12/1 (2014), 55–70
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Teaching Mathematics and Computer Science

Recalling Calculus Knowledge

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Abstract. The main purpose of educational system is not only that the students perform well at the exam, but to remember the learnt material to some degree some time after the learning. This paper investigates students' retained knowledge, focusing mainly on topics concerning derivatives and differentiation, and examines the effect of re-learning in a short period of time. Results indicate that retained knowledge should be taken into consideration in instructional design and curriculum planning for the sequencing courses.

Key words and phrases: calculus, retention, knowledge, re-learning.

ZDM Subject Classification: I40, B40, C70.

1. Introduction

Mathematics is very specific in its nature. It is a cumulative, vertically structured discipline with new concepts building upon previous concepts. This characteristic of mathematics describes the purpose of the educational system where the emphasis is not only placed on students' performance in the exam, but is also connected with retaining the learnt material to some degree some time after the learning has finished. The knowledge gained in some mathematics course becomes prior knowledge in the sequencing or related courses, and it forms the knowledge base necessary for developing new knowledge. However, prior mathematical knowledge is not used only in mathematics courses but also in many science and engineering curriculums, which are building upon this knowledge. Therefore, it is not enough to assess students' knowledge and skills only at the end of the course, but it is also important to examine quality of the retained knowledge when starting a new course. The retained knowledge influences on learning and students' achievement in both positive and negative ways. Good quality retained knowledge has a positive effect on new knowledge acquisition whereas inadequate or fragmented retained knowledge may result in rote memorization and surface learning when dealing with new material [1].

Moreover, investigating knowledge retained after passing a course is an important issue to consider in several ways. It shows the effectiveness of the teaching methods and learning environment, the quality of students' learning and can predict students' performance in advanced courses [2]. Also, investigating the retained knowledge from earlier course in some sequencing course is very helpful for educators. If there is a mismatch between the teachers' expectations of students' knowledge and the students' actual knowledge base, learning may be hindered from the start of the studies. In this study, we are particularly interested in examining the durability and quality of calculus skills and knowledge gained in the first year calculus course.

2. Theoretical background

2.1. Forgetting and retention of knowledge

The processes such as memorizing and remembering happen in human cognitive architecture, constituted from working memory and long-term memory. In order to successfully use material after the learning process has finished, new information that is entering in working memory must be adequately integrated in pre-existing schemas in the long-term memory [3]. Although we are forgetting knowledge and skills that we do not use over time, proper coding and integration of new information enable recalling or remembering pieces of knowledge, processes, or skills that were learned earlier in time [4]. Thus, the retention of knowledge can be described as the extent to which someone can successfully access and use the information from the long-term memory [5].

The long-term retention of mathematical knowledge is one of the major goals of the educational system; it is not expected that a student immediately remembers an answer when asked, but to be able to reconstruct the knowledge with little effort using some hints. This can be regarded just as good as retention and is in line with a discussion of Karsenty [6], where it is argued that recalling some mathematical knowledge is a reconstruction process that yields to an altered version, in contrast to a reproduction, where details are coded in memory, and they re-appear as so-called 'copies'.

Semb et al. [7] showed that sometimes students remember a great deal of what they learn in college courses (of psychology). Particularly, students who have served as tutors retained more knowledge four months after the course than the students they tutored. This suggests that tutoring, which is a type of overlearning, positively affects the long-term retention. Overlearning is common for mathematics teaching, and closely connected with massed practice which is frequent in university courses. However, studies of Roher & Pashler [8] and Roher & Taylor [9] showed that overlearning and massed practice do not benefit long-term retention of knowledge in mathematics.

Forgetting is a natural process for every person, and not a failure of memory. It is necessary for the efficient functioning of memory since it is impossible to remember all learned information and retain free access to them [10]. This would severely impair a person's ability to learn and recall information that is new and currently relevant. Anderson et al. [11] noticed that the act of remembering may cause forgetting. Remembering a certain item increases the likelihood that it will be recallable again at a later time, but the other items that are associated to the same cue for retrieval become very sensitive to forgetting. But there is a way to influence on forgetting, and that is re-learning. A study by Storm et al. [12] gave very surprising results from a common-sense standpoint. It showed that re-learning is very beneficial indeed, but the items that were relearned benefited more from re-learning if they had previously been forgotten. This result provided evidence that forgetting is not always bad itself - sometimes it is an enabler of future learning, creating conditions that enhance the effectiveness of learning.

2.2. Procedural and conceptual knowledge

Two essential types of knowledge that students acquire through mathematics education are conceptual and procedural knowledge. Conceptual knowledge is the type of knowledge which is rich in relationships and provides an understanding of the principles and relations between pieces of knowledge in a certain domain. Procedural knowledge is what enables us to quickly and efficiently solve problems. This knowledge focuses on skills, consists of sequence of actions and can be learnt with or without meaning [13]. Here, one might argue that knowledge, of any kind, learnt without meaning is not knowledge at all. Referring to Illeris [14], there exists the so-called cumulative learning (mechanical learning) which is an isolated formation, something new that is not attached to something one already knows. Procedural knowledge without meaning (i.e., learnt mechanically by heart) we argue is therefore an example of this. One might also ask if procedural knowledge learnt with meaning is not in fact conceptual knowledge. In this discussion, we would argue that it is not the "depth" of knowledge that determines if it is procedural or not, but instead what it is knowledge about.

Results of many studies showed that procedural knowledge in mathematics is very sensitive, meaning that it is quickly forgotten or remembered inappropriately (e.g. [15, 16]). Engelbrecht et al. [17] investigated students' knowledge in basic techniques from a first-year calculus course. Comparing their results from the pre-test and the post-test given two years after the instructions, they found a significant decline in students' performance.

The effect of the teaching style on the retention of mathematical knowledge has been investigated in a small number of studies with different outcomes. Studies examined durability of procedural and conceptual knowledge days, months or years after instructions. Some studies showed that students from the student-centered courses retained conceptual knowledge better than procedural, and students from teacher-centered courses retained procedural knowledge better than conceptual (e.g.[18, 19]). Other studies showed that students from student-centered courses retained conceptual knowledge better than procedural one, but they had retained procedural knowledge at an equal level as those students coming from teacher-centered courses [15].

The long-term retention of the students' knowledge is not always linked with the actual course grade. In a study by Jukić & Dahl [20], the students' retention of procedural and conceptual knowledge was examined at one Croatian and one Danish university two months after the students had passed a similar Calculus 1 course. The obtained results showed that for both countries a large portion of knowledge was forgotten and that the passing grades of the Calculus 1 course did not predict the results in the test two months later. In fact, often students with the lowest passing grades had the better results two months later, or there was no difference.

2.3. Intended learning outcomes

In order to properly evaluate the long-term knowledge of calculus, we take the intended learning outcomes (ILOs) of Calculus 1 as a starting point. The ILOs describe what students will be able to do with gained knowledge upon completing the course. This calculus course covers topics of functions review, sequences, limits, continuity and derivative. In this section we will briefly describe the ILOs for the calculus course taken by our sample of students.

The successful student should be able to apply the following competencies to many important classes of functions of one variable, including polynomials, rational, algebraic, trigonometric, inverse trigonometric, exponential and logarithmic functions:

- to differ and provide typical examples of convergent and divergent sequence of real numbers, continuous and discontinuous real functions of one variable, differentiable and not differentiable real functions of one variable;
- to calculate limit of sequences of real numbers, and limits and derivatives of real functions;
- to approximate value of derivative at a point given a table of function values;
- to recognize conditions on the functions that allow application of the basic theorems of differential calculus and give appropriate geometric interpretation; and
- to interpret application of calculus in simple optimization problems.

2.4. Research questions

It should not be expected that students have the same level of knowledge several months after the exam as they had immediately after the exam in terms of details, but it should be expected that students have retained some basic skills and knowledge. Therefore, we formed following research questions: (1) What calculus skills and knowledge have students retained three months after the exam? and, (2) Can the students, with a limited effort, recall/reconstruct their calculus skills and knowledge, and to what extent?

3. Methodology

3.1. Participants and context

We have found 20 students who were willing to participate in our study. The participants belonged to the electrical engineering study program from one Croatian university, and had obtained various grades in the calculus course. This means that participants represented poor, average and good achieving students. The calculus course in the electrical engineering study program is divided into three courses - Calculus 1, 2 and 3. These are enhanced mathematics courses where elements of mathematical analysis are added to emphasize the theoretical background. The courses consist of lecture and exercise lessons where the teaching approach is teacher-oriented. Lectures are given in a traditional form to a large group of students and emphasize conceptual knowledge. Exercises are organized in groups of 30 students and based mostly on direct instructions and on individual work. A problem-solving or performance of procedure is shown to the students, and conceptual ideas are taught in the context of procedural methods. In order to obtain a passing grade, students have to pass both a written and an oral exam. Besides solving tasks, students' knowledge in the formal mathematical theory is also examined.

Knowledge from Calculus 1 is a prerequisite for the course Calculus 3, which deals with function of several variables. Hence the students' retained knowledge from the first course is in fact the prior knowledge for the next calculus course. This motivated us to design and conduct a study which consisted of two parts: the examination of students' retained knowledge after passing Calculus 1 and the experimental part of knowledge refreshment.

3.2. Evaluation of knowledge

Students' knowledge was evaluated before and after a knowledge refreshment using tests with an array of mathematical items. The test given before knowledge refreshment will be called a pre-test, and the test given after knowledge refreshment will be called a post-test. The pre-test was given to the students three months after the Calculus 1 exam, and it examined what knowledge students had retained, i.e., which learning outcomes were forgotten. After a repetition of certain calculus topics, students were given the post-test where we examined the effect of re-learning on their knowledge. The pre-test and the post-test were given to the students two weeks apart. This was done due to students' busyness and obligations in their study program. Since the participation in the study was voluntary, and we wanted students to take part and put some effort, we decided to do a second part of the study when students had more free time.

The items in the pre-test and the post-test were in the open-ended form and were mostly the same. This means that the post-test contained all items from the pre-test, but it also contained some additional items to avoid direct recalling from previous testing. In this paper, we will report results of the joint items to compare students' results before and after knowledge refreshment.

The design of the mathematical items was guided with the ILOs and was done in collaboration with the lecturers and teaching assistants of the course.

The items asked students to: a) give a formal definition of derivative of a function at the point, b) differentiate given function, c) approximate the derivative of a function at the point, d) determine intervals of monotonicity of a given function, e) determine intervals of concavity of a given function, f) find local extreme values of the function, g) apply the geometric interpretation of the derivative and h) examine continuity and differentiability of the function at the specific point. The following items were given to students:

(1) Define the derivative of a function $f : \mathbb{R} \to \mathbb{R}$ at x_0.

(2) The table shows values of the function f. Estimate the value of $f'(6)$.

x	0	2	4	6	8	10	12	14
$f(x)$	23	26	29	32	33	33	32	33

(3) Find the slope of the tangent line to the graph of the function $f(x) = (3x)^2$ at $x = 1$.

(4) For the function $f : \mathbb{R} \to \mathbb{R}$, defined with $f(x) = |x - 3|$ and given with the graph given below, answer the following questions:

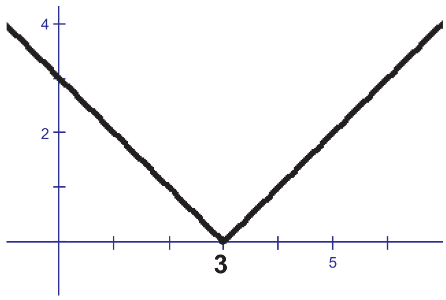

Figure 1. Graph of function $f(x) = |x - 3|$

(a) Explain whether or not this function is differentiable at $x = 3$.

(b) Explain whether or not this function is continuous at $x = 3$.

(5) Differentiate $f(x) = \ln\left(\dfrac{1+x^2}{1-x^2}\right)$.

(6) Observe the graph of the function f in the figure below, and answer the questions.

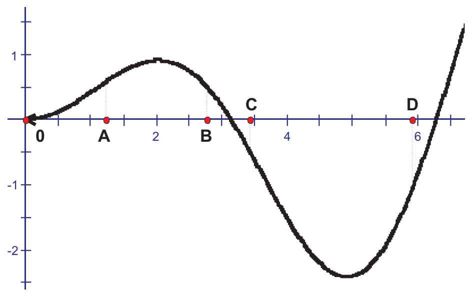

Figure 2. Graph of the function f

(a) What is the sign of f' on the interval $\langle 0, A \rangle$?

(b) Consider the following intervals $\langle 0, A \rangle, \langle A, B \rangle, \langle B, C \rangle, \langle C, D \rangle$. Where does f' change from positive into negative or vice versa ?

(c) What are the signs of f'' on the intervals $\langle A, B \rangle$ and $\langle C, D \rangle$?

(7) Find the local extreme values of the function $f : \mathbb{R} \to \mathbb{R}$ defined with $f(x) = x^2 \mathrm{e}^x$.

The items in the pre-test and post-test can be placed in the procedural or conceptual category. The concepts of procedural and conceptual knowledge in mathematics are not absolute since these types of knowledge show up intertwined with each other, and it is difficult to measure them validly and partly independently of each other. Some items in the tests could be placed in both groups, since they involve both types of knowledge. For instance, several differentiation rules have to be connected in Item 5, and this, at least in some cases, can be considered as conceptual knowledge. On the other hand, it is possible that some students had experienced tasks like Item 3 and thus their solution could be based only on recalling the method without any conceptual knowledge. Schneider and Stern [21] explained that procedural knowledge is usually assessed with routine tasks familiar from practice, and conceptual knowledge is examined by new problems. Therefore, due to the significant number of tasks in the course where students practiced differentiating functions, finding the local extreme values of the function or finding the tangent line on the graph of a function at a given point, we placed Items 3, 5 and 7 in the procedural category, whereas the other items were placed in the conceptual category.

3.3. Knowledge refreshment

The knowledge refreshment was done through discussion with students, using slides from the calculus course and asking the students to interpret what was shown on the slide. This way we wanted to engage students in active knowledge refreshment instead of just showing the slides with information. Generating information, rather than reading it, substantially better improves memory (e.g. [22, 23]). Also, generating information enhances encoding of the item and relations between the item and the cue. The first slides gave a formal definition of derivative and an interpretation of the derivative as the slope of the tangent line to a curve. The table of derivatives for basic functions, as well as the rules for differentiation (powers, sum, product, quotient and chain rule), were shown on the slides, and afterwards students were asked to interpret a certain rule by giving an appropriate example. Then we raised the questions of sketching a graph of the function. The students were asked how the derivative can be used here and, through discussion, the following areas were addressed: the monotonicity and concavity intervals, the extreme values and the inflection point. Here we provided some cues to enhance reconstruction of knowledge. For instance, in the case of concavity, we gave an example of the parabola being concave upward or downward. The question that followed asked how derivative can be used to determine this property. Slides provided formal mathematical statements, and students were asked to describe how each statement is used in practice.

Some topics from calculus were not repeated deliberately, like approximating the value of the derivative at the point given a table of function values, or continuity and differentiability of a function at a point. Some studies showed that the benefit of generating answers may be extended to unstudied items (e.g.[23, 24]), thus we wanted to investigate if knowledge for unstudied topic can be activated with knowledge that is closely related with that topic, and to what extent. The knowledge refreshment session lasted an hour, and after a break students were given the post-test.

We were not in a position to measure the learning effort that students had invested in re-learning. This would be very significant and interesting, but it is impossible in practice [25]. Since we had 20 students in the study, we cannot make strong generalizations, hence we replace the generalizability with the fittingness i.e., "the degree to which the situation matches other situations in which we are interested" ([26], p. 207). Goetz and LeCompte [27] use the notion 'translatability' to denote if the theoretical frames and research techniques are understood by other researchers in the same field, and the notion of 'comparability' to mean if a situation has been "sufficiently well described and defined that other researchers can use the results of the study as a basis for comparison with other studies addressing related issues".

4. Results

First, we will report on the results of the pre-test, where we evaluated students' retained knowledge. After that we will describe the results of the post-test, conducted after the re-learning, i.e., knowledge refreshment.

4.1. Retained knowledge - results of pre-test

The concept of derivative was retained in the geometric form. In Item 1, students associated a derivative of the function with the tangent line belonging to the graph of the function. However, the minority interpreted the derivative as the slope of this tangent. No student wrote a formal definition of the derivative. Further, students did not know how to calculate required derivative for the given table of data in Item 2. Moreover, they did not know if anything can be done with this data at all. There was a small number of correct answers in Item 3; problems in finding the slope of the tangent line were connected with inappropriate differentiation or with inserting a value of x directly in the algebraic formula of the function.

Item 4 examined the differentiability and continuity of the given function at the point $x = 3$. A significant number of students was not able to explain why the given function is or is not differentiable at the given point in this item, even though eight of them gave correct answer. The same problem was encountered when students were asked if the function is continuous at the same point. Their answers were fuzzy and not articulated clearly although traces of good reasoning were found. In the case of differentiability, this reasoning was connected with the shape of the graph which the function of an absolute value has in the Cartesian coordinate system, and in the case of the continuity, with a value of the function at the given point.

Students experienced problems with differentiation of a composite function in Item 5. The main problem was connected with the fact that students were not sure when differentiation stops, i.e., what functions constituted this composition.

Various responses were found in Item 6. Students determined the sign of the first derivative for the given interval correctly, but they were not equally successful in determining where the first derivative changes its sign. Several students gave the correct answer (six students), while others gave incorrect or inconclusive answer. Similarly, students wrote incorrect or inconclusive answers for the sign of the second derivative for the given intervals.

The last item examined students' knowledge in the application of derivative. Here students should have used derivative to find the local extreme values of the given function. However, students differentiated the function incorrectly, and it seemed that they did not know how to apply the product rule. Only three of them were able to differentiate the given function properly, but they stopped after a computation of critical points.

4.2. Re-learning - results of post-test

In Item 1, most students defined the derivative as the slope of the tangent line to the curve/graph of some function. Several students also gave a formal-like definition together with the geometric interpretation. This formal-like definition contained the difference quotient in the limit notation and was directly connected with the slope. Only four students gave the required definition instead of the geometric interpretation. Their definition was accurate formal definition that contained not only the limit of the difference quotient of some function, but also the key part of the definition "if this limit exist" or "provided this limit exists".

In Item 2, students made some calculation to determine the derivative using the table of data, although those calculations were not entirely correct. Further, all students explained whether or not the function is differentiable in the given point in Item 3. The explanations were closely connected with the shape of the graph of the given function, which has a "corner". This "corner" was highlighted as the reason why this specific function cannot have only one tangent line at the point $x = 3$. But when it comes to the continuity of the same function, most students answered without explanation. Several students explained that the graph of the given function can be drawn without lifting the pencil from the paper. Only three students discussed what the continuity at the specific point means for the function of absolute value, checking if the limit of function f exists at $x = 3$ and if it is equal to $f(3)$.

Item 4 was solved correctly, meaning that students made no mistakes when they differentiated the composite function. When it comes to Item 5, the students calculated the slope of the tangent line correctly. They differentiated the function and inserted the value of x in the derivative f'. However, one student still inserted the value of x in the original algebraic formula of the function f.

The responses in Item 6 were correct. The students correctly interpreted the signs of the first and second derivative on the given intervals and detected where the first derivative changes from positive into negative and vice versa.

In Item 7, the students found the first and second derivative and computed the critical points. The majority stopped here, and only three students proceeded further, discussing the extreme values in terms of the computed critical points.

5. Discussion and conclusion

The participants in our study frequently practiced the differentiation of composite function or finding a local extreme value of some function within the calculus course they took. Therefore, we categorized the knowledge for those items as the procedural knowledge. This type of knowledge turns out to be fragile in the long run, deteriorates faster with time and is usually remembered inappropriately (e.g. [15]). The fragility of this knowledge can be seen in the pre-test, especially within the items that required the process of differentiation. After the knowledge refreshment, the results of the post-test showed that procedural knowledge for differentiation was successfully re-activated. However, the knowledge for finding the local extreme values was not evoked in complete form. This means that students did not recall the whole method they learnt when they were taking the calculus course. The method consists of finding the first derivative, the critical points as the zero points of the first derivative and the second derivative which is used in the second derivative test, i.e. for checking whether the critical point is in fact the extreme value or not. Most students had found the first and second derivative and computed the critical points, but they did not perform the second derivative test. This indicates that the method for finding the local extreme values was learnt and coded in the long-term memory purely as a procedure, without sufficient conceptual understanding, wherefore this knowledge could not be recalled from students' long-term memory in appropriate form or re-learned in a short period of time. When the student tries to make this type of knowledge re-appear, we argue that it is usually a process of reproduction since the student tries to re-trace the exact steps, i.e. to find the exact "copies" which are needed to perform the method. We based this conclusion looking not only at the results of the last item but also looking at Item 6 which used the properties of the first and the second derivative, relying on the conceptual approach.

Although we placed Item 3 in the procedural category, there emerged some misconceptions related to the conceptual knowledge. Incorrect differentiation belongs to problems of procedural knowledge, but inserting x value directly in the algebraic formula of the given function shows conceptual misunderstanding. However, with the exception of one student, other students have overcome these problems after the knowledge refreshment.

Solving tasks that include a dozen problems of the same type, like differentiation of composite function, guarantees overlearning. Studies of Rohrer and Taylor [9], and Roher and Pashler [8] indicated that overlearning does not benefit the long-term retention, and results of pre-test in our study support their findings. On the other hand, there is a natural decline if knowledge is not used regularly, and this can also be seen within the conceptual items before the knowledge refreshment. The results of the pre-test showed that students' conceptual knowledge was incomplete and fragmented in some areas. The application of derivative for determining monotonicity and concavity intervals was not entirely forgotten and some aspects retained in the students' long-term memory. After the knowledge refreshment, this knowledge was fully activated, and students were able to successfully apply it in the given context. Although students were able to recall and apply their conceptual knowledge to most of the items, there are some exceptions that need to be discussed. After re-learning, the knowledge for the items that were not included in the process of refreshment was recalled more or less successfully. Even though students were not able to solve the item with the approximation of the derivative, their answers revealed that they recalled some necessary information. Similarly, they reconstructed some parts of the knowledge related to the continuity and differentiability of the function at some point. Students recalled the property of differentiability better than the property of continuity. We believe that was connected with the context where the emphasis was on the derivative and the differentiation, and not on the limits and the continuity. Thus, this knowledge was not entirely re-activated.

According to Semb et al. [7], the retained knowledge depends on the original learning, meaning that a student cannot recall something different from what he had learnt. For example, if the student had interiorized the geometric interpretation of the derivative as the formal definition of the derivative, he cannot recall a limit definition. This can be seen particularly from the students' results before and after the knowledge refreshment. Many students wrote the geometric interpretation of the derivative as the required definition since this interpretation was learnt and coded in their long-term memory as the formal definition. Confusing a definition of the derivative and its geometric interpretation has been well established in mathematics education, and this study confirmed this finding also.

However, we do not place the emphasis on this misconception, but on recalling. Even though the item asked for the formal definition, we noticed that there was a positive effect of the knowledge refreshment in terms of the precision - from the interpretation of derivative as the tangent line to its interpretation as the slope of the tangent line, and from the interpretation as the slope to the formal definition.

Shortly, in the pre-test, students struggled with the given items showing fragmented procedural and conceptual knowledge, but in the post-test their procedural and conceptual knowledge improved. We concluded that the students, with the help they got, could recall their skills and knowledge in differential calculus in a short period of re-learning. However, they could not recall/reconstruct procedural knowledge learnt without underlying conceptual understanding. We did not create procedural items that tested what has been learnt with or without understanding, but we were able to detect this knowledge comparing students' written work from the pre-test and post-test.

Many times teachers of sequencing courses complain on students' retained mathematical knowledge (e.g. [28]). Also some educators expect that students themselves repeat subject matter from previous courses and refresh their knowledge, but often students are not aware which concepts and procedures they should pay attention to. A repetition of concepts and procedures necessary for the new courses would enable a good refreshment of old knowledge as well as better acquisition of new knowledge. Our study showed that it is possible to recall mathematical skills and knowledge in short period of time. Hence, we argue that students' retained knowledge should be taken into consideration in instructional design and curriculum planning for the sequencing courses. Moreover, the integration of knowledge from previous mathematics courses should be imbedded in new courses in a meaningful way when it is possible. This would help students to build a coherent knowledge base. However, this embedding should not apply only to mathematics courses but also to non-mathematics courses that use mathematical knowledge from the past courses like calculus. A study by Hailikari et al. [2] found that a good performance in the advanced courses can be connected with the knowledge retained from the basic courses such as mathematics. Consequently, we believe that cooperation between mathematicians and educators from different disciplines should be established for the repetition of mathematical knowledge within the advanced non-mathematics courses. It is important not to diminish the rigor of mathematics, but it is also important to repeat mathematical procedures and concepts within the physical problems in an appropriate manner.

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(Received July, 2013)