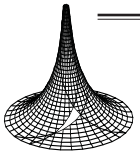

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tmcs@science.unideb.hu
http://tmcs.math.unideb.hu

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Developing a method to determine teachers' and pupils' activities during a mathematics lesson

Maija Ahtee, Erkki Pehkonen, Anu Laine, Liisa Näveri, Markku S. Hannula and Pirjo Tikkanen

Abstract. Third-graders from nineteen classrooms ($N = 316$) were asked to draw a picture on a mathematics lesson. Based on these drawings we have developed a data analysing method that allows us to find out how pupils present both their teacher's and their classmates' activities in their drawings. Two inventories were formed that contain, respectively, teachers' and pupils' activities during a mathematics lesson as seen in the pupils' drawings. The first inventory contains 14 separate items organized into six groups that contain teacher activities like asking questions and giving feedback on mathematics. Ten of the items are related to teaching and the rest contain items like keeping order in addition to the teacher's location in the classroom. Respectively, pupils' activities are organized into five groups that contain altogether 22 items. These contain the activities of a single pupil, and also pupil-teacher and pupil-pupil discussions on mathematics.

Key words and phrases: pupils' drawings, mathematics lesson, teachers' and pupils' activities.

ZDM Subject Classification: C70, C80.

Classroom is a social environment, where pupils spend around 20-30 hours per week during the six-year primary education in Finland. Thus, the classroom environment is significant in shaping pupils' perceptions,with both the teacher and the activities allowed by her/him forming the key influences. The teacher is responsible for all the activities in the class and she/he orchestrates learning processes. Communication between the teacher and her/his pupils is central to pupils' formalization of mathematical concepts and procedures [1].

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Usually, activities in mathematics teaching have been studied by direct observing e.g. [2], by interviews e. g. [3], and/or by questionnaires e.g. [4]. On the contrary, free hand drawings have seldom been used to study pupils' images of teaching. However, third-graders' responses in answering questionnaires are not always reliable due to their young age, and furthermore, interviews and classroom observations are time demanding methods. Additionally, drawings could be a way to get some information about pupils' deep conceptions which cannot be exposed by using observation, questionnaire or interview. In this paper we concentrate to find out how young pupils describe in their drawings their teacher's activities, and their classmates' activities.

The aim of this study is to produce a method how to reveal what kind of teachers' and pupils' activities during mathematics lessons are seen in pupils' drawings. This kind of subjectively perceived classroom of Finnish primary pupils in mathematics has rarely been studied. A reason for the small amount of research may be the lack of suitable measurement instruments for young children.

Theoretical framework

First we present the concept of mental image and introduce its use in research of pupils' drawings. We also introduce some research on mathematics teaching based on pupils' drawings. Finally we describe shortly a typical Finnish primary lesson in mathematics.

Mental images

In cognitive science, it has been proposed [5] that a mental image is a mental representation of an object, an event or a situation whose features are spatially and temporally organised. The formation of such a representation can be based on direct experience with its referent. In the theory of imagery [6] it is explained that the images may have depictive, picture-like qualities that could not be regarded as purely propositional, language-like mental representations. This theory suggests that the images are short-term memory representations generated from long-term memory representations that may have a depictive or propositional form. The depictive form may be used less, as propositional knowledge increases and deduction becomes easier. If pupils have few propositional linguistic representations of the image's referent they tend to use their visual image.

The studies in [7] demonstrate that pupils in comprehensive school have among other things general, specific and episodic autobiographical images both of mathematical contents and of mathematics learning and teaching. General image

represents a concept without any reference to a particular example or to specific characteristics of the item. A general image of a 'table' could be described as a surface with four legs. A specific image represents a single well-defined example of the concept without reference to a specific episode. For example specific images of a 'table' might be a dining table, a coffee table or a kitchen table. An episodic-autobiographical image represents the occurrence of a single episode in the subject's life connected to the concept. Our data consists of drawings about mathematics lessons. The drawings produced by pupils are thus mainly episodic autobiographical images. An episodic autobiographical image of learning mathematics could be for example "Last week I succeeded to solve hard problems in mathematics with my classmates".([8], [9]).

Image-based research of mathematics teaching via drawings

Drawings help pupils to overcome the difficulties in disclosing their thoughts, feelings and opinions to an adult researcher [10]. Drawings are useful, because they require little or no language mediation. According to [11] pupils' classroom drawings form rich data to study children's conceptions on teaching. Pupils' drawings have made an alternative and complementary contribution among conventional research methods by conveying their images about mathematics, mathematics teaching, their teacher, their peers and classrooms in mathematics lessons.

According to [12] primary pupils conceptualise and clearly distinguish the professional of teachers among scientists and veterinarians via drawings. Lower secondary pupils' images of mathematicians were compared in five countries (USA, UK, Finland, Sweden, Romania) using drawings with a questionnaire [13]. About 20% of the drawings portrayed a school teacher, and classrooms which basically looked the same from country to country with only small differences. Drawings of the teacher showed that she/he sometimes neither mastered the teaching group nor the topics to be learned. Sometimes she/he seemed to be cleverer than her/his pupils, but sometimes she/he seemed to lack common sense, style and calculation skills.

In the Turkish study [14] drawings with writings were used in order to clarify fifth-graders' views of changes in mathematics teaching. The data revealed that mathematics teaching had become more student-centred. In order to understand what pupils value in their mathematics learning process, 118 primary pupils' drawings were studied in Australia to find out pupils' individual impressions of effective mathematics lessons [15]. The data revealed that mathematics lessons featured a co-valuing of fun as the teacher and the pupils experience, and the teacher's explicit explanation or instruction on the board.

In the framework of motivation theory, second- and fifth-graders' conceptions on mathematics and mathematics teaching via drawings were compared with the written questionnaire in Sweden [16]. All pupils presented mathematics teaching as an individual activity with a focus on the textbook. Most of the second-graders had a positive attitude toward mathematics whereas a larger proportion of the fifth-graders had a negative attitude.

Typical primary lesson in mathematics in Finland

Since the gathered data here is from the Finnish schools, we will here give a brief description on mathematics teaching in primary school (the lower level of the Finnish comprehensive school), where class teachers teach all school subjects. This description is also additionally supported by our own long experience as teacher educators in mathematics.

On the whole mathematics teaching is rather traditional in primary classrooms [17], [18]. It is mainly teacher-centred teaching with the teacher standing or sitting in front of the class. However, the pupils are active and work keenly with their tasks. Usually a mathematics lesson begins with a short 5-minute session of mental calculations or some other orientation activity like 'Today's nut' (a mathematical problem new to the pupils to motivate and lead the pupils into mathematical thinking). This is followed by checking the homework that is given at the end of each mathematics lesson to the pupils in order that they would practice the main points of the previous lesson. Most teachers also check that all the pupils have completed their homework.

The lesson continues with the teacher introducing and teaching new topics. The teachers use mostly the well-known and conventional teaching methods when dealing with the subject that is new to the pupils. The teachers proceed slowly in the subject as they want to guarantee learning opportunities to all their pupils. This phase is followed by individual work through tasks that help the learners study and acquire the knowledge set in the lesson aims. The pupils work at their own pace, and the teacher helps those who need support. A large proportion of mathematics lessons are devoted to silent, individual work. Some teachers may have a special mathematics lesson with problem solving or project work once a week. The primary teachers follow very closely mathematics text-books and teacher's guidebooks [19]. The guidebooks highlight some essential pedagogical ideas that the teacher should take into consideration when discussing a topic. The text-books also include the tasks that will be given to the pupils as homework at the end of the lesson to promote the learning process.

The purpose of the study

About 5 years ago we began our research on drawings with the case of classroom communication and teaching methods (cf. [20]). In that study we were interested in communication used in the mathematics classroom and therefore, the following categories were chosen: Teacher-pupil communication; Internal classroom communication; Quality of classroom communication. Internal classroom discussion contained the cases when the pupils are discussing with each other, and, respectively, quality of classroom communication contained how the pupils feel about mathematics. The typical teaching scenario seemed to depict a teacher-led class, in which the teacher sits quietly behind his or her desk and supervises the joint practice of the teaching group. Three themes appeared in the pupils' communication: mathematics is nice/easy, mathematics is boring/difficult, and a pupil is competent in mathematics.

However, this kind of classification does not give a comprehensive picture of what is happening in a classroom. What is the teacher usually doing during a mathematics lesson, or how do the pupils behave in the classroom? When we know these things, we might be able to answer the questions like: How do the teachers' activities differ, or how does the pupils' behaviour differ in the different classrooms? A drawing gives a "snapshot" of the activities of the teacher as well as of the activities of the pupils experienced by the drawer. We wanted to find out what is happening during a mathematics lesson as seen by the third-graders. How do the third-graders describe in their drawings what the teacher is doing? How do the third-graders describe what they and their classmates are doing during a mathematics lesson? Therefore, we needed to create a method to analyse young pupils' drawings in order to find answers to the following questions:

1) How can we identify a teacher's activities during a mathematics lesson as seen in the pupils' drawings?
2) How can we identify the pupils' activities during a mathematics lesson as seen in their classmates' drawings?

Methodology

Participants and data gathering

The data of our article consists of third-graders' drawings that were collected in the autumn of 2010 in Finland (Helsinki area). Drawings were content analyzed from a total of 316 third-graders (about 8-9-year-olds) from the classes taught by 19 different teachers in 11 primary schools in Great-Helsinki. The pupils did

the drawing task during their mathematics lessons under the supervision of their teachers in the beginning of the school year. The task for the pupils was, as follows:

Draw your teaching group, the teacher and the pupils, in a mathematics lesson. Use speaking and thinking bubbles to describe discussion and thinking. And mark "me" in your drawing.

The drawings collected by the teachers were obtained from 165 boys, 150 girls and one pupil who did not indicate the gender. In about two thirds of the drawings the speaking and thinking bubbles were used. Especially the contents of these bubbles enabled us to investigate both the teachers' activities (Table 1) and the pupils' activities (Table 2).

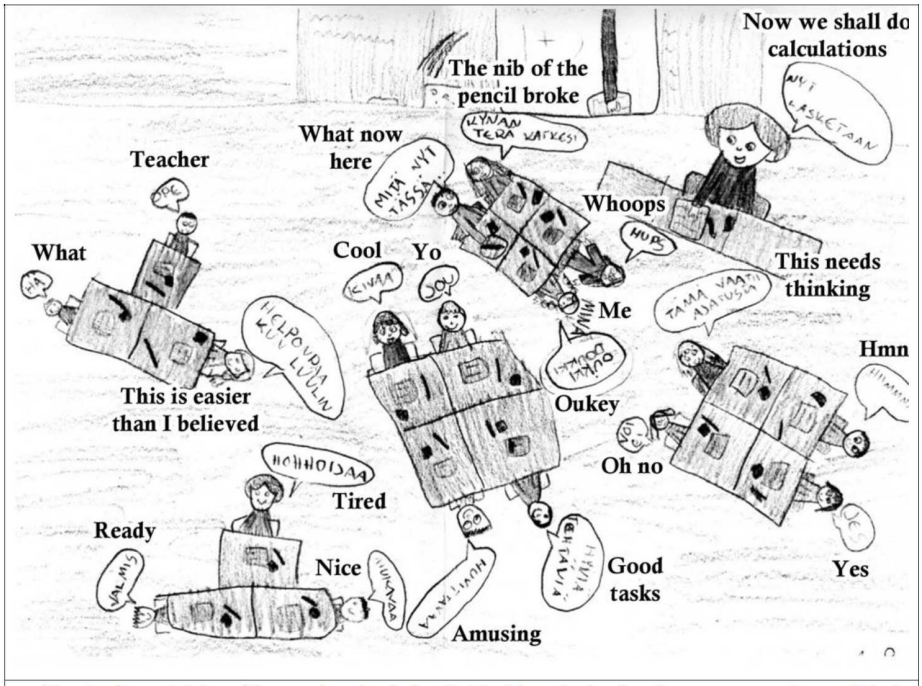
Data analysis

First, all of us looked and listed together from all the drawings in one classroom all possible teachers' activities and pupils' activities we could see in the pupils' drawings. After that, two of us completed these two inventories (Table 1 and Table 2) by going through all the third-graders' drawings from the nineteen classes. The 14 separate items found about the teacher's activities were organized into six groups, and two extra items namely, No teacher and Teacher is quiet were added in the inventory. Respectively, the 23 separate items found about the pupils' activities were organized into five groups. Furthermore, the number of pupils was marked on each item, and an extra item No pupil was added in the inventory. In this way, two inventories were formed: 1) Teachers' activities during a mathematics lesson as seen in the pupils' drawings (Table 1); 2) Pupils' activities during a mathematics lesson as seen in the pupils' drawings (Table 2).

Some drawings and their analyses

Here we have chosen three drawings that will illustrate the coding. These drawings have been selected based on their rich information on teaching situation. They do not represent any kind of prototypes. For example, all these three drawings contain speech bubbles whereas some of the drawings were very simple ones e.g. having only a schematic picture of the class with the pupils substituted by their desks. In the description of the drawings we have used the coding system presented in Table 1 (teacher's activities) and Table 2 (pupils' activities). For example, TA11 refers to the first subcategory in the content category TA1 (teacher's position) given in Table 1. Respectively, PA21(11) refers to the first

subcategory in the content category PA2 (pupil's remarks/thoughts contain no mathematics) given in Table 2. The number in brackets gives the number of the pupils belonging to this subcategory.



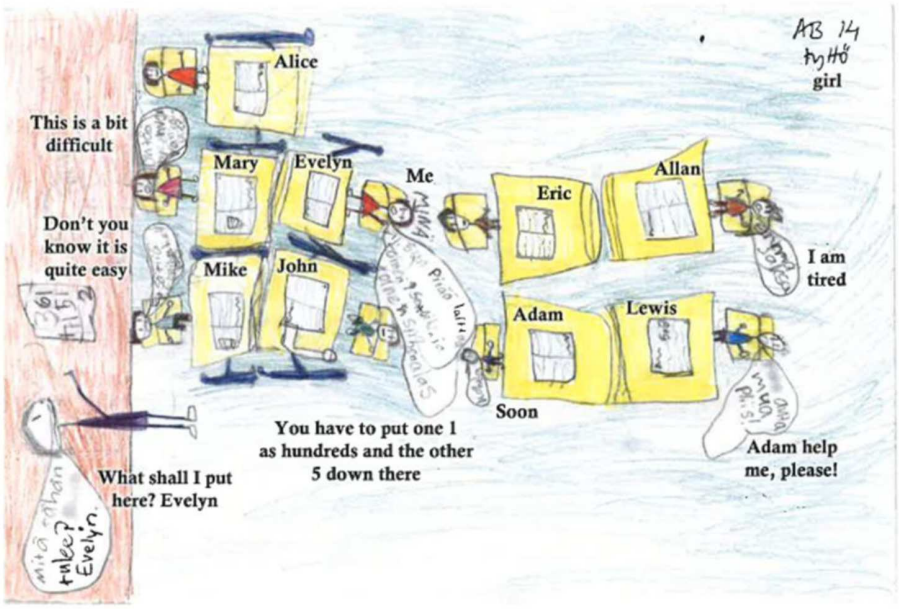
Teacher's activities: The teacher is sitting behind her desk. On the screen on the wall it is shown how she points with her finger the tasks in the book.

Pupils' activities: 18 pupils are sitting in their desks in small groups. All of them are commenting or thinking something. Five of the pupils are finding the tasks difficult or boring (*What, What now here, This needs thinking, Hm, Oh no*), whereas nine of the pupils are finding them nice or easy (*Amusing, Good tasks, Nice, This is easier than I believed, Oukey, Cool, Yo, Yes, Ready*). One pupil is asking help from the teacher (*Teacher*) and another pupil is tired. Two of the pupils are saying something not related to mathematics (*The nib of the pencil broke, Whoops*).

Coding of teacher's activities: TA11: Teacher's position; behind the desk. TA21: Information on mathematics; a task on the blackboard.

Coding of pupils' activities: PA11(18): Pupil's position; sitting in his own desk. PA22(2): No mathematics; Pupil is saying something. PA33(4): Pupil doing mathematics; Mathematics is difficult. PA32(1): Pupil doing mathematics; Mathematics is boring. PA35(3): Pupil doing mathematics; Mathematics is nice. PA34(6): Pupil doing mathematics; Mathematics is easy. PA44(1): Pupil-Teacher discussion on mathematics; Pupil asks help from the teacher.

Figure 1. The first example about coding of a teacher's and her pupils' activities



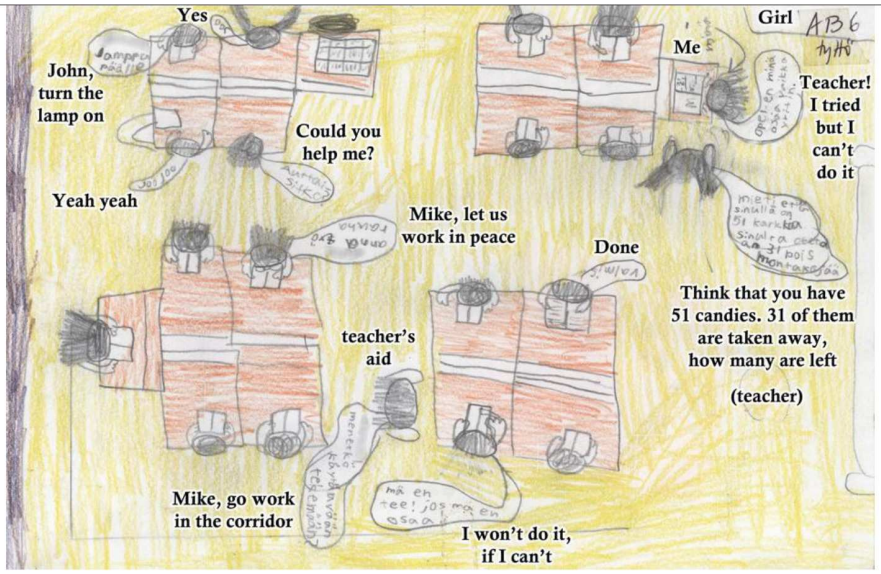
Teacher's activities in the drawing: The teacher is standing near the blackboard. She has written a task of addition algorithm (361 + 151) on the board. She shows the task and asks Evelyn, the drawer of this picture, what should be done.

Pupils' activities in the drawing: Nine pupils are sitting in their desks. Three pupils are sitting quietly neither talking nor thinking. One pupil is saying that the task is difficult; another is saying that it is easy. The drawer herself has her hand raised and is answering the teacher's question. Another pupil is saying that he is tired. One pupil is asking his classmate to help him. This pupil answers to him.

Coding of teacher's activities: TA12: Teacher's position; Near the blackboard. TA21: Information on mathematics: Teacher has written a task on the board. TA31: Instructing: Teacher gives instructions related to studying mathematics. TA41: Questioning; Teacher asks about a task in mathematics.

Coding of pupils' activities: PA11(9): Pupils' position; Sitting on her own desk. PA21(3): No mathematics; Pupil is quiet. PA23(1): No mathematics; Pupil is tired. PA33(1): Pupil doing, thinking mathematics; Mathematics is difficult. PA34(1): Pupil doing, thinking mathematics; Mathematics is easy. PA43(1): Pupil-Teacher discussion on mathematics; Pupil says about the task on the blackboard. PA51(1): Pupil-pupil discussion on mathematics; Pupil asks help from his classmate. PA52(1): Pupil-pupil discussion on mathematics; Pupil answers to another pupil.

Figure 2. The second example about coding a teacher's and her pupils' activities



Teacher's and pupils' activities: The teacher and the teacher's aid are among the pupils. In the right upper corner the teacher is giving advice to the drawer, who cannot do the task. In the group in the right lower corner the teacher's aid is keeping order because Mike is disturbing. The pupil in the group in the left lower corner wants to work in peace. There are altogether 18 pupils sitting in their desks in four groups. Ten pupils are quietly neither thinking nor talking. One pupil in the group in the left upper corner asks John sitting next to him to turn the lamp on. John who has bad eyesight agrees. Another pupil in this group asks help from his classmate who agrees to help. Three of the pupils have difficulties with these tasks, and one has already done them. Pupils are doing group work.

Coding of teachers' activities: TA13: Teacher's position; Among the pupils. TA23: Information on mathematics; Teacher is helping. TA33: Instructing; Teacher keeps order.

Coding of pupils' activities: PA11(18): Pupil's position; Sitting in his desk. PA21(10): No mathematics; Pupil is quiet. PA22(3): No mathematics; Pupil is saying something. PA33(3): Pupil doing mathematics; Mathematics is difficult. PA34(1): Pupil doing mathematics; Mathematics is easy. PA44(1): Pupil-teacher discussion; Pupil asks help from the teacher. PA51(1): Pupil-pupil discussion on mathematics; Pupil asks help from the classmate. PA52(1): Pupil-pupil discussion on mathematics; Pupil answers to his classmate.

Figure 3. The third example about the coding of a teacher's and her pupils' activities

Results

The two inventories Table 1 Teacher's activities during a mathematics lesson as seen in the pupils' drawings and Table 2 Pupils' activities during a mathematics lesson as seen in the pupils' drawings are given and explained here.

Teacher's activities

Six different teacher's activities divided into the subcategories are shown in Table 1. In addition, some comments and examples are included. In the end of Table 1 there are two extra categories. Category TA01 contains those pictures in which the pupils had drawn no teacher. In the pictures of category TA02 the teacher has no thinking or speaking bubbles only his/her location in the classroom could be noted. It was impossible to know for sure whether a pupil had made a difference in her/his drawing between speaking bubbles (whole line) or thinking bubbles (dashed line). Therefore the cases in which the teacher is saying or thinking something cannot be demerged. For example, in subcategory TA51 when the teacher is giving positive feedback "You are good", she may say it aloud or just think it in her mind.

The three subcategories in the first category TA1 'Teacher's location' indicate the three different teaching activities. When the teacher is sitting or standing near her desk the drawer may imply that the teacher is observing the pupils working independently with their tasks. When the teacher is standing near the blackboard (screen, whiteboard, or similar) the drawer may imply that the teacher is teaching the whole class, she/he may be questioning, or she/he may be referring to something written on the board. When the teacher is drawn among the pupils she/he may be giving advice to a group working together. However, one has to be careful in making such conclusions. Only when the majority of the pupils in a class have drawn pictures in a certain tendency such a conclusion can be drawn but then also the hints from the other categories has to be taken into account. In this article we have not made conclusions about the teaching activities.

The second category TA2 is related to how the drawer sees the teacher dealing with mathematics. In the first subcategory TA21 there is some mathematics written on the blackboard like in Fig. 2 whereas in the second subcategory TA22 the teacher is pointing toward the blackboard but there is no mathematics written on it. On the third subcategory TA23 the teacher is among the pupils giving mathematical advice to a pupil or to a group like in Fig. 3. The third category TA3 is associated with the comments needed during the course of the lesson. In subcategory TA31 the teacher gives instructions related to studying mathematics, like in Fig. 2 the teacher gives Evelyn the permission to answer. In subcategory TA33 the teacher or like in Fig. 3 the teacher's aid is keeping order. Categories TA4 'Questioning' and TA5 'Feedback' give information about common practise in classrooms. In subcategory TA41 the teacher asks about mathematics like in Fig. 2 whereas in subcategory TA42 the teacher asks questions that are not related

Table 1. Teachers' activities during a mathematics lesson as seen in the pupils' drawings

Code	Title	Comments and examples
TA1	**Teacher's location**	
TA11	Behind or close to the desk	When difficult to decide between TA11 and TA12 choose TA12 if there is a task on the board.
TA12	Near the blackboard	
TA13	Among the pupils	
TA2	**Informing on mathematics**	**Teacher gives information on mathematics**
TA21	Teacher has written a task on the blackboard	Teacher not necessarily near the blackboard
TA22	Teacher is teaching near the blackboard	Teacher is pointing to the blackboard but there is no mathematics on it.
TA23	Teacher is helping	Teacher is giving advice to a pupil or to a group.
TA3	**Instructing**	**Teacher gives instructions**
TA31	Teacher gives instructions related to studying mathematics	"Start doing calculations." "Go and check the result." Teacher gives a pupil the permission to answer e.g. by saying the pupil's name.
TA32	Teacher gives instructions not related to studying mathematics	"Open the door, please." Teacher says a pupil's name but the reason is not clear.
TA33	Teacher keeps order	"Be quiet."
TA4	**Questioning**	**Teacher asks questions**
TA41	Teacher asks about a task in mathematics	Also a test in the class is included here.
TA42	Teacher asks something else	"Who will come to do the calculation on the blackboard?"
TA5	**Feedback**	**Teacher gives feedback**
TA51	Teacher gives positive feedback	"That is correct." "You are good."
TA52	Teacher gives negative feedback	"Wrong"
TA6	**Reflecting**	**Teacher is reflecting**
TA61	Teacher is reflecting on mathematics	
TA62	Teacher is reflecting something else	"What should I do?", "I wonder how those pupils are managing."
TA01	**No teacher**	No teacher can be identified from the drawing
TA02	**Teacher is quiet**	Teacher is neither speaking nor thinking

Extra comment. When a pupil has drawn a cartoon, the categorizing is made from the first picture which contains both the teacher and at least one pupil.

straight to mathematics like "Does anyone need help?" In subcategories TA51 and TA52 the teacher is giving either positive or negative feedback usually related to mathematics. But also the more general remarks or thoughts are included in this category e.g.that the teacher is thinking that she is pleased with her pupils. Category TA5 'Reflecting' is connected to more general issues what the drawer thinks that the teacher is thinking in her/his mind. Just for the sake of completeness we included also subcategory TA61 in which the teacher would be reflecting on mathematics even that we did not found any example about it in our data.

Pupils' activities

Pupils' activities were divided into five categories with three to five subcategories as shown in Table 2. When the data is collected also the number of the pupils in each subcategory has to be noted. In the end of the table there are two extra categories. Category PA01 contains those cases when the drawer has drawn no pupils at all. Respectively, category PA02 contains the cases when the drawing contains no speaking or thinking bubbles connected to any pupil. Because the cases whether a pupil is speaking (whole line bubble) or thinking (dashed line bubble) could not be determined definitely, we use the term saying for both cases in Table 2.

The number of pupils is marked in each subcategory. Each pupil's remarks are tried to be connected to the teacher's or other pupils' activities or interpreted case-by-case e.g. like short expressions "Jee". If a pupil has made many remarks that belong to the same category, they are marked as one remark. If a bubble contains the name of the pupil like "Mike" or "I am John", the subcategory is PA21, in other words, it is not a remark. If the shapes of the speech and thought bubbles are different in a drawing, the comments in the thought bubbles are not interpreted as answers to the teacher. If all the bubbles look the same in a drawing, then the comments are interpreted as answers to the teacher.

In Table 2 there are four main categories about the pupils' communication. In category PA2 'No mathematics' the pupil may be thinking or speaking aloud by her/himselfor she/he may be communicating with the teacher or with her/his classmates but these expressions do not contain mathematics at all. In category PA3 'Pupil talking about mathematics' the pupil is communicating neither with the teacher nor with her/his classmates. S/he is thinking or speaking aloud about her/his own ideas or feelings. For example, in Fig. 3 one pupil is categorized in subcategory PA34 'mathematics is easy' as she/he simply says that she/he

Table 2. Pupils' activities during a mathematics lesson as seen in the pupils' drawings

Code	Title	Comments and examples
PA1	**Pupil's location**	
PA11	Sitting on her own desk	Also when pupils are drawn without desks
PA12	Near the teacher's desk	Or some working point
PA13	Near the blackboard	
PA14	Near another pupil	e.g. asking for help
PA15	Somewhere else	e.g. sharpening a pencil near the waste paper basket
PA2	**No mathematics**	**Pupil's activity is not related to mathematics**
PA21	Pupil is quiet	Neither talking nor thinking
PA22	Pupil is saying something	"The tip of the pencil broke."
PA23	Pupil is tired	"Hohhoijaa"
PA24	Pupil is says something inappropriate	e.g. is swearing
PA25	Pupil says something unclear	impossible to track it down
PA3	**Pupil talking about mathematics**	**Pupil is talking/thinking by him/herself**
PA31	talks about mathematics, calculations	
PA32	says mathematics is boring	"Oh no"
PA33	says mathematics is difficult	"What", "This needs thinking", "Hm"
PA34	says mathematics is easy	"Easier than I thought", "Ready"
PA35	says mathematics is nice	"Yes" or "OK" without anybody making a comment or a question, "Good tasks"
PA4	**Pupil-Teacher discussion on mathematics**	
PA41	Pupil answers the teacher's question	
PA42	Pupil asks the teacher about mathematics	
PA43	Pupil talks about the task on the blackboard	
PA44	Pupil asks help from the teacher	"Teacher"
PA5	**Pupil-Pupil discussion on mathematics**	
PA51	Pupil asks from the classmate	"Could you help me?"
PA52	Pupil answers to another pupil	"I can help"
PA01	**No pupil**	Only teacher or classroom is drawn
PA02	**All pupils are quiet**	Pupils are neither speaking nor thinking

has "Done", whereas three pupils have been categorized in subcategory PA33 'mathematics is difficult', because they have problems with the tasks like the pupil who asks help from her/his classmate "Could you help me". In subcategory

PA41 'Pupil answers the teacher's question' the pupil is answering to the teacher's short question like "What is 2 times 4?"; whereas in subcategory PA43 'Pupil talks about the task on the blackboard' the pupil explains something like in Fig. 2. In category PA5 'Pupil-pupil discussion on mathematics' the discussions that we found in our data were mostly about helping each other like in Figures 2 and 3.

We have collected almost all cases of the pupils' different behaviour in the subcategories. However, many of these could be combined to bigger entities depending on the research interests. For example, in category PA1: Pupil's location the five subcategories could be reduced to two subcategories like appropriate behaviour or disturbing behaviour. In doing so one could also look what the pupils' average behaviour is like in a certain classroom.

Discussion and conclusions

To give an idea what kind of information can be obtained by using the inventory given in Table 1 we give some results of the analysis how third graders describe in their drawings their teacher's activities during mathematics lessons. Teacher is a central element in the mathematics lessons. Only about 10% of the pupils did not draw the teacher at all, and only in 15% of the drawings the teacher was drawn without saying or thinking anything. In 50% of the drawings the teacher was sitting or standing near the desk and in 60% of the drawings she was giving information about mathematics. Half of the instructions the teacher was giving concerned mathematics and a third about the order in the classroom. Nearly all the feedback that the teachers were giving is positive.

In the earlier articles we have looked at several aspects in the drawings: what kind of teaching methods and communication [20], emotional atmosphere [21], and level of mathematics [22] can be found in the pupils' drawings. A typical communication in a Finnish class seems to be built mainly in three themes: mathematics is nice/easy, mathematics is dull/ difficult, and pupils can do mathematics [20]. In the Finnish third-graders' drawings the mode value of the emotional atmosphere in mathematics lesson is positive. The most interesting result is large differences between the emotional atmospheres in different classrooms [21].

With the drawings we have also looked at the differences of teaching methods and communication found in third-graders' mathematics lessons in Finland and Chile [23] and Finland and USA [24]. In the Finnish drawings, teachers are typically drawn sitting behind the desk and pupils working often in groups. In Chile, the drawings show most frequently teacher-centred teaching methods the teacher standing in front of the class. Regarding communication, there are no

significant differences between both countries [23]. The U.S. students have greater self-confidence with respect to knowing/learning mathematics than their Finnish counterparts. In addition, U.S. comments during class were more related to the lesson (teaching) and their teachers gave more feedback [24].

In this article we introduce a tool to analyse teachers' and pupils' activities from young pupils' drawings. In their drawings pupils describe how their teacher is giving information on mathematics, how she/he is giving instructions to the pupils related to mathematics as well as on how to behave during the mathematics lesson. Some of them also write down the teacher's questions and the kind of feedback she/he is giving. Respectively, the drawings contain incidents when the drawer describes her/himself and her/his classmates dealing with mathematics or something else, discussions between the pupils and the teacher, and also discussions between the pupils. From these incidents we collected two inventories from third-graders' drawings. With the help of these two inventories it is possible to identify a teacher's and her/his pupils' activities as seen by the pupils. To really understand what the drawer has meant it is important for example to join the teacher's and the pupils' comments in each drawing and not just mechanically code the teacher's activities. Furthermore, in order to avoid including the coder's own interpretations we think that it is important that at least two persons are doing the classification. Pupils did the drawing task in the beginning of the third school year. One of the researchers (LN) followed once a month a mathematics lesson from each of nine teachers. According to her the general overview obtained from the drawings in the corresponding classes is in accordance with her observations.

Pupils' drawings reveal important information on the pupils' behaviour that is difficult to be obtained from young children using more conventional methods (cf. [13]). Especially by connecting words and images the drawers reflect their feelings and attitudes towards their teacher, other pupils and situations. They also express the group values that are prevalent within their specific environment. Thus the two inventories in Tables 1 and 2 give us a tool to find out how young pupils see the teachers' as well as the pupils' activities in mathematics lessons. Thus it gives the researchers and school authorities a possibility to have a look what is happening in classrooms. It also gives a possibility to compare for example the changes between different grades, and also between different teaching systems even between different countries. The drawings contain rich information from which one can select an interesting part according to the purpose.

An analytical system PAIR uses drawing as a tool to access children's ideas [25]. PAIR assumes only that the child knows something about the social world and can express this pictorially. According to PAIR drawing offers children an alternative means of communication, it is familiar to and enjoyed by most children, and it sets very little restrictions on children's freedom of expression. It is based on the children's ability to create detailed and flexible models of social experiences, and can be reliably applied to subjects between the ages of 6 and 14.

Altogether, pupils' drawings seem to be a versatile way to collect information about teachers' and pupils' activities in mathematics lessons. Children should be told as clearly as possible what they should draw [25]. In this study, the drawing instruction given to the pupils was quite open, thus there is a large variability among the drawings. In some classes there was a big variation in the teacher's and the pupils' activities. There are, however, several other factors that may influence the drawings pupils will produce. Some pupils may have difficulties to draw complicated pictures and, therefore, they might draw only such objects or situations that are easy for them to draw. They have used, for example, stick figures or have written only their classmates' name on the desk. Also the fact how the task is given may have an effect on how the pupils will concentrate on working and how they understand what they are expected to do. In our case the pupils were motivated in working as they were told that the drawings will be part of an international research.

One has to be careful in making definite conclusions from the drawings. Different pupils pay attention to different happenings. Our inventories will cover most teachers' and pupils' activities in Finland, because the pupils are studying in heterogeneous groups, the primary school core program is virtually identical for all the pupils, and less than 10% of the variation of mathematics scores in PISA or TIMSS can be explained by the school variables [26]. Also the standard of the Finnish primary teachers is very high; the students in the primary teacher education take a higher academic degree, with their main subject in education. Furthermore, only some 10-15% of the applicants can be admitted to the programs [27]. However, one has to notice also that there are so many different ways for a teacher or pupils to act that our inventories may not include all the possibilities. This means that when this analyzing method is used in other countries, these inventories should be checked in each new case.

References

[1] M. K. Stein, R. A. Engle, M. S. Smith and E. K. Hughes, Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move beyond Show and Tell, *Mathematical Thinking and Learning* **10** (2008), 313–340.

[2] A. B. Smith and T. L. Glynn, Contexts for Boys and Girls Learning Mathematics: Teacher Interactions and Student Behaviour in two Classrooms, *New Zealand Journal of Psychology* **19** (1990), 9–16.

[3] P. Chaviaris and S. Kafoussi, Developing Students' Collaboration in a Mathematics Classroom through Dramatic Activities, *International Electronic Journal of Mathematics Education* **5** (2010), 91–110.

[4] K. W. Kosko and J. M. L. Wilkins, Mathematical Communication and Its Relation to the Frequency of Manipulative Use, *International Electronic Journal of Mathematics Education* **5** (2010), 79–90.

[5] S. M. Kosslyn, Aspects of a Cognitive Neuroscience of Mental Imagery, *Science* **240** (1988), 1621–1626.

[6] S. M. Kosslyn, *Image and brain*, Cambridge, MA, Mitt Press, 1996.

[7] D. Pitta, *Beyond the obvious: Mental representations and elementary mathematics*, Unpublished PhD, Mathematics Education Research Centre, University of Warwick, 1998.

[8] R. De Beni and F. Pazzaglia, Memory for Different Kinds of Mental Images: Role of Contextual and Autobiographic Variables, *Neuropsychologia* **33** (1995), 1359–1371.

[9] R. De Beni, F. Pazzalgia and S. Gardini, The Generation and Maintenance of Visual Mental Images: Evidence from Image Type and Aging, *Brain and Cognition* **63** (2007), 271–278.

[10] D. Zambo and R. Zambo, Using Thought Bubble Pictures to Assess Students' Feelings about Mathematics, *Mathematics Teaching in the Middle School* **12** (2006), 14–21.

[11] S. Weber and C. Mitchell, Drawing Ourselves into Teaching: Studying the Images that Shape and Distort Teacher Education, *Teaching and Teacher Education* **12** (1996), 303–313.

[12] S. C. Losh, R. Wilke and M. Pop, Some Methodological Issues with 'Draw a Scientist Tests' among Young Children, *International Journal of Science Education* **30** (2008), 773–792.

[13] S. H. Picker and J. S. Berry, Investigating Pupils' Images of Mathematicians, *Educational Studies in Mathematics* **43** (2000), 65–94.

[14] M. Bulut, Curriculum Reform in Turkey: A Case of Primary School Mathematics Curriculum, *Eurasia Journal of Mathematics, Science & Technology Education* **3** (2007), 203–212.

[15] W. T. Seah, Qualities co-valued in effective mathematics lesson in Australia: Preliminary findings, in: *Proceedings of the 31st Conference of the International Group*

for the Psychology of Mathematics Education, Vol. 4, (J. H. Woo, H. C. Lew, K. S. Park and D. Y. Seo, eds.), Seoul, PME, 2007, 161–168.

[16] A. Dahlgren and L. Sumpter, Children' Conceptions about Mathematics and Mathematics Education, in: *Proceedings of the MAVI-16 conference June 26-29, 2010*, (K. Kislenko, ed.), Tallinn, Estonia, Tallinn University of Applied Sciences, 2010, 77–88.

[17] N. Norris, R. Aspland, B. MacDonald, J. Schostak and B. Zamorski, *An independent evaluation of comprehensive curriculum reform in Finland*, Helsinki, National Board of Education, 1996.

[18] P. Andrews, A. Ryve, K. Hemmi and J. Sayers, PISA, TIMSS and Finnish Mathematics Teaching: An Enigma in Search of an Explanation, *Educational Studies in Mathematics* (2014), DOI 10.1007/s10649-014-9545-3.

[19] L. Pehkonen and H. Krzywacki-Vainio, Mathematics Teaching in Primary Schools, in: *How Finns Learn Mathematics and Science*, (E. Pehkonen, M. Ahtee and J. Lavonen, eds.), Rotterdam, The Netherlands, Sense Publishers, 2007.

[20] E. Pehkonen, M. Ahtee, A. Laine and P. Tikkanen, Pupils' Conceptions on Mathematics Lessons Revealed via their Drawings, in: *Current State of Research on Mathematical Beliefs XVII: Proceedings of the MAVI-17 Conference*, (B. Rösken and M. Casper, eds.), University of Bochum, 2011, 182–191.

[21] A. Laine, L. Näveri, M. Ahtee, M. Hannula and E. Pehkonen, Emotional Atmosphere in Third-graders' Mathematics Classroom - an Analysis of Pupils' Drawings, *Nordic Studies in Mathematics Education* **17** (2013), 101–116.

[22] L. Näveri, A. Laine, M. Ahtee, E. Pehkonen and P. Tikkanen, Kolmas-luokkalaisten matematiikan oppituntipiirrosten yhteys laskutaitoon [The Connection of Third–graders' Drawings on Mathematics Lesson to Calculation Skill], in: *Lahja yhteiseksi hyväksi, Professori Eira Korpiselle omistettu juhlakirja [Gift for common good, Celebration book for professor Eira Korpinen]*, *Journal of Teacher Researcher 1*, (P. Tikkanen and E. Törmä, eds.), Jyväskylä, Tutkiva opettaja - yhdistys ry (2014).

[23] L. Varas, E. Pehkonen, M. Ahtee and S. Martinez, Mathematical communication in third-graders' drawings in Chile and Finland, in: *Proceedings of PME-36*, Vol. 4, (T.-Y. Tso, ed.), University of Taiwan, 2012, 195–202.

[24] L. Hart, E. Pehkonen and M. Ahtee, Young Students Drawings Reveal Communication and Beliefs About Mathematics Class in Finland and the USA, *Research in Mathematics Education* **16** (2014), 18–37.

[25] A. S. Bombi, G. Pinto and E. Cannoni, *Pictorial Assessment of Interpersonal Relationships (PAIR). An analytical system for understanding children's drawings*, Firenze University Press, 2007.

[26] P. Kupari, P. Reinikainen and J. Törnroos, Finnish students' mathematics and science results in recent international assessment studies: PISA and TIMSS, in: *How Finns Learn Mathematics and Science*, (E. Pehkonen, M. Ahtee and J. Lavonen, eds.), Rotterdam, The Netherlands, Sense Publishers, 2007.

[27] A. Laine and R. Kaasila, Mathematics education in primary teacher program, in: *How Finns Learn Mathematics and Science*, (E. Pehkonen, M. Ahtee and J. Lavonen, eds.), Rotterdam, The Netherlands, Sense Publishers, 2007.

M. AHTEE, E. PEHKONEN, A. LAINE, L. NÄVERI, M. S. HANNULA
UNIVERSITY OF HELSINKI
HELSINKI, FINLAND

E-mail: maija.ahtee@helsinki.fi

E-mail: Erkki.Pehkonen@helsinki.fi

E-mail: Anu.Laine@helsinki.fi

E-mail: Liisa.Naveri@elisanet.fi

E-mail: Markku.Hannula@helsinki.fi

P. TIKKANEN
UNIVERSITY OF JYVÄSKYLÄ
JYVÄSKYLÄ, FINLAND

E-mail: pirjo.tikkanen@hirspek.fi

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