

Teaching Probability Theory by using a Web Based Assessment System together with Computer Algebra

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Abstract. In the course of Maths Basics 2, the Faculty of Economic Science students of Kaposvár University learn the classical chapters of Probability Theory, namely random variables and the well-known probability distributions. Our teaching experiences show that students' achievement is weaker in case of problems concerning continuous random variables. From school year 2012/13 we have had an opportunity to take Maple TA, the web-based test- and assessment system, into the course of education. It is sufficient for the users of Maple TA to have a browser. Maple computer algebra system, which runs on the server, assesses students' answers in an intelligent way, and compares them with the answers that are considered correct by the teacher. In our presentation we introduce some elements of Maple TA system, the didactic considerations the test sheets were made by, as well as our research results concerning the use of Maple TA.

Key words and phrases: learning mathematics using computer, representations of knowledge levels, calculus, teaching probability theory, Maple Computer Algebra System, Maple Test and Assessment system.

ZDM Subject Classification: B40, D40, I50, K50, N80.

1. E-learning, IKT in education

The Canadian Tony Bates, an acclaimed researcher and theorist of distance education and information technology (University of British Columbia), investigated the thrift of e-learning, while confronting the expectations concerning e-learning with practice and development trends. He found that the introduction of the big e-learning frameworks (WebCT, Blackboard) is still very expensive,

their integration in administration is also costly and wearisome - only few institutions of education have decided in favour of using it. Several minor institutions choose an in-house development framework or a smaller one.

Applying advanced technologies in order to support interesting learning situations is on the borderland of research and education.

An essential question is whether students have an access to computers and to the Internet. In developed countries this condition is fulfilled in case of middle-class users, but for those who are in worse social circumstances, the digital gap means a huge problem.

According to last years' researches the main difficulties lie in the high IKT-costs, and in the lack of basic knowledge and home infrastructure. In Hungary, internet service is still very expensive compared to wages, the necessary infrastructure is not to be found, although more and more institutions are able to develop their IT structure in the framework of EU tenders. [6]

However, as opposed to the previous expectations, e-learning is realized in the form of a slow development rather than of a revolutionary change. According to the opinion of the critical part of professionals, the quality and the cost-effectiveness of education through the net is not proved yet, the reasoning propagating e-learning often goes beyond realities. Connecting systems (network resources) seems to be more difficult than was thought. It is still uncertain whether the technology is able to change teaching methods of teachers and learning methods of students. [6]

The economic viability of electronic distance education is also questionable: the often quoted cost-effectiveness is a complex question, and it can be justified hardly in general-, and partly in particular educational situations. It is worth remembering that there were no e-learning experts - everybody started to practice this area as a beginner, may they have arrived either from the area of IT or education.

Between 2000 and 2002 Hungary's e-learning classification is average or worse. [6]

Howell, Williams and Lindsay's research concerning distant education takes 32 important trends into consideration, the most relevant parts of their list are: increasing interest in higher education; choice of courses suitable for the interests of students and the environment; shifts in the traditional faculty roles; extension trainings for faculty-workers; increasing number of profit-based institutions; fusion of traditional institutions; decentralisation of training structures; the fact

that education and training are becoming more open; shifting of stress of trainings towards competencies; need of knowledge of IKT-tools. [4]

Decision-makers in education have to be aware of the economical and political trends, as well as the rapidly developing technologies, the demand for trainings and training types, and of course the needs and interests of students. While trends themselves are not sufficient to find the solution, the decision-makers' quick reaction, the adoption of well-proved trends may considerably contribute to the gradual shaping of institutes, may insure their competitiveness, their survival. [4]

2. Experiences in using Maple TA during the process of preparations

We use the computer algebra system Maple, the dynamic geometric and algebraic teaching and learning system GeoGebra, and Excel spreadsheet in Calculus lessons for illustration and for solving a diverse range of problems. The tools of Statistics Package of Maple provide an excellent opportunity for the discussion of Probability Theory. The declaration of random variables with a given distribution is possible there. This is especially useful in teaching continuous random variables, because it can determine the distribution of random variables obtained as the result of operations. The only disadvantage of Maple is that it requires programming, and our students do not have such preliminary training. That is why Maple TA, the Maple Testing and Assessment, which is built on Maple, is particularly useful; it is used in education for assessing knowledge, and does not need direct mastery of programming tools. At the same time the teacher can use Maple to assess the answers.

Maple TA is an easy-to-use web-based system. It supports compiling tests, assignments, exercises. The system automatically assesses students' responses and summarizes their performance. After the web-based entering, the user can see mathematical formulae, equations and figures in the same way as he is accustomed to in the test-books. A limited degree of 2D formula-entering can also be used. The system assesses the answers in an intelligent way, which means that the Maple program written by the teacher checks the answer in the background. Thus, the tool is ideal for those courses of science, technology, engineering in which teaching mathematical topics is unavoidable.

University of Guelph Ontario (Canada) is a long-standing user of Maplesoft products. Recently they made an experiment to statistically determine the effect of Maple TA on students' performance. They chose students of economy for the

experiment. Half of the students were randomly chosen to use Maple TA, the other half used the traditional tools. 20% of the final grade was made up from the assignments using Maple TA and other tools. Instructors experienced that in the group of students using Maple TA the drop-out rate was lower than in the group using conventional tools.

“Besides being an excellent learning tool for students, Maple T.A. is a wonderful aid for teachers too,” said Dr. Sadanand, professor of the university in the Department of Economics and Finance. [10] It frees instructors’ and teaching assistants’ time, makes work more productive. “We can provide more personal attention to students which was limited due to time constraints. The technology in Maple T.A. also helps us to delve deeper into the students’ thought processes and see how they approach their problems and assignments.” Powerful, algorithmically-generated questions can be created in Maple T.A. One question template may generate hundreds or thousands of similar questions for students. We can provide individual homework assignments and lots of practice questions for students. “This is one of Maple T.A.’s biggest strengths,” commented Dr. Sadanand. “It is a time-saver and an extremely useful tool for students and teachers.” [10]

The institute of University of Waterloo came across a much too well-known problem: the increasing number of students and decreasing budget. As students are frequently tested in mathematics, assessment is a great burden on instructors. The staff of the faculty was not willing to decrease the frequency of tests at the expense of quality, so they had to find another solution. They decided to automatize assessment on their mathematics-based courses, so they introduced Maple TA. 9000 students use this tool in a year (there are such periods of time when 800 students are simultaneously using the system). The quality of education improved with its use, and students get an immediate feedback. [11]

3. Maple TA in Hungary

In academic year 2012-2013, within the framework of TÁMOP tender with reference number 4.1.2.A/1-11/1-2011-0098, described as “E-learning based course material and methodology development of basic mathematics skills for engineering and economic trainings”, our university obtained the professional computer algebra system, Maple 16 version, and the web-based testing and assessment system, Maple TA 8.5 version. In our country four institutes of higher education are

testing the introduction of Maple TA into education: the University of Debrecen, the University of Pécs, Kaposvár University, and Eszterházy Károly College.

Mathematics instructors of the four institutes participated at courses where they acquired the use of the basic functions of software Maple 16. In the above mentioned project, version 8.5 of Maple TA was purchased. The acquirement of the web-based testing and assessment system and an introduction into question-editing were the topics of the 2-2-day courses, one of which was led by the instructors of The Vienna University of Technology. They reported on their experiences, and their well-proven editing practices.

The main problem of the system appears when the teacher starts question-editing unaided. At first a great amount of time is needed to edit an exercise or a question. The accomplished exercise must be tested several times because of the random generating of variables, to find the occasional mistakes, and then eliminate them.

There are only 40-50 first-year, full time students of Finance-Accounting and Trade-Marketing at The Faculty of Economic Sciences of Kaposvár University. This number would not justify the introduction of the system; we still think it is important to get it into the teaching-learning process, because these students can reach several exercises, they do not have to buy the exercise books.

Introducing Maple TA was a great challenge. At the beginning much time had to be expended on producing assignments. In the spring semester we “learned together” with the students, they studied how to solve problems, and we studied how to make exercises. We kept in touch with students on Facebook; the forum and chat functions offered by Coospace did not work, as students did not use it. The system of relationships between students, instructors and Maple TA can be seen below:

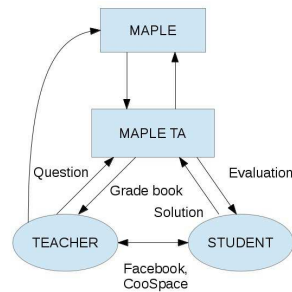


Figure 1. Relationships between the instructor and students in the use of Maple TA (Source: self-editing)

The most important element of the system of relationships is that Maple TA does not solve problems, but sends them to Maple on the university server; it computes and sends for assessment to Maple TA, and this assessed answer is seen by the student. That is why the student does not have to own the Maple software, only an entrance code to the TA server. Syntax is shown to them at seminars, thus entering the correct answer causes few problems. Maple helps the instructor's work in editing questions, in converting commands into one dimension, and in making the graphs of functions. It is emphasized that the teacher has to find the solutions of the problems with the help of Maple, instead of calculating them explicitly. It would be impossible because by generating random parameters hundreds of solutions should be given. [5]

When students got stuck in solving the problems, we used the chat function of Facebook.

4. Question editing in Maple TA

The following types are offered by the Question Designer: Adaptive Question Designer, Clickable imagemap, Essay, Fill in the blanks, Maple -graded, Matching, Mathematical formula, Multipart question, Multiple choice, Multiple selection, and Numeric. When compiling practice sheets we paid attention to include as many of them as possible. The following standard problem often occurs in tests and exams:

EXERCISE 4.1. *Determine the value of parameter a such that function $f(x)$ be the density function of a continuous random variable [2].*

$$f(x) = \begin{cases} a \cdot (x^2 - x), & \text{if } -1 < x < 0 \\ 0, & \text{otherwise} \end{cases}$$

The majority of our students solve this kind of problem insufficiently, defectively, or they do not solve it at all. That is why we chose to assign this type of problem in Maple TA.

The strength of Maple TA is that 100 exercises may also be produced from one question with variables and randomly generated parameters. Searching question banks I did not find exercises related to probability theory, thus we had to invent suitable questions.

When making test exercises we have to think along different didactic viewpoints than in case of the conventional test-making. The majority of our students

got accustomed to click simply on the right answer in case of questionnaires. Thus, we had to keep in mind during the experiment that we could assign only those types of exercises that require relatively little time but still can measure the acquirement of the material. When the result is numerical, we have to give the students guidance in which form the real numbers have to be entered.

The exercise mentioned above is considered difficult. Its generalization for Maple TA:

EXERCISE 4.2. *Determine the possible value of parameter c , such that function $f(x)$ be the density function of a continuous random variable.*

$$f(x) = \begin{cases} cx - d, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$$

Four parameters occur in the exercise, three of which, a , b and d are generated randomly, while c has to be determined by the student. For the domains of parameters a and b , which mean the bounds of integration, we defined disjoint sets of integers, and every member in the domain of parameter a is less than the elements in the domain of b . One of the clues for the solution is the property of the density function, that its improper integral from $-\infty$ to $+\infty$ is 1. Using this property we get a calculation scheme for c :

$$c = \frac{[1 - d \cdot (b - a) \cdot 2]}{(b)^2 - (a)^2}$$

(Names of algorithmic variables in Maple TA start with character \$.) In many cases students concentrate only on this condition, they perform integration, state the equation and accept the result without checking. At first we made the same mistake, but the next exercise built on this, where the variance had to be calculated, which turned out to be a negative number, highlighted this error. Thus, a plus condition had to be installed: the density function has to be non-negative.

[5]

Using the above results, the algorithm of the problem is:

```
$a=range(-3,4);
$b=range(5,10);
$d=range(-5,0);
$condition=maple("($d)*(($b)-($a))^2+2*($a)");
condition:gt($condition,0);
$da=maple("abs($d)");
$f=maple("f(x)=piecewise(($a <= x) and (x <= $b), c*x+($d), 0)");
$fnice=maple("MathML:-ExportPresentation($f)");
$c=maple("2*($d)*(($a-$b)+1)/($b*$b-($a)^2)");
```

Our students still had difficulties with the problem, so we installed hints:

Hint 1: $\int_{-\infty}^{\infty} f(x)dx = 1$ and Hint 2: $\int (a \cdot x + b)dx = a \cdot \frac{x^2}{2} + b \cdot x + c.$

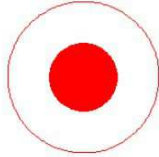
After this there were students whose results were correct.

EXERCISE 4.3. *Geometric probability helps introducing the concept of continuous random variables.* [9] *We constructed questions of this type as well:*

Question:

The figure shows a circular target. Given that the dart will always land within the circle, find the radius of the red area, if the radius of the target is 4 ,

and the probability of hitting the inner circle is 1/5.



The radius of the red area:

```

$dr=range(3,12);
$denominator=range(2,5);
$P=maple("1/$denominator");
$R=maple("$dr*sqrt($P)");
$Rnice=maple("printf(MathML:-ExportPresentation($r))");
$graphicgood=plotmaple("plots[display]([plot([$dr*cos(t), $dr*sin(t), t = 0 .. 2*Pi]),plot([$R*cos(t), $R*sin(t), t = 0 .. 2*Pi],filled=true,color=red)],axes=None),plotoptions='height=200,width=200'");

```

Figure 2. Question and algorithm in the topic of geometric probability
(Source: self-editing in Maple TA)

The following question is not difficult, but students have to use their theoretical knowledge.

EXERCISE 4.4. Determine of the variance [1]

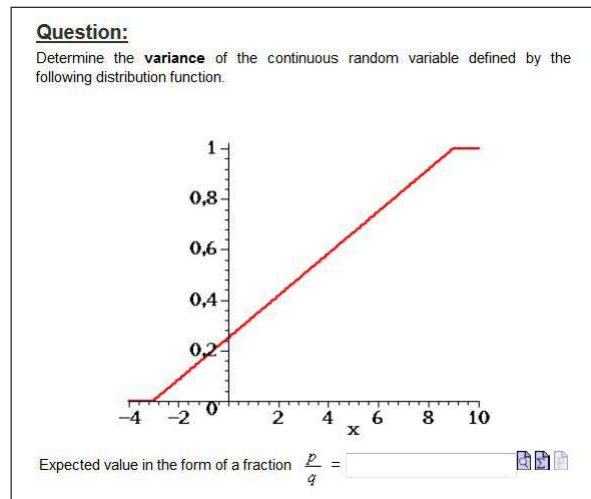


Figure 3. Question for practicing rectangular distribution in Maple TA (Source: self-editing in Maple TA)

There was one student whom I helped by leading him through the solution on the Facebook chat:

T: Which probability distribution has a distribution function like this? If we have found it, how do we calculate its variance?

S: Rectangular distribution?

T: That's it!!! How do you calculate the variance of a rectangular distribution? It is on your formulae list.

S: A square of D equals to $(b - a)$ square divided by the square root of 12.

T: Without the square root, only divided by 12.

S: Oh yes, the square root is not needed

T: Read the value of b and a then calculate it. It must work.

S: It's OK now.

EXERCISE 4.5. *In the exercise concerning exponential distribution students had to pay attention to the way of giving the answer:*

Question:

The expected lifetime of a calculator is 1 602 hours, the lifetime is an random variable with exponential distribution.
Determine the probability $P(x < 4806)$

Give the result on the basis of the definition of the distribution function. Function e^x is denoted by $\exp(x)$




P=   

Figure 4. Question of exponential distribution in Maple TA (Source: self-editing in Maple TA)

EXERCISE 4.6. *Distribution function properties 1 [8]*

Question:

Choose the properties of the distribution function of a continuous random variable.

- Its limit at negative infinity is 0.
- Its limit at positive infinity is 1.
- Its range is the interval $[-1;1]$
- Its values are non-negative.
- It is continuous at every point.

Figure 5. Question on distribution function properties (Source: self-editing in Maple TA)

Most students could not find the four properties of the distribution function. They have to apply this knowledge when the distribution function is determined from the density function. 10 – 30% of the students are able to solve exercises of such type; that is why we thought it important to emphasize the properties of the distribution function and to install them in the test.

When teaching Probability Theory we put emphasis on being picturesque, on familiarising methods of solution and on teaching the density- and distribution functions of special distributions (they are expected to recognise the distribution from their density- and distribution functions).

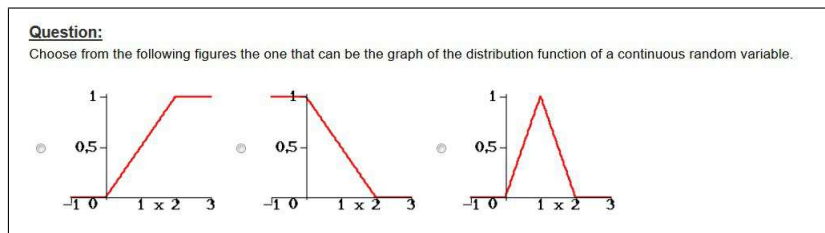
EXERCISE 4.7. *Distribution function-graph*

Figure 6. (Source: self-editing in Maple TA)

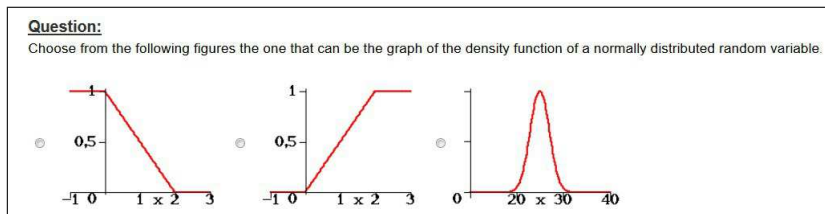
EXERCISE 4.8. *Density function-graph (normal distribution)*

Figure 7. (Source: self-editing in Maple TA)

EXERCISE 4.9. *Distribution function-definition- this is an evaluated answer (Fig. 8)*

We compiled assignments and test sheets from the questions. We just picked up the questions, letting the permutation of them. After solving the test sheet, the student gets an immediate feedback about his good and defective solutions.

In the Gradebook the teacher can see statistics about the solutions of test sheets, as well as the answer a given student gave to a question, the number of cases he solved the test and the amount of time he spent with the solution.


Choose the correct definition of the distribution function of a continuous random variable, where $f(x)$ denotes the density function.

a) $\int_{-\infty}^x f(t) dt$

b) $f(x) \cdot x$

c) $\frac{d}{dx} f(x)$

d) $\int_0^x f(t) dt$

 Correct

Choice	Selected	✓/✗	Points
b	No		
a	Yes	✓	+1
c	No		
d	No		
the good answer is not here	No		

Figure 8. Evaluated answer (Source: self-editing in Maple TA)

5. Results

The experiment aimed at finding the answer to the questions of whether applying Maple TA in the teaching-learning process increases students' performance, and whether its use makes students study regularly.

We had taught the subject Maths Basics 2 traditionally, using the paper-pencil method. Half of the group, which consisted of better and weaker students as well, used Maple TA for practice.

The practice sheets contain exercises that cause difficulties to students according to our many years' educational practice. We aimed at improving students' performance in these problems.

Students had the best performance in recognizing the graphs of distribution functions; this was followed by the question regarding the knowledge of the properties, then the question concerning the definition of the distribution function. Their performance was the worst in case of the above mentioned parametric exercise relating to density functions. We could observe that finding the expected value with a given density function showed a 20% better performance.

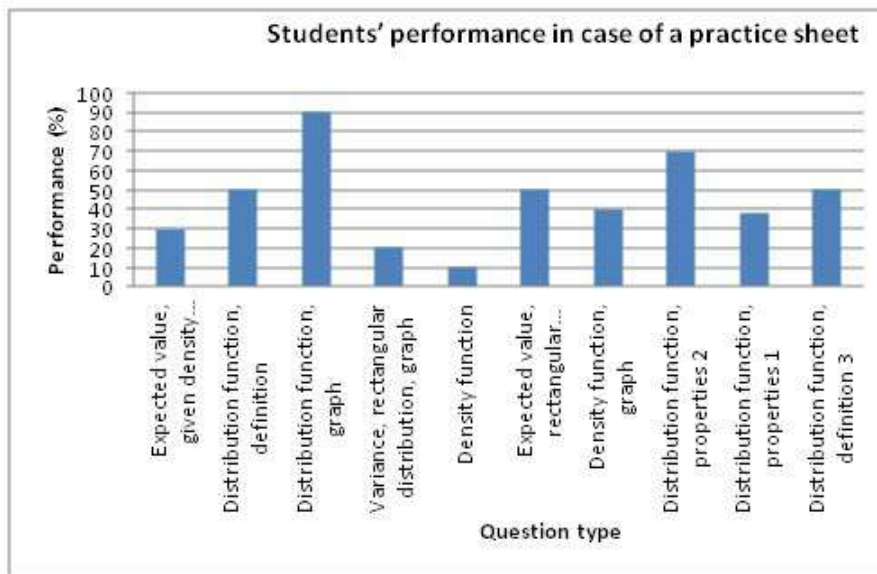


Figure 9. Students' performance in case of a practice sheet Source: self-editing according to students' solutions

By choosing different types of questions we can compile more diversified practice sheets.

There were 15 students in my group of Finance and Marketing in the academic year 2012/2013, 53% of them took upon themselves to practice problem solving in Maple TA. These students were the ones who had difficulties with the traditional problem solving. After the second test we can sum up the following results:

20% of the students had one mark better than their previous one, 26,6% of the students had the same mark, and 6,6% of the students got worse marks.

It turned out that those 20% of the students who improved their mark had practiced Maple TA 3-4 times a week, the other 26,6% just occasionally, and the students with the worse results "had not been allowed" to enter the system. The students who had improved their mark could pass the exam on the first try, although they had failed twice during the first semester. According to their report, Maple TA helped them and they liked using it. The teacher cannot be neglected; he or she took part in the course of teaching-learning as a guide, with helpful questions.

6. Consequences, suggestions

In the test sheet discussed, the average performance is 44,75%, the standard deviation is 23,29%, thus 60% of all data fall within 1 standard deviation of the mean. Due to the low number of the participants of the experiment at Kaposvár University, this can only be considered as a “try-out” (the low number of students is given): the sample was small and not representative (not every student was chosen with the same probability, as they voluntarily chose to participate and solve exercises in Maple TA).

Despite the low number of students we consider the experiment to be useful and successful. Students and teachers could work in cooperation. The teacher learned how to construct questions, the student gave an immediate feedback while solving the problem, and exercises could be improved.

By constructing a suitable amount of questions and practice sheets we can accustom students to prepare and study more regularly. They might as well achieve higher performance by solving one practice sheet every week. Thinking through the questions makes their knowledge more systematic.

Our aim for next academic year is to use Maple TA in both of our bachelor’s courses, in the course of Maths Basics 2. One of the issues of the experiment is to find out whether we can get students to study regularly with the use of Maple TA, and whether their results will improve owing it.

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