during the mathematics lessons and individual learning.

Another goal of our research was to get reliable information about students learning methods and their awareness of self-efficiency, furthermore their achievement in the subject of Engineering and Economic Mathematics.

Detailed assistance was provided for the students in the e-learning portal. Knowledge tests, questionnaire and personal interviews with the students were also used.

During the semester four topics have been discussed: linear programming, graph theory, game theory and differential equations. In this article I will describe the lesson preparations, the help for examinations and the students' achievement on game theory.

Key words and phrases: lesson preparation for Engineering and Economic Mathematics on game theory, matrix game, bimatrix game, prisoner's dilemma.

ZDM Subject Classification: D45, K95, U35.

Introduction

In 2015 we made a case study with Mechanical Engineering Management MSc level students at Szent István University, Gödöllő. The 48 participants of the case study attended the Engineering and Economic Mathematics subject taught by the author. This subject is located in the first semester.

The students were involved in the case study during the whole semester. They had four learning units in mathematics, on four Friday mornings. Each unit took 5 hours, and had one of the next topics: linear programming, graph theory, game theory and differential equations. The content of the subject was mainly reinterpreted and structured regarding to the didactical aspects described in the previous article [4]. The most important changes were made by the author in game theory.

In chapter *"The importance of game theory from the didactic point of view"* we describe our arguments why to teach this difficult and abstract topic. In chapter *"Selection and arrangement of the mathematical content for the subject"* we describe the structure and content of the lessons. In chapter *"Some tools of learning support"* we demonstrate the tools with which we have helped the students to prepare. The most important of these tools is the practice worksheet set out in section *"The practice worksheet for game theory"* In chapter *"Evaluation of the Learning Unit, the written exam"* we analyze the students' achievement on game theory and the realization of our own objectives.

The importance of game theory from the didactic point of view

Bruner ([2]) introduced fundamental ideas as a compass for curriculum development. Our students learn not „a mathematical theory", they recognize only some examples during a couple of hours. „In order for a person to be able to recognize the applicability or inapplicability of an idea to a new situation and to broaden his learning thereby, he must have clearly in mind the general nature of the phenomenon with which he is dealing. The more fundamental or basic is the idea he has learned, almost by definition, the greater will be its breadth of applicability to new problems. Indeed, this is almost a tautology, for what is meant by „fundamental" in this sense is precisely that an idea has wide as well as powerful applicability." ([2], p.18) In sense of Schweiger ([10], p. 68) the fundamental ideas of mathematics are:

– Time dimension (recur in the historical development of mathematics).
– Horizontal dimension (recur in different areas of mathematics).
– Vertical dimension (recur at different levels).
– Human dimension (anchored in everyday activities).

Game theory is considered a theme that can be used to meet almost every fundamental idea. The arousal of interest of game theory may be served by talking about Hungarian aspects and international successes. John von Neumann laid the foundations of game theory ([7]), and as an independent theory it was born in 1944, when John von Neumann and Oskar Morgenstern wrote and released their book ([8]). John Harsányi worked out his key ideas from 1967 to 1968, later received the Economic Nobel Prize mainly because of them. Harsányi was the first who distinguished in its present form the cooperative and non-cooperative games. He created the game theory based on imperfect information, in which the competing player knows in a limited way his opponents purposes and available strategic assets. Since then this theory has changed the economic and political fundamentals of thinking, starting from the international disarmament negotiations through the Soviet-American nuclear deterrence through the auctions of the licenses of oil fields by the government until the allocation of radio and television frequencies. Game theory can be used to model emerging problems in many areas of life. Not only in games, but also in many of the decision-making situations where we did not even think to do it with mathematics and game theory.

- For example, if a good position job vacancy is expected a year from now and two capable colleagues compete for it, there is the game theory to model the situation. The potential strategies of the ,,players" are learning language, the development of computer literacy, enrollment in a driving course or getting a master's degree. Which one should they choose depends on the size of the expected profit for each possible strategy pairs.
- When a small town has only shops belonging to two department stores, and there are no expected customers from other settlements, then any of the following strategies could be able to attract customers: establishment of free parking, free recycled bags, longer opening hours or issuing loyalty card discounts. Which one should the shops choose, depends on the customers' interest in case of the shops possible strategy pair.
- If two political parties compete for the votes in a country, then a strategic objective can be to develop education, to improve the state of health, to increase pension or to support small businesses. Which one should the parties choose, in the event of their possible strategy pairs will depend on the interest of voters.

The great advantage of game theory is that number of problems can be solved by common reflection and knowledge of the four basic operations (which causes no problem to anybody). So everyone has equal and 100 % chance to start on their journey of discovery. The problem to be solved is usually not mathematical, but it comes from everyday life and the curiosity for the result is a large driving force. This differs significantly from the most branches of mathematics, where it is often not easy to arouse the audience's interest towards the given problem. In terms of methodology a big advantage of the game theory is to solve problems arising from entirely different part of life leads to the same mathematical model. Thus, the need and benefits of modeling can be seen. The aim of teaching is much more than just to teach students to solve a particular problem in the known method. The real benefit is when the real problems of their own lives are solved with the help of the learned method.

Selection and arrangement of the mathematical content for the subject

The unit consisted of five 45 minutes lessons for students to get familiar with the essence of game theory through applications.

Our aim was to expand the scope of concepts, procedures and applications gradually, and in parallel increase the complexity of the strategies to be applied:
– Two-person zero-sum matrix game – first and second lesson;
– Two-person bimatrix game – third and fourth lesson;
– Applied game theory – fifth lesson.

First lesson

The target was to get the students acquainted with the two-person zero-sum games based on many specific tasks and they are able to recognize the diverse application possibilities. The aim of the selection of examples has been to review the knowledge about matrices. The situation will be a real game and help to guide the students in the direction of abstract ideas. A suitable opening exercise is for example the puzzle card game (Task 1). The second task requires the opposite direction; there the matrix must be filled based on the text, so the text must be interpreted and the meaning of the matrix values as well. After that follows the task, where only one optimal strategy pair exists and the first player's gain is not zero, than another in which the game is fair, than a third one, where more than one pure saddle points exist. Important to point out at least in one task that if one player has unilaterally deviated from the equilibrium strategy, then neither of any other strategy would be better. This is the case where the students can detect why the saddle-point strategy is worth pursuing. It is useful to illustrate the interchangeability and equivalence properties if the matrix has more than one saddle points. That can be generalized.

Task 1: There are two players and each of them has 4-4 aces in the Hungarian (Tell) cards. At the same time they put 1–1 aces on the table. According to the Table 1 they pay to each other. Which player should expose which card on the table if he wants to achieve maximum profit (which strategy should he choose)?

	Red	Nut	Pumpkin	Green
Red	-3	-1	4	1
Nut	3	-2	0	1
Pumpkin	2	-1	2	3
Green	1	-4	-2	6

Table 1. The payoff matrix of the game

Each pair of strategy is characterized by a row and a column. The number at the junction of the appropriate row and column of the table expresses the first player's gain in case of the selection of the given strategy pair (Table 1). It is necessary and useful to talk about the concept of ,,zero-sum game". This means that if one wins, the other loses the same amount, so the combined profits will be zero. Therefore, the second player's gain can be read from the table, it is the minus one times of the first player's gain.

			second player			
			1	2	3	4
			Red	Nut	Pumpkin	Green
	1	Red	-3	-1	4	1
first	2	Nut	3	-2	0	1
player	3	Pumpkin	2	-1	2	3
	4	Green	1	-4	-2	6

Table 2. Extended payoff matrix

Table 2 is the extended version of the same payoff matrix. It is advised to number the players' strategies and since this is the first task on game theory it is useful to mark which the first and the second player is. The concept of ,,pure strategy" means that it is clear for the players which strategy is worth choosing. Other conditions are:

– players are selfish and rational, they only want to increase their own profits
– they possess all the information regarding profits for both parties
– they have to choose their own strategy at the same time, without worrying about which strategy the other chooses.

They can find the most preferred strategy to minimize the potential loss. That means that the first player has to find the maximum of the row-minimums and the second one has to find the minimum of the column-maximums. If the maximum of the row-minimums and the minimum of the column-maximums are equal then it is the ,,value of the game". The strategy pair which results this value is called optimal.

> ***Task 2:*** Two players have fire-red (f) and pitch-black (p) cards. The first player gets a pitch-black one, two, three and a fire-red five; the second gets a fire-red one, three, five and a pitch-black five. They show one of their cards at a time. If they show matching color, then the first wins the difference of the numbers on the two cards. If they show ones in different, then the second wins the same amount. Is there an optimum strategy, saddle point and what is the value of the game?

Knowing the rules of the game the extended payoff matrix can be created. According to the procedure used in the first task, we read the solution from the matrix (Figure 1).

The maximum of the row-minimums and the minimum of the column-maximums are equal:

$$\max\{-4,-3,-2,0\} = \min\{4,2,0,4\} = 0,$$

the game is optimal for the strategy pair (4;3).

			II				
			1	2	3	4	
			f1	f3	f5	p5	min
I	1	p1	0	−2	−4	4	−4
	2	p2	−1	−1	−3	3	−3
	3	p3	−2	0	−2	2	−2
	4	f5	4	2	***0***	0	**0**
		max	4	2	**0**	4	**0**

Figure 1. The extended payoff matrix

Second lesson

Starting with a simple and symmetrical game, for example, the two-finger Morra we can discover that it has no pure strategy solution. After that we are searching for general solution for any two by two matrix game. If the probability of the player's first strategy are chosen as variables the required matrix inequality or a system of inequalities can be written with these variables. We can show for example by using Geogebra software that the optimal solution to the inequalities is the same then the optimal solution of equations. We can solve it in case of two by two matrices even parametrically. For larger matrix sizes we get a linear programming problem. Finally we are talking about the fundamental theorem of game theory which states that any matrix game has a solution using pure or mixed strategy.

Task 3: The two-finger Morra game Two players show one or two of their fingers at the same time. If the number of the fingers is even, then the first player wins, otherwise the second.
a) Can the winning strategy be defined according to the procedure used in previous tasks?
b) What can be the winning strategy if we play the game several times?

Knowing the rules of the game the extended payoff matrix can be created (Table 3).

		II		
		1	2	min
I	1	1	−1	−1
	2	−1	1	−1
	max	1	1	

Table 3. The payoff matrix of the two-finger Morra game

Comments:

a) Trying to use the method which was successful in Task 1 and 2 the students can recognize, that in Task 3 there is no optimal pure strategy for any of the players (the maximum of the row-minimums is less than the minimum of the column-maximum).

b) Assuming that the game is played several times the strategies can be mixed. To win, the strategy must be randomly mixed with a constant probability. Let's assume that the first player chooses with probability p the strategy 1 and $(1-p)$ the strategy 2 while the second player chooses with probability q the strategy 1 and $(1-q)$ the strategy 2. The gain of the players can be described by two systems of linear inequalities and because of the symmetry the solution is $p = q = 0.5$ (Figure 2).

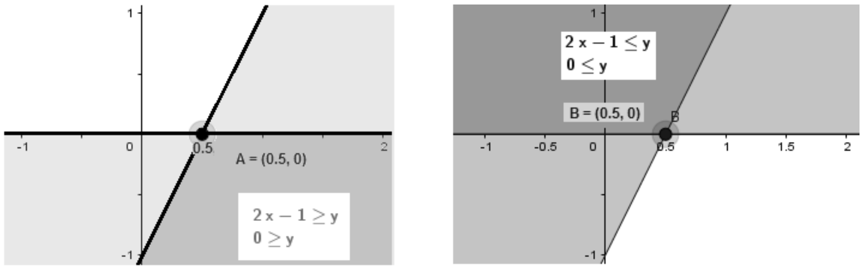

Figure 2. Visualization of the geometrical solution by GeoGebra

The use of the model requires that the player's tactics are not predictable, that is, the player has to choose a strategy randomly and ensure constant probability. For probability 0.5 such a model can be, for example, throwing a coin. If in the two-finger Morra game the first player chooses between the two possible strategies with 0.5 probability then the value of the game will be zero and it is not affected by the other player's choice of strategy. A result better then zero can not be guarantied in a zero-sum symmetric game.

Third lesson

The content of the third lesson is the principle of dominance, which is a part of the curriculum, but in such a short time, it cannot be transferred into problem solving.

In matrix games can be useful to introduce the concept of dominance. The principle of dominance states that if one strategy of a player dominates over the other strategy in all conditions, then the latter strategy can be neglected. In certain cases the application of the principle of dominance the size of the matrix can be reduced and thus the solving of the problem may be technically easier. Task 4 serves to raise awareness of it.

It is also interesting to note that the dominance principle is applicable not only to a pure strategy, but in case of a mixed strategy, too. (Because of lack of time we did not solve specific task for mixed strategy.)

Additional contents of the lesson are:

– textual task where the possible strategies can be arranged in the form of ordered pairs;

– construction of infinite solutions of a matrix game as convex linear combinations of strategies;

– definition of interchangeability and equivalency property (because of the next topic).

Task 4: Reduce the size of the matrix A given in Table 4.

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 6 & 4 & 2 & 4 & 3 & 3 \\ 6 & 5 & 3 & 5 & 4 & 4 \\ 6 & 9 & 9 & -3 & 3 & 4 \\ 6 & 5 & 6 & 1 & 4 & 4 \\ 6 & 5 & 5 & 0 & 4 & 4 \end{bmatrix}$$

Table 4. Example for an easily reducible matrix ([6], p. 28)

We can conclude that the matrix game does not have a pure saddle strategy. The 3rd row of the matrix dominates the 1st and 2nd rows, so lines 1 and 2 can be omitted. The 5th column dominates the 1st, 2nd and 6th columns, so these three columns can be omitted. Continuing the reduction, a simple (two by two) matrix is generated, which can be solved easily.

Finally, we turn to the general solution of matrix games. John von Neumann theory theoretically provides that every matrix game can be solved. In practice this is done in three steps:

– We find the saddle point/points in case of pure strategy, if they exist.

– If there are no saddle points, the concept of dominance helps to reduce the size of the matrix of the game in some cases.

– Solving a linear programming problem or in the case of two by two matrices using the right formula for substitution we get the proper probabilities and the value of the game.

Forth lesson

The bimatrix games fundamentally differ from the matrix games, the players in these not only increase their profits at the expense of each other, so the profit of one does not determine the other's. It is possible to represent a bimatrix game by two different payoff matrices or by a single matrix where the elements describe the profit of both players' in ordered pairs.

We suppose (as by each matrix game) that the players are selfish, rational, etc. The solution of a bimatrix game means to find the strategy with optimal payoff. Since the bimatrix game is not zero-sum, we need another method to find the solution. (For demonstration see the Task 3 of the practice worksheet in this article.)

First step: We search a solution with pure strategy.

We used the two matrices representation instead of ordered pairs. (According to our experience, the students understand easier the two matrices representation.) In the matrix ,,A" we sign the places of the maximums of the columns, in the matrix ,,B", the places of the maximums of the rows. The places where we can find both marking are the pure saddle points.

Second step: We search a solution with mixed strategy.

The mixed-strategy saddle points of the game can be obtained by solving quadratic programming problem. For the solution one can use computer programs or a web page for example [1].

We have to mention that in contradiction to matrix games the interchangeability and the equivalence property are often not true for bimatrix games.

We discuss the dominant strategy and the concept of the dominant point.

We formulate without proof the theorem that for a bimatrix game with

– the interchangeability property

– equivalency property

– dominant saddle point

cannot be achieved a better solution then the saddle point. All three properties are necessary.

Although any bimatrix game has its saddle point, the cooperation between the players can give a better result than the saddle-point strategy, if one of the properties is not true.

Fifth lesson

After presenting the mathematical model, we point out the versatility of the bimatrix games. We are examining well-known practical problems that may be relevant to managers' lives and work (arms race, environmental pollution, etc.).

Classical examples for coordination game

In the coordination game it is not worth having the chance to choose the strategy randomly, more information and a coordinated decision is necessary in order to find the best solution. We chose the simplest model (matrix, see Table 5) of the coordination games, which leads to the same strategy in our three examples and yields equal gains for the parties, and the outcome for the different strategies is equally unfavorable. One of the main cases of the coordination game where for both players have two equal optimal solutions. In these problems the situations need to be regulated despite that there are no conflicts of interests between the players. We discussed the following problems:

Coordination game 1: The technological standards. Two companies (Player 1 and Player 2) develop electrical devices. There are two types of standards (Strategy 1 and Strategy 2). The payoff matrix is described in Table 5. How to choose a standard?

Coordination game 2: Right-wing traffic. Two drivers (Player 1 and Player 2) want to choose between right hand traffic and left hand traffic (Strategy 1 and Strategy 2). The payoff matrix is described in Table 5. How to decide?

Coordination game 3: Secret date. Two Mafia bosses (Player 1 and Player 2) have to decide, that they go out to dinner on Friday or on Saturday (Strategy 1, Strategy 2) with their girlfriends and the other day with their wives. The payoff matrix is described in Table 5. How can they avoid the meeting of any wife with a girlfriend?

	Player 2		
Player 1		Strategy 1	Strategy 2
	Strategy 1	(1;1)	(0;0)
	Strategy 2	(0;0)	(1;1)

Table 5. The common bimatrix of Coordination games 1, 2, 3.

The other discussed game was the prisoner's dilemma (Table 6).

	Player 1 cooperates	Player 1 defects
Player 2 cooperates	(1;1)	(2;-1)
Player 2 defects	(-1;2)	(0;0)

Table 6. The common bimatrix of a classical Prisoner's dilemma

The prisoner's dilemma is a basic problem in the history of mathematics (time dimension of fundamental ideas). The interesting part of its solution is that pursuing individual reward logically leads both of the prisoners to betray, when they would get a better reward if they both kept silent. Humans display a systemic bias towards cooperative behavior, much more so than predicted by simple models of ,,rational" self-interested action. It is another kind of rationality, where people have information how the game would be played if they cooperate and then maximize their payoff.

The prisoner's dilemma is a prototype of many problems in the society and economy (horizontal dimension of fundamental ideas): to change or not money on the street corner, arms race in two countries, or decision-making position of Tosca. It has a moral attitude (human dimension of fundamental ideas). The failure of dominant strategy choice is shocking. It is sad but undeniable that free, independent people often do not decide properly due to lack of trust in others. Only external pressure, laws can ensure the public interest protection ([9]).

After discussing the basic problem, it is also worth considering the multiplayer prisoner's dilemma. Here, the individual is the first ,,player" and the ,,others" is the second one. Such situations, for example, are the tax-fraud, the doping, the tragedy of the commons or overfishing of the seas while pollution and climate change. The repeatedly played prisoner's dilemma shows that there is hope for resolving the conflict between the individual and the community.

Axelrod's competitions have shown that in the long-term the most important characteristics of profitable strategies are being ,,nice" and ,,forgiving". ,,The simplest successful algorithm is ,,Tit-for-Tat" (TFT), which begins by cooperating, and continues by doing each turn what its partner did last turn. It has all three of Axelrod's desirable qualities, and does very well at the task. A community of TFT players will always cooperate with one another, but since each one will retaliate immediately in response to defection, a defector cannot prosper by exploiting any of them." ([3], p. 665) The success of Tit-for-Tat suggests that at the beginning one has to trust in the partners, has to forgive for them, but do not let them to take advantage of himself.

Some tools of learning support

In order to support the students' learning there is an E-learning Portal at our university (SZIE) with the description of the subject, the e-learning material used in the lesson (in more detail), the list of recommended reading materials and the practice worksheet for self-control in each topic with solution aids. (See for example [5], [11].)

The practice worksheet is a very useful tool to point out the most preferred concepts, algorithms and connections (see Table 12). We wanted to avoid that the students unintentionally looking at the solution, so we presented the tasks and the solutions separately. The solution contains not only the results but also orienting questions and explanations. It is also useful to read the solutions for them, who have received the correct result, in order to reinforce the correctness of the thought. In addition, they can see how detailed the solution of the given task needs to be described.

During the test of the previous topic, it turned out that the students could not live well with the possibility that all written materials could be used, so we proposed them to collect the most important formulas on an A4 paper, which will be allowed to use during the test.

The practice worksheet for game theory

The practice worksheet does not include a mixed-strategy task at the bimatrix game because it is very difficult to calculate without internet connection (it is not available during the test) and the concept and meaning of the mixed-strategy comes to light at the matrix games.

Although the students received all the tasks first and then the solving instructions for the tasks, for easy reading, this article gives the most important parts of the instructions in italics immediately after the task or part-task.

Task 1 of the practice worksheet: Consider the matrix games given in Table 7.

– a) Determine (all of) the pure saddle point(s).

– b) Give (all of) the optimal strategy pair(s).

– c) Decide if the game is fair. If not, who benefits from it? Why?

– d) How much would the first/second player's gain be if he unilaterally deviated from the saddle-point strategy.

-4	1	-7	8
-3	0	-6	7
5	2	3	4
0	2	6	7

-4	1	-7	8
-3	0	-6	7
-1	-2	3	4
0	2	6	7

-4	5	-7	8
-3	0	-6	7
5	5	6	7
4	5	6	7

Table 7

Results: We give the solution of the first matrix, the others can be solved similarly (Table 8).

-4	1	-7	8	-7
-3	0	-6	7	-6
5	2	3	4	2
0	2	6	7	0
5	2	6	8	

Table 8. Extended payoff matrix

a) There is only one saddle point: (3;2) (see the gray row and column in Table 8).

b) (3;2) is the unique optimal strategy pair, which means that the first player

should choose the third strategy, the second player should choose the second one.
c) The value of the game is 2, which can be seen from the payoff matrix as the common gain of the two optimal strategies.The first player will win 2 HUF, which means that the game is not fair.
d) The first player's possible gain could be 1, 0 or 2 if he unilaterally deviates from the saddle-point strategy (which gain is clearly not better than the current 2).

The common features of the three matrix games are that they have pure saddle points. The three matrices describe three different types, namely the game in the first one is not fair, in the second one is fair, the third has two saddle points. The task can be solved on the first lesson's content. In a) the students have to apply the learned method. In b)–d) they have to interpret the values received in a). By wording of the task we used single and plural as well because we do not know the number of saddle points.

Task 2 of the practice worksheet Consider the 2 by 2 matrix games given in Table 9.
a) Are there any pure saddle points? Why?
b) If there are no pure saddle points, determine the probability of the first and second strategy worth choosing for the first and second player respectively.
c) What is the expected value of the game from the point of view of both players, if they choose the optimal strategy pair?

1	2
4	-2

1	3
2	-4

Table 9

Results: We give the solution of the first matrix; the second one can be solved similarly.
a) The maximum of the row-minimums is 1 and it is not equal to the minimum of the column-maximums, which is 2. There is no pure saddle point.
b) The first player should choose the first strategy with 6/7 probability:

$$\frac{d-c}{a+d+b+c} = \frac{-2-4}{1+(-2)-2-4} = \frac{-6}{-7} = \frac{6}{7}.$$

c) In case of selecting the optimal strategy pair, the expected value of the game for the first player is:

$$v = \frac{ad-bc}{a+d-b-c} = \frac{1\cdot(-2)-2\cdot 4}{1+(-2)-2-4} = \frac{-10}{-7} = \frac{10}{7}.$$

Matrices in the second task have no pure saddle points (second lesson's content). The students have to see clearly, that the strategy must be chosen depending on from which player's point of view we look at the game. The sign of profit also depends on the point of view.

Task 3 of the practice worksheet Consider the bimatrix games given in Table 10.
a) Determine (all of) the pure saddle point(s).
b) Give (all of) the optimal strategy pair(s) for (all of) the pure saddle point(s).
c) How much would the first and second player's gain be if he unilaterally deviated from the saddle-point strategy?
d) How much is the first and second player's gain for each saddle point?

First exercise

$$A=\begin{array}{|c|c|c|}\hline -4 & 4 & 4 \\ \hline 1 & -2 & 3 \\ \hline -3 & 1 & 0 \\ \hline \end{array} \qquad B=\begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline -4 & 1 & 2 \\ \hline 1 & 2 & -4 \\ \hline \end{array}$$

Second exercise

$$A=\begin{array}{|c|c|c|}\hline 1 & 0 & 3 \\ \hline -2 & 3 & 4 \\ \hline -3 & -2 & 1 \\ \hline \end{array} \qquad B=\begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline 4 & 0 & 0 \\ \hline 1 & -2 & 5 \\ \hline \end{array}$$

Table 10

Results: We give the solution of the first bimatrix game, the second one can be solved similarly. (Table 11)

$$A=\begin{array}{|c|c|c|}\hline -4 & 4 & 4 \\ \hline 1 & -2 & 3 \\ \hline -3 & 1 & 0 \\ \hline \end{array} \qquad B=\begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline -4 & 1 & 2 \\ \hline 1 & 2 & -4 \\ \hline \end{array} \qquad \begin{array}{|c|c|c|}\hline & 1 & 1,2 \\ \hline 1 & & 2 \\ \hline & 2 & \\ \hline \end{array}$$

Table 11

a) We mark the maximum of the column in each column in matrix ,,A" and the maximum of the row in each row in matrix ,,B". The common points will be the pure saddle points.
b) The optimal strategy pair is (1;3). So the first player should choose the first strategy, the second player should choose the third strategy.
c) If the first player unilaterally deviates from the saddle-point strategy, then his possible profit could be 0 or 3 (which are clearly not better than the current 4).
d) The first player's gain is 4, the second player's gain is 5 in the saddle point.

The third task contains bimatrix games with pure saddle point (fourth lesson's content). To solve a bimatrix game one has to use different strategy from that, which was used by matrix games, because it is a non-zero sum game. The students have to know, that the selection of a row or column of a matrix – from which we read the correct answer – depends on, whose perspective we consider the game.

Task 4 of the practice worksheet Exercises for the application of game theory.
– Write an everyday problem for a coordination game in a sentence or two.
– Give the bimatrix of your problem described above using only the digits 0 and 1.
– Write an everyday problem for a classical, a multiplayer and an iterated prisoner's dilemma in a sentence or two.
– Give the bimatrix of your problem described above using only the digits 0, 1, 2 or 3.

In the fourth task we asked for problems occurring in everyday life and demanded to give the bimatrix of the game (the fifth lesson's content). There are only two applications on the worksheet, which were discussed during the lesson.

Evaluation of the learning unit, the written exam

Game theory was one of the two topics of the written exam. The students had four possibilities to write a successful test. Each worksheet was similar, so we present the tasks of the first test which are connected with the game theory.

The relevant tasks of the first test

Task 1 of the first test (8 points) Consider the matrix game given in Table 12.
a) Determine (all of) the pure saddle point(s).
b) Give (all of) the optimal strategy pair(s).
c) Decide if the game is fair. If not, who benefits from it? Why?
d) How much would be the first and the second player's gain if he unilaterally deviated from the saddle-point strategy.

1	4	0	3
-3	0	-2	7
1	0	2	0
3	5	4	7

Table 12

Task 2 of the first test (8 points) Consider the 2 by 2 matrix game given in Table 13.
a) Are there any pure saddle points? Why?

b) If there are no pure saddle points, determine the probability of the first and second strategy worth choosing for the first and second player respectively.
c) What is the expected value of the game from the point of view of the second player, if they choose the optimal strategy pair?

2	0
-1	1

Table 13

Task 3 of the first test (9 points) Consider the bimatrix game given in Table 14.
a) Determine (all of) the pure saddle point(s).
b) Give (all of) the optimal strategy pair(s) for (all of) the pure saddle point(s).
c) How much would the second player's gain be if he unilaterally deviated from the saddle-point strategy?
d) How much is the first player's gain for each saddle point?

A=

3	2	-1
0	-2	3
3	1	0

B=

1	5	2
-4	1	0
1	3	-4

Table 14

Experiences based on the performance of the exam

Since this was the first exam time, despite the sample test, students did not know what the worksheet would be like. 14 of the 17 students reached the required score for passing, questions on game theory achieved 73% performance. Everyone accepted the first test's grades and they did not use the offered improving option. One can see that there is greater emphasis on preferences ,,Knowledge of the concept" and ,,Rule-following competency", but the other preferences are also occurring in the tasks (Two-person zero-sum matrix game with pure and mixed strategy, Two-person bimatrix game with pure strategy, Applying of game theory, Computational skills, Communication, Connection to the everyday life, Creativity).

The results on the tasks:
The solving rate of Task 1 was 93%, there were almost no mistakes. It means, that they can handle competently the pure strategy.
The solving rate of Task 2 was 75%. The students are also competent in mixed strategy games. They lost some points by writing a statement without reasoning, using an incorrect formula or using a correct formula incorrectly.
The solving rate of Task 3 was 50%. It was a typical mistake in the third task, that some students tried to solve the bimatrix game using the method relevant for matrix games. The half of the students did not deal with the task.

Summary

The program implemented as part of the course is considered successful because the students' feedback and test results show that students are not averse towards this particular topic. It is significant methodologically because of the fact that the modeling process is implemented almost unconsciously. It is possible to work with simple mathematical tools, so the students made less counting mistakes than usual.

We asked the students' opinion about game theory during the lessons; their answers were given without names. The interestingness of the topic reached 66%, the utility of the topic 60%, the interestingness of the lessons 56%. Considering these results on the last lesson we pointed out the versatile applicability of the game theory.

After this course we are right to believe that students will recognize some of the typical applications, for instance the prisoner's dilemma in changing flats, having an agreement or prepayment. By changing flats each owner has the interest in showing his flat better than it is in reality. Using the model for the situation it comes out, that the best for both is to tell the pros and cons of their own home honestly, because in this case they both get what they expected. For more examples see [12].

In the next courses I would like to give more time for students themselves to find an appropriate model for the given problem already during the lessons.

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