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Research Studies in Didactics of Mathematics supported by the Operant Motive Test

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Abstract. The present paper reports a case-study which took place within an EU-supported international program organized for research and development of multi-grade schools (NEMED, [16] [26]). One of the main goals of the research was to develop the connection between disadvantageous social situations and the efficiency (success or failure) in learning mathematics especially from the point of view of average and above-average (talented) students: Why does the talent of children with socially disadvantageous background remain undiscovered? How can we make school mathematics more aware of hidden talents?

The author was looking for a didactical solution that compensated for social disadvantages without restricting the development of "average" students by using sociological, educational, psychological and mathematical (experimental and theoretical) studies in interaction with a series of experimental (hypothesis testing and exploratory) investigations.

We constructed tools and methods for exploration and experimental teaching, adapted to Hungarian conditions (Curriculum Development, teacher training, materials, interviews, Kuhl's motivation test, Malara's "researchers and practicing teachers in cooperation" method, etc., see [18], [20]).

The teaching materials and methodological guidelines are based on Bruner's representation theory (see [5]). The empirical research took place in 16 multi-grade schools located in different parts of the country. The author co-operated with nearly 250 students and 25 teachers for 3 years. In this paper we try to demonstrate how an Operant Motive Test can be involved in this research (see [18]).

Key words and phrases: sociological aspects of learning (disadvantaged students), affective aspects (motivation), manipulative materials and their use in the classroom, picture stories.

ZDM Subject Classification: A49, C20, C69, U62.

1. Background – students with social disadvantages

In Hungary 2 percent of children learn in multigrade classes but the background of these classes is entirely different from those of average classes. Multigrade classes can be found mostly in villages with disadvantageous background where children usually come from families who are struggling with many problems and are unable to move (see [17]). Pupils' outcomes in these small schools are weaker than the country's average (see National Assessment of Basic Competencies [25] and Results of the National Assessment of Basic Competencies in Hungary, [1] p. 35). We found that this result was contradictory to the assumption that talent was independent of social background and ethnicity. We believe that weaker outcome of disadvantaged students is due to disparity in access to information and knowledge, not to lesser ability. Therefore the author was looking for a didactical solution that compensated for social disadvantages without restricting the development of "average" students by using sociological, educational, psychological and mathematical (experimental and theoretical) studies in interaction with a series of experimental (hypothesis testing and exploratory) investigations.

The author took part in an EU sponsored program for investigating and developing multigrade schools (NEMED, [26]) and organized the research concerning mathematics education. 243 pupils from 16 multigrade schools participated in the present study. The documentation of the study contains documents written (or drawn) by the students, by their teachers and by the observer (see documentation in the research archive).

One of the most important aims of the field study was to seek the connection between socially disadvantageous background and efficiency in learning mathematics, especially from the point of view of average and above-average (talented) students: Why does the talent of children with socially disadvantageous background remain undiscovered? How can we make school mathematics more aware of hidden talents? Which methods of talent protection are most suitable for the special population?

We use the word talent in subject-independent sense as derivation from general personality characteristics (according to the new trend in the pre-service teacher training for primary schools, e.g. [2], [23]). We decided to work without prior talent selection because of the variation in age and social status of the children. We did not want to pick out some children from their schools, so we decided to choose the so-called enrichment method (grouping-lessons and outdoor activities).

Our method of optimal activation is apt to identify highly talented students and helps teachers find them during the course of the program. In order to be integrated in traditional Hungarian talent education, even highly talented students need special support. This demands compensational socialization, for example engagement in mathematical problem solving. For these talented students need increasing of institutional training in depth.

2. Conditions of learning – the Operant Motive Test

We applied traditional IQ and creativity tests within the framework of Hungarian NEMED project. These tests showed that our pupils were in the normal range (NEMED archive), although their achievements were very low. There are no obvious differences; weakness and strength are hidden (as it is also shown in some international studies [10], [29]). In order to know fine-graded intellectual skills and emotional states there are some effective test methods (e.g. face to face interviews and other oral tests), but these should be performed by specialists. This was not possible because of the high number of test persons. Therefore we had to find an examination method which can be administered by classroom teachers.

J. Kuhl, German psychologist accomplished in 1999 a test to map the motive-structure of the personality. He designed strong, provoking graphics and simple questions. He could draw conclusions from the answers about the motivational system of the examinee ([3], [18], [19], [28]). The test includes 15 pictures with 4 questions: 1) Who is in the picture? 2) What is (s)he doing? 3) How does (s)he feel? 4) What is the end of the story? The evaluation happens (analogously to the projective key motive of the Thematic Apperception Test) by content analysis of written stories (operant). The answers belonging to a picture are divided into motive classes and levels in accordance with the self regulations grade of the person. The most frequented motive class and the most frequented level in this class together determine the type of the person. A general insight can be gained with reference to the following evaluation table. Columns define needs ("what") and lines (levels) define mechanisms ("how").

A – social contact Development Hypothesis: Low family cohesion (emotional distance or low heat): Frustration of the need for closeness	L – achievement Development Hypothesis: Parental claim to independence (i.e., confrontation with difficulties): Frustration of goal attainment	M – power motive Development Hypothesis: little influence of the father/teacher on the child: frustration of the need for structure/hierarchy
A1: Personal encounter Featured by reciprocity; warmth, love, joyful exchange; abandonment of control	L1: Flow Absorption in a task, knowledge, enjoying the challenge	M1: Management Influencing others through strength or self-evident superiority
A2: Sociability Shared by contact and fun entertainment	L2: Grade scale Orientation to quality standards or objectives adopted, the person's aim is to do something well	M2: Acquired recognition Located at the centre; status; confirmation by others; also helping others but focusing on a common aim
A3: Networking Contact by independence; overcoming distance through competence, one has problems in social relationship, but is able to handle it	L3: Failure management Recognizing errors and problems and action-oriented battling them	M3: Self-assertion Prevail against the resistance of others; decision making under uncertainty; leadership despite opposition
A4: Connection Prevent loneliness or exclusion; seek shelter and security; persistently seek closeness / connection	L4: Performance pressure Persevere under stress and not fail; vie; withstand pressure; requirements withstand;	M4: Management Recognition of the negative side of power and difficulty in implementation of power (implicitly); unilaterally seeking of control, fight for power
A5: Liability Feelings of loneliness and fear; recognition of distance; ask for help; "clinge"	L5: Self-criticism Recognize and admit their own mistakes, perceived failures; accept help; being helpless and passive	M5: Subordination Make themselves aware of their own powerlessness; ineffective enforcement strategies; recognition of being suppressed

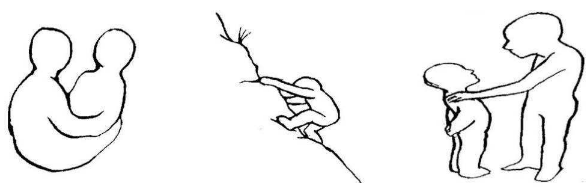
An important argument for using OMT was that the evaluation can be done even on the base of little communication, on the few words commenting the pictures. OMT has been adapted for different populations, especially for pedagogical application ([13], [30]). The pedagogical adaptation of OMT means that from the possible conclusions regarding to the personality only those aspects were selected, which directly support the educational application. Based on the complex analysis of data one can see long-term development tasks and also proposals for organisation of classroom work. According to the major motive class and the level of a student there is a description of suggested instructions about how the student can get help to be able to participate in group work and in other forms of the learning process. The test helps teachers find out students' need of assistance, and helps the majority of students make independent decisions. The tension for students and the pressure to make decisions can be reduced (if necessary) without reducing mathematical complexity. The significance of pedagogical application of psychological tests is to make learning enjoyable and successful, building on students' motivational system.

OMT was earlier used in Hungary to correct the misleading results of the IQ test when examining a pupil from low SES family ([30], p. 73–75.). According to Austrian, German and Hungarian studies we can presume that the achievement motivation of children beginning school is higher also in our population than expected by teachers, from which further pedagogical, didactic questions arise.

2.1. OMT classroom study

The young students around the age of 9, received the drawings, and wrote or dictated their answers. We organized a pre- and a posttest. Now we show some data from the pretest and the most important connections with the posttest. The complex analysis of the answers was made by the author. Herber and Vásárhelyi made a parallel evaluation for OMT with 90% correspondence.

The analysis of writing skills, technical quality of the texts was made according to the categories introduced by Nagy (see [24]). We classified the students' written work according to the first 4 categories. (There is also a level 5 which includes those students who dictated their answers, these student's works were not evaluated by the quality of writing skills.) On the basis of Nagy's classification approximately a quarter of the students belong to the acceptable category. Communicating expressive power of the students is high, many of them showed considerable creativity. The technical level of the written work is low.



We quote some answers precisely as they were written by the pupils (with poor grammar and mistakes):
"egy ember hegyet mász. bátornak érzi magát. mert bátor. végül fel mászot a hegyre."
"a man climbs a hill. he feels himself brave. because he is brave. at last he climbed the hill."
"Bátor erös volt mert okosvol hogy felmaszot a hegy csucsra mert olyan erős volt és így volt a vége."
"He was brave because he was clever to climb the hill because he was so strong so that is the end."
"Miki hegyet mászot. Egy kicsit fél. Attol hogy leesik. megcsúszott és leeset. Beviték a korházba és egy hétmolva haza mehetet Miki."
"Mike climbed the hill. He was a little scared. That he might fall off. he slipped and fell off. He was taken to hospital and Mike was released after a week."
"A gyerek bátor volt. Egy nagy hegyre mászott fel."
"The child was brave. He climbed a high hill."
"Nagyon fél. Rosszul érzi magát. Mert nagyon fél."
"He is very scared. He feels bad. Because he is scared."
"Andris felmászot a hegyre. nagyon bátra viselkedet és feltudot mászni a hegytetejére."
"Andrew climbed the hill. he acted very bravely and could climb the hill."

In spite of the technical problems of writing answers clearly reflects that children imagine themselves into the situation, all of them struggle, some with joy, some with fear and anxiety.

Summing up: The form of students' written work does not meet the performance expected from 8-9-year-olds. The nasty letters, the words-limits do not coincide with the breakdown of their true limit, of the possible use of punctuation. Nevertheless, we can understand the responses of pupils if we know the situation. They are good for the purpose of communication, almost without exception. The expressive power of the children's texts is on a high level of creativity (which we could see in pupils' drawings, too).

Conclusions regarding the experimental learning program

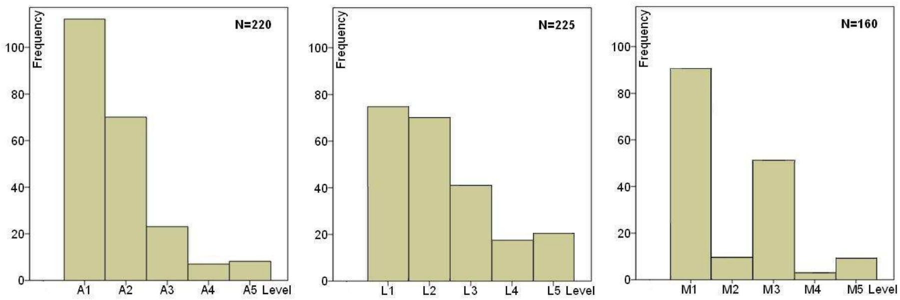
C1) Formal elements of the communication should be exercised both in writing and orally.

C2) We need many different tasks which require communication among children in the mathematics classroom. Moreover a lot of time should be devoted to the discussion of experiences.

2.2. Descriptive analysis of the data

We analyze three (most characteristic) pictures from the viewpoint of mathematics education.

Distribution of the answers within the motive class		
A – social contact (affiliation)	L – achievement	M – power motive



Good relationship skills, the joy of an intimate relationship. Most of the kids do not require the school to solve their emotional problems; this gives possibilities to concentrate on intellectual tasks.	225 from 237 pupils gave an answer to the accomplishment question meaning that achievement is pleasing and a source of joy. It was rarely interpreted as a source of anxiety.	They see mostly the hierarchy of power as a chance to help each other. They empathically convince others to do "the right things".

The OMT showed that choosing teaching forms should be dominated by the didactic features, it does not seem necessary to indulge into emotional or volitional therapeutic work at the mathematics lessons.

Conclusions regarding the experimental learning program

C3) Most members of our population have high level of achievement motivation. We need open-ended problems which contain useful items for internal differentiation, because children look for interesting and challenging tasks, they are not afraid of solving problems, because children are not averse to problem solving, but, on the contrary, they are eager to find interesting and challenging tasks.

3. Planning the experimental teaching program with respect to the OMT results ahead of an example, teaching polyhedron

The preconception that children from low SES families with bad results cannot and will not learn, was disproved by the results of the OMT test. The vast majority of the 243 children from 16 multigrade classes showed healthy motivational system, could learn to write and could use it for communicational purposes. Disadvantaged students with low level of linguistic-symbolic skills and high achievement motivation can recognize problem-solving strategies and expand the problem-solving routine, if the richer vocabulary of everyday situations can be used for mathematical discussions. Based on Bruner's representation theory and C1, C2, C3 we need intellectually challenging problems using hands-on activities. We should organize movement between enactive, iconic and symbolic levels.

For the operationalization of these principles we made a complex plan. It was necessary to select the learning content, to establish the learning tools, to make the teachers familiar with the new methods of classroom work.

During the first phase of experimental teaching we sent the methodological instructions and materials to the teachers via internet. The teachers taught 1 to 3 lessons in each of the four topics picked in agreement with the team. The selected topics were Cups, measuring, units; Excursions, spatial orientation; Wheel of time, Egyptian numbers, first steps toward a mathematical proof; Travel, data management and Polyhedron Safary. The summarized and edited reports written about the lessons were sent back to them so that they could build in the conclusions into their next lesson. (Apart from mathematics we organized esthetical development programs and competitions outside of school and other activities were organized for them as well.) We worked in cyclically repeated phases (preparation, slide show and conversation, problem solving with the help of concrete manual activities, drawing and at a symbolic level, conversation and reflection, students' written feedback). After a lesson the teacher wrote on the lessons, they received the feedback from the organizer, we prepared the new topic together.

In the second phase the topic was the introduction to the concept of polyhedrons. This example shows a way of solving the problems arising from transformation of mathematical content into school curriculum. Through this example our aim was to demonstrate some prototypical steps for primary school teachers (who are not specialized in mathematics) some prototypical steps, which have to be must be done to prepare the students for more complex spatial geometry tasks. In the mathematics lesson it is necessary to make simplifications (especially at the beginning). How can we avoid the impression that mathematics examines simple objects which do not exist in real life? We wanted to show how the intuitively rich content of the concept germ "geometric body" can be preserved and be transformed into an exact concept.

3.1. Selection of learning content – polyhedron from curricular, didactical and mathematical aspects

The word polyhedron does not occur in the curriculum. The students learn cube, brick, pyramid, prism and truncated pyramid. They learn also volume and surface calculation methods for some solids. Some knowledge, which is very important for developing of concept of polyhedron, can be found within other topics. For learning spatial geometry and for everyday use of knowledge the developing of spatial orientation is essential. This plays an important role for grades 1–12 in the NAT (Nemzeti Alaptanterv, National Core Curriculum). The names of directions by spatial orientation belong to the requirements in primary school (see [14]). The geometric solid in the primary school is a germ of concept with expressive content, which will be enriched through a variety of tasks during the whole curriculum. (Dealing with building games: free construction, based on model, construction of reflected shapes, classification by 1 or 2 properties; construction the net of the solid. Some of these are physical activities; as such, they are related to the iconic and symbolic (verbal) representation.

The Core Curriculum (NAT) only describes the arc of the development and the teachers are free to choose the content. The students from better social situation have special experience which is the ground of the school work (Board games, and constructive games, domestic work, sport and recreation experiences). They can discuss their experiences in the family. Lack of early reflected experiences is very important in our population. Therefore teachers dealing with low SES students would need more aid for their daily work.

The topic of polyhedrons seems to be rich enough to support the specific learning competences of mathematics and the subject independent development of learning competences at the same time (see [6], [7], [4]). There are many possibilities for connecting mathematical contents with other fields of life and culture. The polyhedron soon aroused the interest of people. The first mass-produced polyhedron was used for buildings, namely clay bricks, whose form, size and proportions were determined by the criteria of high stability. At prehistoric archaeological sites amulets have been found, sometime in the shape of a regular polyhedron. The macroscopic crystals, rocks and minerals provide opportunity for geometrical characteristic observation. The polyhedrons have also symbolic meaning in the astronomy. Dealing with solids and discussing their (not necessarily mathematical) properties can be a very good demonstration for mathematical concept building.

The word is used in everyday language and in school to describe geometrical bodies with planar faces (see [9]). The concept of polyhedron for the mathematics teachers is strongly determined by the structure described in the famous textbook of György Hajós ([12], 26–30. p.). The basis of this structure is in accordance with the everyday meaning of the word polyhedron. The polyhedron is defined as "a solid bounded by flat faces and straight edges…" which includes the classification of interior, exterior and boundary points. (This can only be done in school intuitively. We do not use the various subclasses of polyhedrons described by Hajós in school.) The convex polyhedrons are the most important special polyhedrons, which can be defined independently. Arbitrary polyhedrons can be constructed by the union of finite number of convex polyhedrons, moreover by union of tetrahedrons (see [12], p. 212.). This fact can be used as a definition of polyhedron.

From the mathematical point of view the "polyhedron" is also a complex and demanding topic. The concept of polyhedron is an open research topic of mathematics ([11], [21], [22]). The main field of research is the generalization of polyhedron for higher dimensions.

3.2. Polyhedron in action

The *spatial orientation* is undoubtedly useful in everyday life, related to more complex skills such as using maps, representation of bodies in the plane. The orientation of the plane/space is a useful concept by describing of symmetries and transformations.

"By left-right distinctions related to a non-living object the different conventions known from everyday life cause confusion in problem solving. We must determine the position of the observer in every case if we want to get an unambiguous answer." ([14], summary 3.)

Exercises for the left and right

Lift up the left hand! Lift up the left and right hand!

The picture shows a small boy with a flower in his hand. Which of his hands holds the flower?

The question can be answered without any analysis of the image. The students need not to imagine the flower in the same hand and they can answer according to their own feeling.

You see the grandfather on the photo to the right at the edge of the row. Which one is he?

The difficulty of the task is that the student can choose the right side of the row on the photo and the right side of the photo paper lying in front of him.

Discussion with help of power point presentation:

For the experimental teaching we prepared slide shows and software, which can serve multiple purposes. The characters of the slides, Daddy Bear and Teddy Bear discover some mathematical relationships in each part, and learn new concepts. It is served for the teachers as a model of inquiry-based learning. In the frame of the story the pupils' tasks got meaning and significance.

The pictures show examples of hands-on activity. These are all simple, interesting activities, behind which some deep mathematical relationships could have been found.

The images did not only show examples of hands-on activity but they also moved the children forward to symbolic thinking. Children's experiences were described in words, images and symbols just as was done by characters of slide show.

Daddy Bear and Teddy Bear are making an excursion. Daddy Bear asks tricky questions.

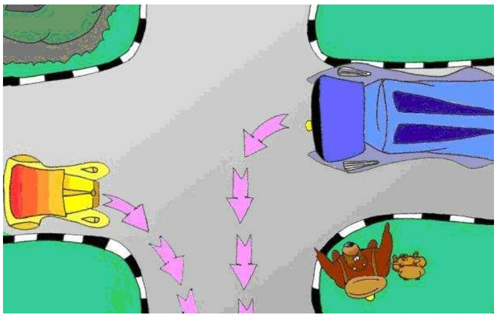
"On which side of the road does the tree stand?"

“On which side of the road does the tree stand according to the direction of travel?”

The new information “according to the direction of travel” made a clear answer possible.

Joyful experiences of this form of discussion encourage students to feel free to request additional information if the task is not clear for them.

Which car turns to the right?



The two cars turn at the intersection to the drawn direction. Which car turns to the right?

The known method – modelling the situation with their own hand – helps find the answer.

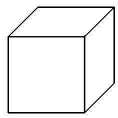
This task was preceded by a task in which a ladybird was driven through the edges of a cube.

Stations of the cube concept development

In addition to hands-on activity pupils got grouping, sorting tasks with pictures and symbols. We tried to optimize the ratio of known and unknown words, because too many new words make learning difficult. However, giving a name has had almost magical quality in the process of acquiring knowledge about the world from ancient times.

We tried to show on a simple polyhedron the noticeable properties of the polyhedron from the mathematical point of view. For this purpose, we used stories and illustrations fitting to the kids’ world, we embedded the polyhedron into a thematic context. For small children definition does not have any significance. They should know some type of polyhedron, mainly the cube, to know the terms face, edge and vertex, and elements of spatial orientation. (See about teaching spatial orientation the research of Herendiné Kónya, E. [14])

For adults the parallel projection of the cube is the most natural image. This form of presentation can be used under age of approximately 10 years just like a symbol of cube, but the children cannot recognize the relations between the elements of cube.



A cube in textbooks

The children from low SES families have fewer experiences with dice and mapping and reconstruction of spatial objects. Therefore, they should collect the experiences even if they are over 10 years of age. For better reorganization we represent the objects in a familiar environment. In our structure the axonometric image of the cube is not the starting point, but the relative closure of a learning curve. The concepts brick, face, edge and vertex are in the focus, because these are essential for describing a polyhedron. The brick with edges of different lengths allows us to communicate better than the cube. They walked first on a cube, along given paths on the polyhedral skeleton by drawing on paper or by a computer program. The main problem in working with real objects or models of solids was that they had to use the known direction words in a mathematics exercise. By the help of the pictures it was easier to understand the exact meaning of terminology which is necessary for spatial orientation.

How many jumps are necessary from the left front leg to the right hip? Which way?

How many jumps forward, up and left?

Knowledge of the polyhedron started by moving on net of the polyhedron.

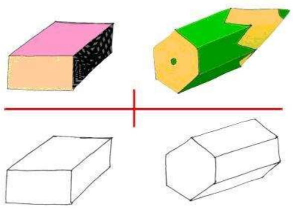
Examination of the spatial directions and using the words left and right, back and forth, up and down also showed the importance of the elements of polyhedrons for the students. Moving on the edges of a polyhedron gives a natural meaning to the elements connected to the experiences.

Bricks are everywhere in everyday environment

The students observed polyhedrons of the natural and artificial environment (partly concrete, partly with help of photos, videos): brick, books, houses, jewelry, minerals, skyscrapers, statues, sculptures. . . . They built an imaginary city from the building blocks of a construction toy. Sometimes they used the experiences of the imaginary trips. The children tried to make their constructions complex without being specifically asked.

Besides having important experiences in statics, they used the recently learned names in communicating with each other. They could observe – making models from paper – the connection between the plane and the space from another aspect. In order to correctly choose the connecting edges of the polygons, they had to activate their spatial experiences.

Back to the geometrical topic



The two types of preparation (moving along edges of a brick and dealing with various polyhedrons) give the foundation to fulfil the requirements of the geometry curriculum. The initial tasks demonstrated the importance and richness of the topic for the students in on an interesting way. They could see the most important bodies and everyday objects transformed into a mathematical object through abstraction.

Apparently we arrived at this point with a long detour, but actually it was not slow. Students clearly see the relationships between the square and the cube; they know the basic properties of the cube. While we have achieved this goal, we solved a variety of other didactic tasks. The students not only caught up, but also gained important new skills. Their problem solving skills were developed, they learned about mathematical models and mathematical modeling. Parallel to the observation of polyhedron morphology and characteristics, new ways towards more abstract topological properties were opened.

Proposing complex problems

For individual work we formulated complex problems on different levels of representation.

Now we show an example of inverse type question (reconstruction of the polyhedrons) formulated on symbolic level.

For which Polyhedron do the data apply?	Number of		
(Write the name or draw a picture!)	edges	faces	vertices
	9	5	6
	8	5	5

The table with some data was not interesting for many students and they left out this task. There was a student who started to work, then gave up, but during the discussion he verified with great interest (and criticism) the "official" solution. Some students solved the problem successfully without help; they wrote the names (triangular prism and square-based pyramid) or drew the figures. They are ranked to the highly talented students on the basis of other difficult tasks as well.

3.3. Experiences

We cannot see the children's intellectual development and improved learning abilities in such a short program. The information – based on the oral and written reports of the teachers and pupils – show that the students worked happily and successfully. We try to illustrate the changes with the help of a simple pre- and post-test.

In the pre-test they gave wrong answers even if they had the model in the hand. Our conjecture was that the reason for wrong answers was not the word "edge" or to count until 12. In accordance with our previous experiences, the children

Pre-test: Counting of edges, faces and vertices B school, 4^{th} grade, December 2007

B school, pre-test	matchbox in hand			imaginary cube		
	edge	face	vertex	edge	face	vertex
Pál	**8**	6	8	**8**	6	8
Laura	**8**	6	8	**8**	**4**	8
Dorina	**8**	6	8	**8**	6	8
Regina	**8**	**4**	8	**8**	6	8
Szabina	**8**	6	8	**6**	6	8
Ágnes	**8**	6	8	**8**	**8**	8

in the sample gave essentially blind guesses for questions about the number of edges, faces, and vertices without control and responsibility. During the pretest the children whispered the number of edges, faces, and vertices of the matchbox in their hands or the imaginary cube into the ear of the schoolmistress.

At the end of the program we made a post-test and everybody could give the perfect solution.

4. Discussion and further pedagogical duties

We made some *correlation calculations* between the answers concerning the different pictures. We found no correlation either between affiliation and achievement or between affiliation and power, but there was weak correlation between achievement and power. The higher achievement motivation may be related to higher respect of authority, or maybe good performance aspiring students are able to see the teacher as a helping people

The analysis of pre- and post-tests shows following changes of the achievement motivation:

by 14% positive, these students reached mostly higher levels(probably due to success experiences by understanding and perform of school duties) within 2%, who started from the lowest level;

by 13% one level drop (probably because of confrontation with given duties and limits);

by 3% drop to the lowest level (probably due to failure experiences by understanding and perform of school duties).

Maybe there is a conflict between the experienced success on the experimental lessons and the mechanical practical duties like homework which are emphasized by the traditional type of lessons. The answer given by a little girl supports this assumption. She wrote 'performing duties' in connection with the bear climbing a hill in the pre-test, but later, in the post-test mentioned a child wanting to climb a hill as a form of physical exercise (s)he likes but has to hurry to accomplish his or her homework. The joy of achievement appeared but did not expand to homework.

These are indicators given by OMT, but the teacher has to find the individual reasons and instructions.

The most important task is to give individual help to 9% students on the lowest level of achievement motivation with inclination to be helpless and passive. These self-insecure, anxious failure-students need help from the teacher and social group. This can be initiated by free choice of group or partners, since in this way help of familiar persons can be requested. If the help on the student level is not possible then the teacher must help individually.

We made *cluster analysis* according to their writing skills, their marks in mathematics and Hungarian language. We excluded the two classes with mentally handicapped children. We chose the hierarchy method and added data up to 9 clusters. In this experiment we did not check common properties of the children with about the same number of points in the same cluster. We wanted to find out whether those attending the same school had similar results in the areas measured. If there was any typical, dominant cluster amongst such children, the answer to the question would have been affirmative. There is a coincidental connection between the children attending the same school and the similarity of their results.

We formed nine clusters based on data. They have no bearing on which schools pupils attend. This result confirms the information from other sources, that our experiences are valid not only for the pupils of schools at issue, but also for other socially disadvantaged students.

It is necessary to develop and try new educational tools with a wide range of applications and to introduce the new devices with detailed descriptions based on teaching experiences.

The significant improvement of the grade in mathematics can only be expected in the long run, but the positive changes in the lessons could be seen immediately.

We laid greater emphasis on the concrete manipulative and the iconic representations according to the principles formulated by Bruner in comparison with the conventional lessons, and regarded the achievement of the symbolic level a process with many stages, the first few of which we tried to walk by the storytelling.

The attention of the teachers was directed to didactic problem solving. The collection of the teachers' reports was also a method for documenting the program.

Follow-up studies are needed to examine the rate of development and durability of the results with methods of the psychology and didactics of mathematics especially for students with different social backgrounds.

The most talented students drew attention by solving difficult problems. This was independent of their social status. The previous "good" and the recently emerged talents were equally active.

There was development in both great sub-areas of the symbolic level. Students became capable of talking and also writing about the events, their memories and experiences at the mathematics lessons. Conversation about mathematics and the learning of mathematics began to emerge. In some cases students felt it necessary themselves to introduce mathematical symbols. The concept map of the students – who struggled with many learning obstacles and appeared to be hopeless in verbal communication – changed significantly; the known but abstract sign and the episodic meaning have been connected.

The difficult problems presented in the context made the creativity possible, the development of the non-identified mathematical talent. For example some students could solve the quite difficult inverse problem given as an optional task, without any tool or picture, at the symbolic level. Apart from logical reasoning, this problem needed such a high level of spatial orientation, that I evaluated as the manifestation of mathematical talent.

At the end of each phase of the development process one needs special sequences of tasks supporting the identification and characterization of talents.

The feedback of teachers and children pointed to the direction that games and puzzles should have been planned explicitly into the phases because in many cases our encouraging suggestions were not enough.

In cases when we sought more effective methods of concept development we analyzed the disadvantageous background and the expected accomplishment of the children in mathematics learning. Educators of such children could have used more help for planning the flow of carrying out the lessons and the assessment of the results. The main interest of teachers involved in the research did not include mathematics (A similar phenomenon could be observed for other teachers); many of the teachers involved in the program became uncertain. We could have helped this problem by giving a more intense mathematical preparation but finding the proper frame for that seems to be difficult. Up to now we can see no institutional frames in which we could find the mutually benefiting long-term collaborative forms of work between researchers and educators. Smoothing away problems could also be done by a closer collaboration between teachers and specialist in talent care.

We have to give in teacher training a more tinged picture about the problems and possibilities of development of children from low SES families, including talent care.

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