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Teaching Mathematics and Computer Science

Development of spatial perception in high school with GeoGebra

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Abstract. In everyday life, on numerous occasions we need to project 3D space onto a plane in order to activate our spatial perception. While our ability in this area can be improved, and considering several national and international research results, the development is even necessary on all levels of education. GeoGebra, as a supplement to previously used tools, has proven to be very useful respective to the development. We have many possibilities to display spatial elements in GeoGebra and to apply such kind of worksheets among 15–18 year old students. I show the results of the 2011/2012 school years connected to the development of spatial perception and the results of an input case survey, which also justifies the need for development.

Key words and phrases: 3d in GeoGebra, IT tools in Mathematical, spatial perception.

ZDM Subject Classification: M10, G10, G40.

1. Introduction

Today and in recent years there has been a continually increasing role of spatial perception development of students studying on various levels. Even currently the spatial perception maturity level evaluation of students and the question of development/ and upgradeability [1], [2], [3], [4], [5], [6], [7], [8], [9] are the subjects of many national and international research projects.

This research was realized in the frames of TÁMOP 4.2.4. A/2-11-1-2012-0001 "National Excellence Program Elaborating and operating an inland student and researcher personal support system convergence program" The project was subsidized by the European Union and co-financed by the European Social Fund.

The psychological background of the development of spatial perception and the subdivision of spatial intelligence [10], [11], are well known, but it seems that the past 10 years' cultural, social, technological, mathematical-didactical changes have modified these psychological/didactical benchmarks.

The above-mentioned studies have reported that engineering university students on an international level and in our country do not have an adequate spatial perception development level, which may cause them a great disadvantage later during their work.

This can be traced back to knowledge acquired on spatial geometry (sufficiently incomplete) in high school. The main reason for this insufficiency may be the number of class hours (Table 1).

Table 1. Average occurrence of spatial geometrical type class hours in high school education

Level of Education	High School level Education			
Grade	9.	10.	11.	12.
Total number of class hours in mathematics during the school year	108	108	108	108
Total number of class hours in Geometry during the school year	39	59	45	35
Total number of class hours in spatial geometry type subjects during the school year	3	6	4	21
Relative percentage of spatial geometry type class hours compared to total class hours	2.7	5.4	3.7	19.5

Based on this table, we can state that there is a very small possibility for development within the framework of class hours. Thus, the fundamental problem arises, how can we decide on the basis of class hours weather a specific student possesses adequate spatial approach for engineering (or another profession requiring good spatial perception). Just as importantly, how can the student decide on this question if in school, he/she does not have adequate opportunity to ascertain it. The curricular framework effectuating the New National Core Curriculum would reduce the minimum number of weekly mathematics class hours, which given the actual budgetary situation of institutions can be considered as the final frame number in the majority of cases. This may worsen the existing situation.

Reviewing the two level graduation exam problem sets, introduced in 2004, we can see that there is a great chance of spatial (calculation) problems occurring in the mathematics graduation exam. Students only encounter problems of this nature in a more focused way in the 12^{th} grade, which proves to be a bit late in the case of many students. It may be mentioned here as a comment, that perhaps the launching of a spatial perception study circle may remedy this situation (the evolution of spatial perception in the history of mathematics, plane-space analogies, spherical geometry and other types of geometry could be discussed ...).

On the other hand, consideration should be given to those students who definitely cannot envision themselves in such fields, but they also need to handle the spatial problems that occur in our daily lives, in the best possible way. Let us think about a user manual for a device, or an everyday assembly drawing, or orientation with the help of a map (Figure 1).

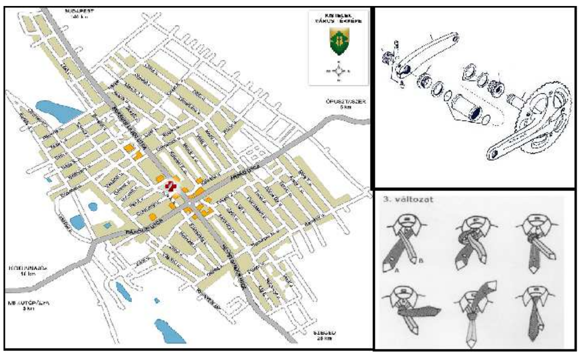

Figure 1. Everyday spatial problems

The above described facts justify the conscious development of the spatial perception of students. After taking into consideration the aforementioned fast social, technological changes, it is practical to complement the already used so-called traditional tools with modern tools (model kits).

2. Why exactly GeoGebra?

According to international research the integration of DGS (Dynamic Geometry Systems) into mathematical education may bring a nearly 10% efficiency increase, not to mention the thus formed motivational base.

Many DGS exist today, but for certain reasons none of them have been able to compete on the high school level with GeoGebra, which is an unrestricted, free and easily accessible software for everyone. Some additional reasons (Table 2).

Table 2. Average occurrence of spatial geometrical type class hours in high school education

62 languages	45000 online study materials
190 countries	900000 visitors/month
122 institutions (82 countries)	400000 downloads/month
43 developers	6.2 million downloads in 2011
200 translators	5.5 million in schools on laptops

Thanks to the high pace of development, GeoGebra gradually can be reached on tablet PCs and smart phones as well. GeoGebra 5.0 (real 3D view) is also developing more and more dynamically and newer useful tool-kits appear on it almost every day. GeoGebraTube (nearly 20,000 shared worksheets) thanks to the Google drive integration, makes real time, online collaboration possible within the class. The interactive tablet support, the drawing of implicit curves, is working in a stable manner on version 4.0. The 4.2 trial version is introducing the CAS view, which is a dynamically updated symbolic view. One of the currently running projects is GeoGebraSTEM, the Lego Mindstorms Robot project constitutes one of its very interesting parts [12].

3. The visualization of spatial objects with GeoGebra

We have two ways to display spatial objects with GeoGebra, the GeoGebra 4.x version and the GeoGebra 5.0 Beta version. The latter has a real 3D view, its disadvantage is, that it is still under fairly serious development, and under certain conditions, the use of the program is unstable. Although there is no built-in possibility to display spatial objects in version 4.x, with a tiny computer graphic trick, we can solve the problem, and taking into account the current deficiencies of GeoGebra 5, it is worth taking full advantage of these possibilities.

We have a number of options to project spatial objects onto the plane. In high school education, in the vast majority of the cases a spatial parallel projection, the orthogonal projection is used for these applications. Thus, in GeoGebra we do the same. This trend is developed further in more detail (GeoGebra Tube contains many worksheets, which were created by projection, also thinking about the university level).

In order to create these worksheets we need to have prior knowledge of the vector concept, the different vector procedures, the concept of matrixes, some matrix procedures, and it would not hurt to know some coordinate system types (primarily the spatial and plane right-angled Cartesian coordinate system). I will not cover these basic skills, more details can be found among others in [13].

First, let us define a spatial right angle Cartesian type coordinate system. This can be done with 4 points-the origin and the three unit points on the axes. In basic cases, we multiply each of the coordinates of the points with the Rzxy rotation matrix (which would allow the rotation around all three axes). We proceed a little bit differently due to the time factor related to the preparation of GeoGebra worksheets and taking into account certain mathematical and didactical considerations. We will multiply with two arbitrary rotation matrixes around the axis, for example, Rxy. To identify the projected image we will need two angles. We may define a so call reference point. The degree of the two rotation angles is specified by the coordinates of x and y given in radians. Rotation around the third axis can also be simulated under certain conditions in the system thus established. Due to the orthogonal projection, we omit the spatial points' z coordinates and we have arrived to the plane projection. The coordinates of the three points in more detail:

$$\mathrm{E_X} = (-\sin(x(\text{reference point})), \cos(x(\text{reference point}))\cos(y(\text{reference point})))$$

$$\mathrm{E_Y} = (\cos(x(\text{reference point})), \cos(y(\text{reference point}))\sin(x(\text{reference point})))$$

$$\mathrm{E_Z} = (0, -\sin(y(\text{reference point})))$$

Based on experience, this solution allows a more convenient use for students than the separately done rotation around the 3 axes. Namely, in that case, we could achieve the rotation only with 4 sliders, but in this case, we can achieve the desired effect merely by grabbing and moving the point. Another advantage may be that in this basic system, the only unengaged object is the reference point; any other object will be a function of this. This way our basic system is ready (Figure 2) [14].

Originating from this basic system, the following can be achieved exclusively with plane geometrical constructions (but not limited to): various polyhedrons, truncated polyhedrons, cones, truncated cones, cylinders, prisms, pyramids, truncated pyramids ... (From this point on our creativity may set the boundaries,

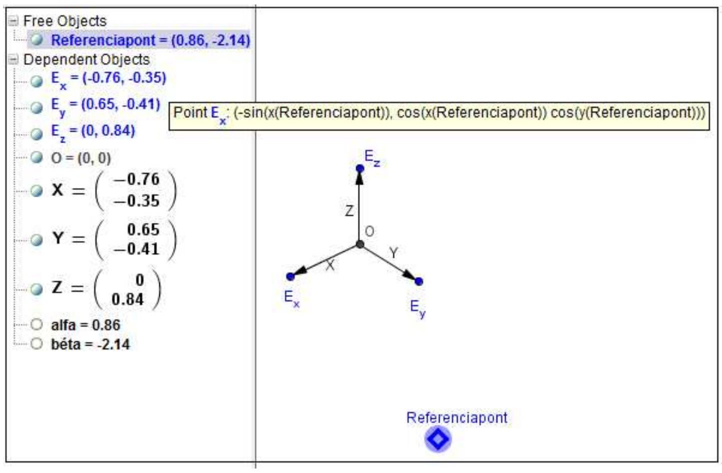

Figure 2. Creation of the basic system

Figure 3). Curved surfaces, such as a sphere can be achieved by creating lists. We can organize stipulated point series into a list through which we can fit conic sections, this way an arbitrary surface can be created.

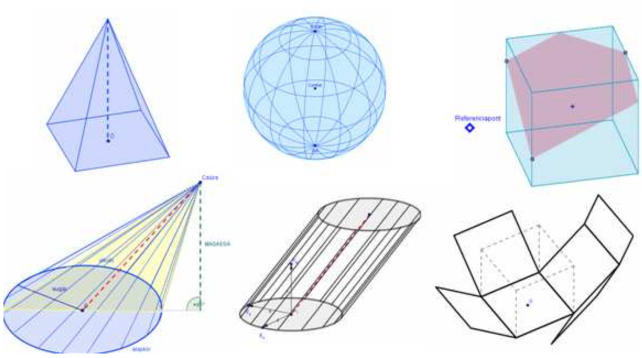

Figure 3. Dynamic spatial objects in GeoGebra

The worksheets actualizing the above-mentioned spatial objects and many other related worksheets can be found on `www.geogebratube.com`. With the use of the basic worksheets, an illustrative/practice worksheet can be created specifically for almost any spatial problem very quickly (even the kind, which is very difficult to create in the traditional way).

4. The research

The research strategy is basically a control group experiment in a classic natural scene. The research method is cyclically operationalized, quantitative (with mathematical statistics methods) and qualitative (continuous monitoring of groups, oral and written questioning of students, and organization of experimental observations ...) in nature.

The research schedule is from school years 2010/2011 to 2014/2015 and contains the processing and ongoing monitoring of the related scientific literature as well as the preparation of summaries of reviews and reports.

The location is a high school in a small city in Hungary (Szécsény in Nógrád County) which has pupils with abilities that reach the national average. It is an eight year secondary school with two or three parallel grades.

During the test sample and the survey on status measurement, the participants were the current 9/a, 9/b, 10/a, 10/b, 11/a, 11/b, 12/a and 12/b grade students (15–18 age group) an average of 200 students. We improved the 10^{th} and 12^{th} grade experimental groups in more detail with GeoGebra and conventional methods (model kits), while the students in the control group were improved exclusively with conventional methods. Thus, about 80 students participate in the research process per academic year.

All of the applied tests are international standardized tests: the MRT (Mental Rotation Test) and a test developed in 2000, by the ELTE University Department of Educational Science on spatial perception level measurements, and a further task is the preparation of a visibility drawing of buildings. It is a 60-minute test, 105 points may be earned, and it can be broken down into items according to various thinking capabilities, which constitute spatial perception. The input and output measurements are equivalent test problems and the intermediate measurements are connected to the current spatial perception topic. The test is designed to measure the abilities necessary to solve the elementary steps of spatial computational problems (interpretation of the text of the problem, preparation of a sketch drawing-projected figure, finding correlations in a given shape, describing correlations, performing calculations, checking).

During the academic years 2010/2011, different teachers taught the 10/a control group and the 10/b experimental group. My role was to help in the design and the conduct of the research. In the case of 12/a and 12/b, I taught the latter experimental group. The research results in that academic year are in line with the results of the presently discussed and presented academic year's results thus, eliminating the Pygmalion effect, and ensuring teacher independence, so the results can be considered objective.

I also consider it essential to emphasize that a high level of targeted curriculum change is necessary in order to; for example, include GeoGebra into everyday classroom life in the 12$^{\text{th}}$ grade. It is important to make a decision about at which topics' expense we should give GeoGebra a role. This requires a lot of preparatory work and prior experience tailored to both individuals and groups. It is important to use them at the appropriate place and time and in the right way, taking the possibilities into consideration.

5. The status measurement

The status measurement survey in school years 2011/2012, which affected exactly 182 students in September of 2011, was concluded with the following results (Figure 4).

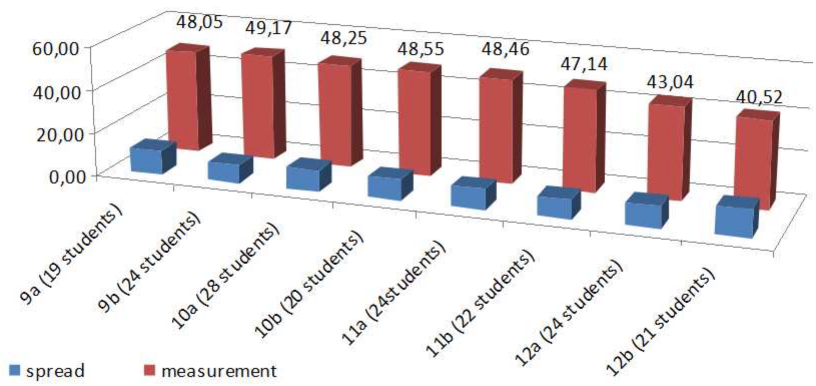

Figure 4. Status survey of high school

Regarding deviation and mean, it can be stated that the students involved possess nearly the same abilities, although 18-19 year old students should be more effective during the test procedure, due to the development of perspective vision, than a 15 year old, but the results demonstrate otherwise. This contradiction was revealed after the completion of a modified Claus's questionnaire and after the interviews with 12$^{\text{th}}$ grade students. The affective factors of students are not appropriate at the time of reaching 12$^{\text{th}}$ grade, and a great lack of motivation is typical. Thus, during the development with GeoGebra, the development of the appropriate motivational base, its consolidation and also its improvement need to be set as a goal, and kept in mind as almost primary considerations.

The results of the 15 year old age group (9/a and 9/b) may still be considered satisfactory, but the further classes according to their age should perform better.

The best result in 12/a was 60 points (57.14%), in 12/b 69 points (65.71%). Both performances remained below the appropriate performance corresponding to the age of students. However, of the 45 students 12 students gave markedly poor performances (around 30 points and below) a number, which is more than a quarter of the students.

10/a and 10/b show similar results based on the 2011/2012 academic year's input measurement.

Let's look in more detail at the 10th and 12th grade MRT input results (Figure 5), which were divided taking into consideration the age characteristics and the psychological features of space perception development.

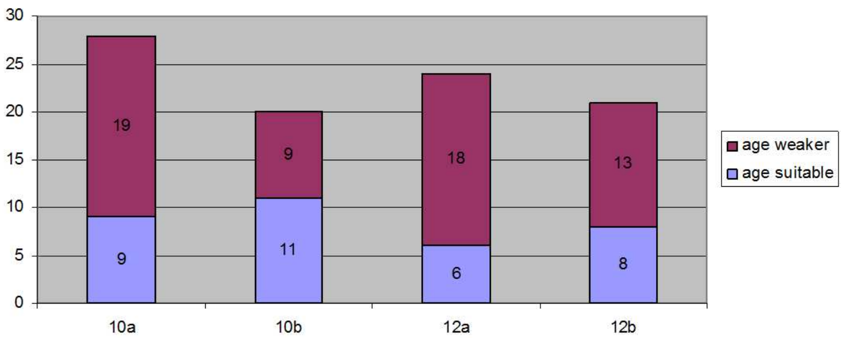

Figure 5. Input results of the Mental Rotation Test grades 10 and 12

In general it can be stated, that, considering the details, the MRT turned out the best. Of the 45 12th grade students, 14 achieved appropriate results in line with their age. Of the 48 10th grade students, this number was 20. This capability is not only indispensable in everyday life but would help in relation to spatial problems, to position objects relative to each other in a problem, to take into account the conditions and to render a preliminary estimate of the result.

The above-mentioned international tendency is characteristic of the results of the other related test problems: the advancement of spatial-perception of the students does not comply with the expected level.

6. The advancement of spatial perception development level

Following the survey of the general spatial perception development status, I applied GeoGebra in a targeted manner, in almost every spatial geometry class, of course complimenting it with the traditional methods. The magnificence lies in the fact that practicing/illustrative worksheets can be prepared for almost every spatial problem. Thanks to the dynamic figures and calculations, we can also cover all the equivalent problems. It is not necessary to construct the worksheet ourselves from the beginning, because a number of worksheets can be found which we can tailor to our own needs very fast.

It was surprisingly easy to raise interest and to develop a motivational base in the case of the 12^{th} grade. The spatial equivalent of the square is the cube. But can we generalize this? The one-dimensional cube is the point, the two-dimensional cube is the square, and three-dimensional cube is the hexahedron. Is there a four-dimensional cube? The answer, to the great astonishment of students: yes. In GeoGebra, a hyper cube can be illustrated which we can even rotate. The hyper cube is such four-dimensional formation, where the cube's each "side" is a three-dimensional cube.

After this, we can explore what kind of projections and plane sections a cube may have. We can observe this with a table lamp and a model kit as well. The students need to collect various cases, based upon their experience. Is it possible for a cube's projection, plane section to be a standard hexagon, rectangle, perhaps a square? Quite a few cases will certainly be skipped. The worksheet appearing in Figure 6 helps students to recognize plane sections, where they try to set out all cases with the help of movable points along the edges.

It is worthwhile to discuss the nets of the cube even in grades 10 and 12. The students should gather them and after this, we can view all cases dynamically as well, in GeoGebra.

After that, it is always worthwhile to demonstrate with modelling, and in parallel with it, also with GeoGebra, the current formations, prisms, cylinders, pyramids, cones and spheres, along with their truncated versions. Practice worksheets can be prepared in connection with these, where a three-dimensional dynamic sketch drawing is given as well as a couple of text boxes for entering the data. The worksheet works out the calculation of the surface and the volume in detail, in a dynamic way. Thus, the basic computation problems are covered by GeoGebra in relation to all the surfaces and volumes and from these we can proceed towards the specifications.

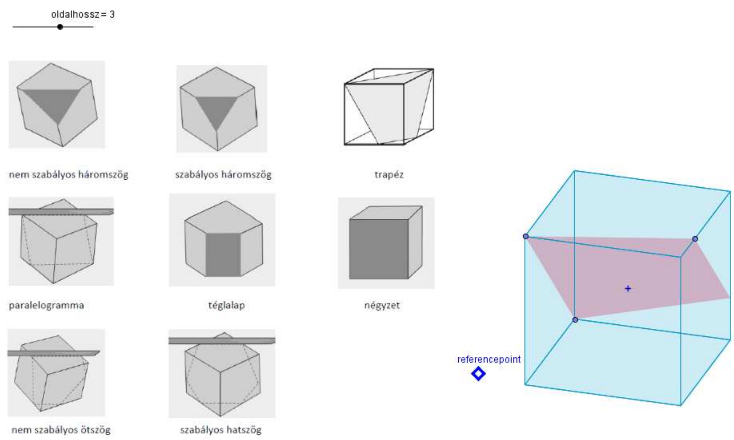

Figure 6. Plane sections of a cube in GeoGebra

GeoGebra was especially helpful in those places, where we had reached the possible limits of the model kits: interpenetrative spatial objects. We can also separate the spatial objects dynamically and the relationship between the volumes can be demonstrated. Practice worksheets similarly can be developed which show in detail the process of problem solving and are also ideal for checking.

Formulas are easier to remember for students if they can connect them to something. A spherical volume formula is not displayable with volume integration as a rotational object however; it can be derived by other means (Figure 7).

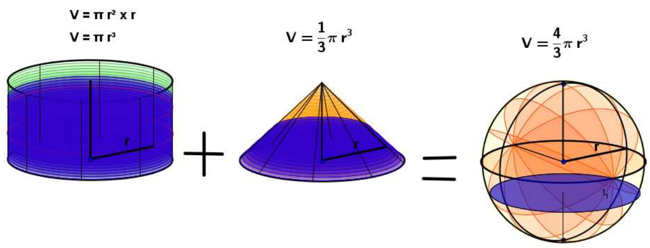

Figure 7. Originating the volume of a sphere with GeoGebra

Thus, even if he/she cannot remember the formulas, he/she can trace them back to the already known shapes and based on that he/she can figure out the formula in a couple of easy steps.

The following was an actual case: given were the radiuses of the two base circles of a truncated cone and its height. Let us calculate the length of the constituents. We need to work with a KLJ right-angled triangle. The question is, how long the LJ section is, which is one of the perpendicular sides of the right-angled triangle. Surprisingly few correct answers were given to this question (Figure 8).

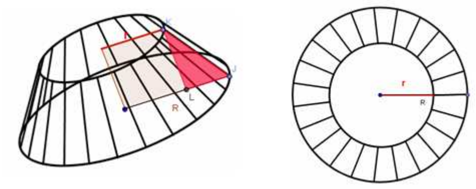

Figure 8. Removal of the right angled triangle from the truncated cone

7. The results of the development

In the case of the group using GeoGebra among others, all the here mentioned worksheets were demonstrated, and the students actively used them. Development was shown already during the intermediate measurements and in January, 2012, in the results of the complex spatial geometrical problem series as well. The test results of the final test in April-, which in its structure was equivalent to the input test-, are summarized in Figure 9.

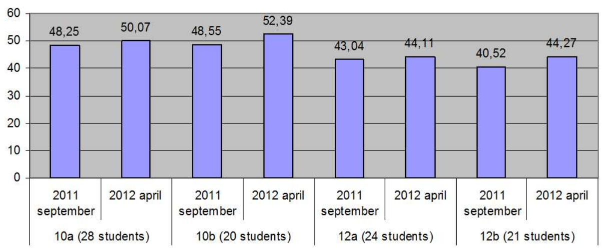

Figure 9. Summary results during the output measurements

In case of grade 10, the control group shows a 3.77%, improvement, while the experimental group using GeoGebra shows a 7.91% improvement. In case of grade

12, these figures are 2.48% and 9.24%. It might be interesting to examine the rate of development in the case of the present grade 10, meaning the improvement when they will be in grade 12, as well as the boy/girl ratio numbers in the different classes [15], [16], [17].

It is worthwhile to highlight the progress made in relation to some of the problems. Let's look at diagram 3 in a more detailed breakdown.

The 4^{th} problem of the test series (Figure 10) is about how to identify the different many angles with the help of the vertex point of a cube and the possible edge bisectors of the cube. This might be the most important ability, which is needed for students to solve a spatial computation problem. To choose an appropriate plane object (right-angled triangles, similar triangles, trapeziums, circles ...) is inevitable for moving forward.

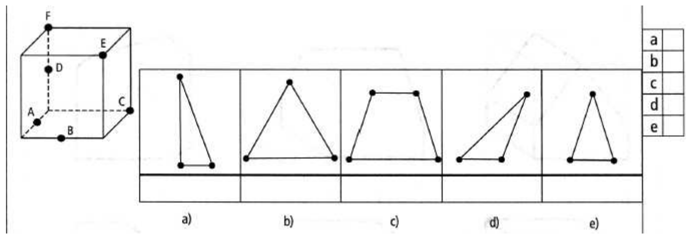

Figure 10. C-4 test problem

Solving the problem requires a very serious mental operation: it is not enough simply to find a similar shape in the cube, but the shape of the front view of the item has to match exactly with the desired searched shape. There can be an arbitrary spatial transverse of the desired shape in the cube, which can cause a more serious problem for students. Here the maximum achievable number of points was 10. Figure 11 shows the input and output results in the case of 4 classes, based on average scores.

In the case of 10/a on average, the rate of development was 10.3%. In the case of the experimental group, it was 20.7%. Based on the input measurement of the control group in the 12^{th} grade, students reached 6.7% better results, this value was reduced to 1.7% during the output measurement. On the other hand, the rate of development in the case of the experimental group proved to be larger. This could be sensed during the problem solving in the classroom. Quantifying: The rate of development of the control group in the case of problem C-4 was 20.8%, while in the case of the experimental group the same value was 26.13%.

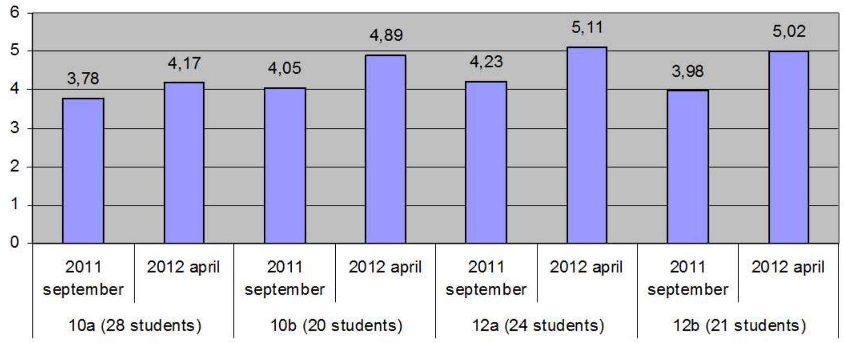

Figure 11. Average scores reached in case of test problem C-4

Although, effectively the final result of the experimental group was just a little behind, compared to the control group, taking into account the initial state, the rate of development and the results of other qualitative measurements (interviews, student's products …), the overall picture was convincing to me.

In the case of test problem C-9 (the maximum number of points that could be earned was 4) students had to create a sketch drawing, which did not exceed the minimal school curriculum requirements from the point of view of its difficulty level. A spatial formation was specified from 3 different viewpoints (front, bottom, side), and based on that, students had to create an arbitrary axonometric drawing of the object. The object was made up of 7 cubes, namely they had to build by using the object, which is one of the most featured in public education. That is why previous descriptive geometry knowledge did not have to be expected from the students.

Figure 12 shows how many student reached 0, 1, 2, 3 or 4 points during each measurement.

In general it can be determined, that in case of the control group the number of worse performers (those who earned 0, 1, 2 points) did not decrease at such a rate as among the students using GeoGebra. In parallel with this, the number of students performing better (those who earned 3, 4 points) significantly increased in the case of the experimental group. In the case of 10/b, from 2 students to 5, and in case of 12/b, from 3 students to 9 students. The result is consistent with the expectations of the school curriculum, since spatial geometry in the 10th grade is not as often present as in the case of the 12^{th} grade. Therefore, the rate of development is proportional to that.

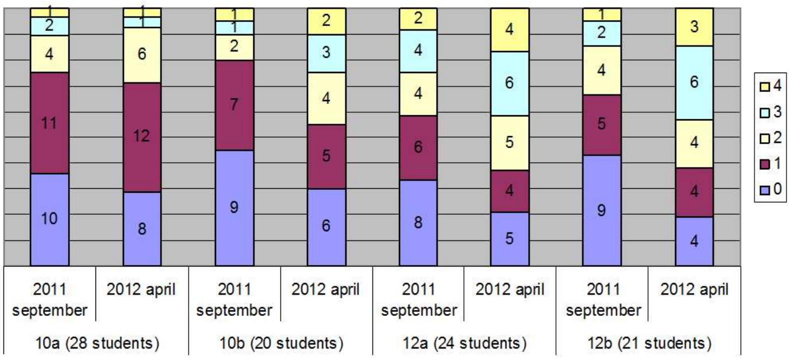

Figure 12. The number of students earning 0, 1, 2, 3, 4 points during the measurements

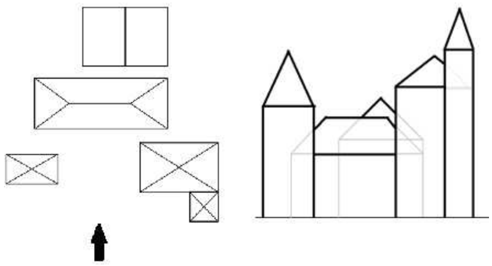

Figure 13. Preparation of a visibility drawing

The preparation of the visibility drawing was an interesting problem (Figure 13). This was the last problem of the test, it may be said to be relatively trivial: a visibility drawing of buildings had to be prepared from the front, based on a view from above. The students were aware, that what might be closer to us was hiding the object behind it, but it seems that even this knowledge was not enough to create an accurate visibility drawing.

Considering that this problem could be given even at primary school age, it produced very interesting results even at the stage of the input measurements. 20% of the 182 students solved the problem correctly, 18% got 0 point, of which 6 students admitted that they did not understand the text of the problem and did not know what they should start doing.

Students analyzed the problems related to visibility questions in GeoGebra 5.0, because it is fully integrated and works consistently. The program provides several options for this theme (representation of hidden edges with dotted lines, full saturation, opacity ...) even in multiple views (axonometric, perspectives ...).

Merging the results of the control group following the development, of the 52 students 12 prepared the figure correctly (23%), and 9 students were given 0 point (17%). In the experimental group with 41 students, 16 students prepared a perfect figure (39%) while 6 students were given 0 point (13%).

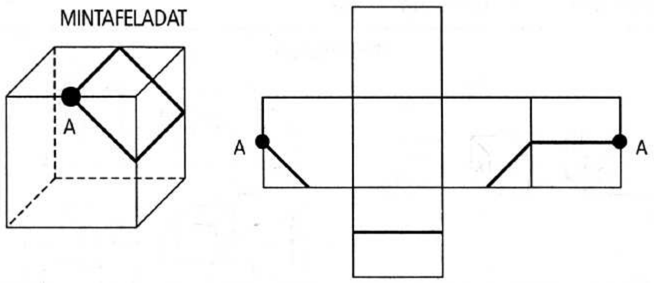

Figure 14. C-5 test problem

The test problem being one of the most problematic was the following: objects (cubes, cuboids ...) were intersected with a plane sheet and the intersecting lines were marked with thick, solid uninterrupted lines on the surface of the object. This line had to be drawn by the students into the outstretched net of the object (Figure 14).

Students managed this problem with surprising difficulty, and this was apparent in the input status survey results as well. In GeoGebra (4.x. individually defined spatial system), it was possible to perfectly adapt the problem, i.e. we could make the static figure completely dynamic, furthermore the cube/other polyhedron could be folded out completely. (The worksheet can be found on geogebratube, and the last shape in Figure 3 shows one similar to that.)

Initially, we modelled the solutions to these types of problems during the solution process in GeoGebra (it is ready in a couple of minutes if a template worksheet is given), after that we viewed the result. We developed this to the point, that students mentally performed the tasks and we used GeoGebra only as a follow-up control.

Surprisingly enough, the most outstanding development was shown, in the case of one of the most troublesome problems. During the input measurements of the 182 students, only 4 students earned the maximum amount of points and 132 students (73%!) were given 0 point. Unfortunately, in the case of the control group no significant improvement could be detected. The proportion in the case of students using GeoGebra was as follows: during the input measurements of the 41 students, 1 prepared it perfectly and 23 students were given 0 point. The final results showed: of the 41 students, 6 students reached the maximum number of points and 9 students were given 0 point.

The result also includes the fact, that students did not use GeoGebra just during class hours, the majority of them also used it at home to solve their homework. (Interestingly, two students according to their own admission, in addition to the compulsory curriculum began independent experimentation and did other designs using GeoGebra.)

The overall evaluation is in line with the partial results, since every partial ability improved to a certain extent. This approach in the case of the 12^{th} grade is presented for example, in Figure 15. This is a self made (spatial problems taken from the previous years' graduation problem sets) input and final examination result, in which I did not consider only the final results as important, but the path leading to solving the problems as well (spatial geometric computational problems).

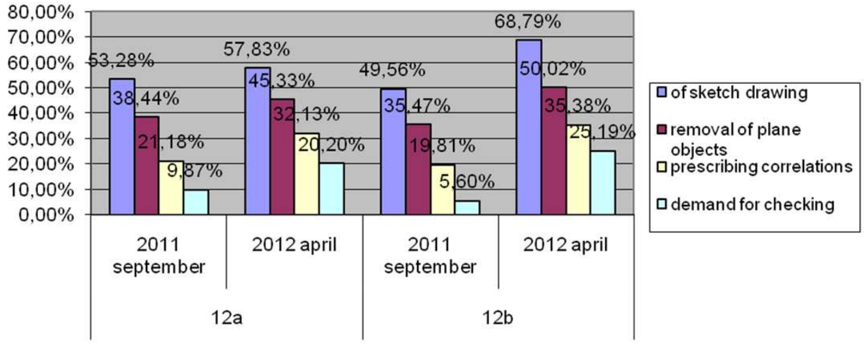

Figure 15. Analysis of the spatial geometric computational problem solving

It was visible and could be experienced, that the 12/b experimental group produced progress at a faster rate in the area of executing certain solution steps.

I consider individual results very important as well, beyond the generally achieved indicators by the GeoGebra group. The raising of proper interest, then beside the consolidation of the motivational base, the individual student performances achieved in the spatial calculation problems, which occurred in the graduation exam, were also remarkable. We learned this from the different qualitative studies, and even based on students' self-proclamations.

The development can be considered successful and this was proven by the results of the examinations written on May 8^{th}, 2012. The students using GeoGebra performed significantly better in relation to problem 18, which was a spatial computation problem: it was to prepare an adequate figure, unfolding the context then discussing the result. We had worked with many wireframe models in GeoGebra during the school year, therefore the problem mentioned was more easily understandable for the experimental group. In the case of the control group, more than 40% of the group skipped the problem.

The involvement of GeoGebra was particularly useful in the case of problem types like "let's rotate the polygon around its one side". On the basis of Table 1 as well as originating from everyday pedagogical practice it is understandable, that for these types of problems neither enough time nor suitable means are available to the average teacher. Most of the students cannot solve these kinds of mental rotation operation problems at all, or only able to solve them with mistakes. The GeoGebra group had the opportunity for the arbitrary rotation of an arbitrary plane object (even a curve) in space with the help of certain worksheets. The results achieved this way by the individual students in relation to these types of problems are convincing to me, since the majority of GeoGebra group students not only could prepare the appropriate figure, but also could size-up the number of potential solutions.

The extent of the development of the GeoGebra group is approaching the level, at which they would be able to use it safely at graduation for spatial computational problems and it would give a proper foundation for possible further study in an area of similar nature (CAD systems).

8. Conclusion

The above research section is in line with the international and national achievements and personal experience. It is very important to note, that we should take into consideration the students' affective and psychomotor factors during the development, providing a basis for further studies and for getting by in life, while meeting the expectations of today's society. GeoGebra, according to my experience, is an excellent tool to achieve these goals, supporting the development of the students' abilities at several points.

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