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Mathematician Judita Cofman (1936–2001)

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Abstract. Judita Cofman was the first generation student of mathematics and physics at Faculty of Philosophy in Novi Sad, Serbia, and the first holder of doctoral degree in mathematical sciences at University of Novi Sad. Her Ph.D. thesis as well as her scientific works till the end of 70's belong to the field of finite projective and affine planes and the papers within this topic were published in prestigious international mathematical journals. She dedicated the second part of her life and scientific work to didactic and teaching of mathematics and to work with young mathematicians.

Key words and phrases: Judita Cofman, finite projective planes, affine planes, teaching of mathematics, didactic of mathematics.

ZDM Subject Classification: A30, G90.

1. Family and childhood

Mathematician Judita Cofman¹ came from a wellknown and formerly wealthy family of Zoffmanns from Vršac, a little town in Vojvodina, Serbia (Yugoslavia). The exact time when the first members of the Zoffmann family moved to the plains of Banat was not documented, but the literature puts their arrival there at the time of the reign of Maria Theresa of Austria (1717–1780) where they

¹Her name was entered into the Official Register as Judit Zoffmann, but in the documents of Yugoslavia of that time her name was written in its Serbian rendition as Judita Cofman, and that was the name she used for the rest of her life.

came from a German region with a strong beer brewing tradition.² Although the Zoffmanns were originally German, they gradually adopted Hungarian identity, so Judita declared herself as a Hungarian from Vojvodina. The family memory goes back to Sebastian Zoffmann and his wife Barbara, Judita’s great-great-grandparents. The first Zoffmann family member to be mentioned in the official documents of the town of Vršac was Johann, Sebastian and Barbara’s son and Judita’s great-grandfather, as one of the initiators of the foundation, development and modernization of the brewing industry in Vršac. In 1859 he applied for and received a permission for beer production, and only two years later opened a brewery tap with a terrace called “Gambrinus” after the legendary king of Flanders and patron saint of beer.³ The brewery remained a property of the Zoffmanns until 1946 when it was nationalised. Over this period of almost one hundred years, through three generations, the family prospered, and gained and enjoyed a great social renown. Johann started and developed an ice production business, first for his own needs, and then for other consumers in Vršac and the surrounding area; he owned a wine cellar, vineyards, and a factory for the production of cement barrels and was one of the most prominent vintners and wine wholesalers. Johann Zoffmann, as well as his son, Judita’s grandfather Sándor (1864–1932), and later his grandsons, went to Germany and/or France to gain new knowledge and for professional advancement in the field of beer and wine production. Johann and Sándor remained in records as pivotal figures in the renewal of Vršac viticulture after the attack of phylloxera, which was done by the process of grafting onto American species.⁴ Sándor Zoffmann graduated from higher schools in Germany and used the acquired knowledge to modernize the factory and the cellar. Besides wine growing and production, he was a wine merchant trading in bottled, aged and mature wine sorts of Silvaner, Sauvignon, Kretzer, Riesling, Burgundy, and Cabernet, and produced champagne.⁵ Sándor was also engaged as a member of the Executive and Supervisory Board of the Vršac People’s Bank (Vršačka Pučka

²Dragana Kuručev, Portraits of Margit and Šandor Cofman, the piece of the Museum of Vojvodina, 49, 157–163, 2007. In the Hungarian version of the History of Vršac there is written that family Zoffmann came from Murnau in the middle of the 18th century.

³Srećko Mileker, Povesnica slobodne kraljeve varoši Vršca II [The History of the Free Royal Town of Vršac II], Pančevo, 1886, p. 92.

⁴Feliks Mileker, Istorija vršačkog vinogradarstva 1494–1927 [The History of Viticulture in Vršac 1494–1927], Vršac, 2003, pp. 87, 123, and 124.

⁵Eugen Klir, Sorte vinove loze koje se gaje u vršačkom vinogorju [The Vine Sorts Grown in the Vineyards of Vršac], in Istorija vršačkog vinogradarstva [The History of Viticulture in Vršac], Vršac, 2003, pp. 95, 96, and 105.

banka)⁶, and being a keen hunter and a renowned community member he became the President of the Hunters’ Society. There are mentions of his name as a donor in charity activities on public gratitude documents. Judita’s grandmother Margit Zoffmann (1883–1957), born Harsányi, moved to Vršac from central Hungary, was a graduate of the Teachers School in Timisoara and married Sándor quite young.⁷ The family spoke Hungarian at home, but they were aware of the fact that the knowledge of foreign languages was necessary for cooperation and trade in Europe, so the children started learning them at an early age. Such an environment of material and cultural wealth marked the life of Judita’s father Ákos Zoffmann (1910–1974). Having received wide education in Germany, he became a great expert in beer brewing, and wine growing and storing industries. During the German Army occupation of World War II the brewery was closed; only a couple of workers were left to maintain the machines for prevention of deterioration. After the war Ákos was the Technical Manager of the Chocolate Factory as well as the nationalized Brewery. The family moved later to Novi Sad and Judita’s father started working at the Food Industry Institute, drawing plans for the breweries and wine cellars throughout former Yugoslavia.

Judita Cofman was born in Vršac on 4th June 1936. Her mother Lujza (1910–2000), born Kozics, comes from a Hungarian family of lawyers from her father’s side, while her mother came from Vršac. Lujza’s grandfather was a mathematics teacher at the Vršac Grammar School, and her uncle the mayor of the town of Vršac. Despite the uncertainty and horrors of World War II, Judita enjoyed a happy childhood at her family home. She went to primary school in Hungarian and later to Serbian Grammar School (since there was no grammar school in Hungarian in Vršac) in her hometown. The family home, full of love and harmony, installed in her a great feeling that work, study, reading and knowledge of foreign languages are necessary preconditions for success in life. Besides being gifted for mathematics, Judita had a talent for languages, so, besides her mother tongue of Hungarian and the official Serbian language, as a child she learned German, Russian and, which was rare at that time, English. She later learned French and Italian.

⁶Aleksandar Bobik, *Vršačko bankarstvo 1868–1994* [Banking in Vršac 1868–1994], Vršac, 1994, p. 72.

⁷The registry of death of the Roman Catholic Church of St. Gerhard in Vršac, Book XVII, p. 386/25.

2. Studies and career

In the summer of 1954, a decision to initiate the first year of mathematical studies at the Faculty of Philosophy in Novi Sad was made. The first curriculum was based on the study programme of the University of Belgrade. There were 66 students enrolled in the first generation and they were all studying to be teachers of mathematics. Among them was Judita Cofman as well. At that time the majority of classes were given by professors from the University of Belgrade – Miloš Radojčić, Anton Bilimović, Radivoje Kašanin and Jovan Karamata (professor in Geneva at the time) – all members of the Serbian Academy of Sciences and Arts (SANU). Among the mathematicians from Novi Sad, there were Mirko Stojaković and Bogoljub Stanković, while the first assistants were Vojislav Marić and Mileva Prvanović. They were all to become eminent Serbian scientists and also the members of SANU. The first elected Assistant Professor (Docent) at the Mathematics Department was Mileva Prvanović (1956), and it was for the field of geometry. After graduating with the highest grades in 1958 Judita Cofman spent the next two years in Zrenjanin (Nagybecskerek) working as a teacher of mathematics, and in 1960. she was appointed as Assistant of professor M. Prvanović at the Faculty of Philosophy in Novi Sad.

Judita had been the best student of mathematics for generations. Her younger colleagues, later professors at the University of Novi Sad, Irena Čomić and Danica Nikolić Despotović, remember that students had great respect for professors but also some kind of fear for them. Despite all efforts of professors to travel from Belgrade, they were not always accessible to their students. The professional literature in Serbian language was still insufficient at that time, and students could not use foreign titles because their knowledge of foreign languages was generally modest. The only person who was able to answer at any moment a variety of questions by curious students was Judita Cofman. As her knowledge of foreign languages was high, she was almost the only one among the students who could use German, English and Russian textbooks and in this way to wide her knowledge of mathematics, which she used to unselfishly share with her colleagues. They had a feeling that she knew all there was to know about mathematics! As soon as she was made Assistant, in collaboration with students she published the lecture notes *Ruler-and-compass Constructions*. This was the first publication in the field of mathematics issued at the Faculty of Philosophy in Novi Sad and it heralded what was to become an abundant publishing activity at the University of Novi Sad.

The following year, in 1961, she left for postgraduate studies in Roma. There she studied with well-known Italian mathematician Professor Lucio Lombardo–Radice (1916–1982). As Lombardo–Radice contributed to finite geometry and geometric combinatorics together with Guido Zappa (1915) and Beniamino Segre (1903–1977), and wrote important works concerning the Non–Desarguesian Plane, Judita Cofman chose the Non–Desargues Planes as her field of scientific work. In 1963 she returned to Novi Sad and defending her Ph.D. thesis under the title *Finite Non–Desargue Projective Planes Generated by Quadrangle*, she took the first doctoral degree in mathematical sciences from the University of Novi Sad. The committee for her thesis defense consisted of Lombardo–Radice, and professors Mirko Stojaković (mentor) and Mileva Prvanović. As the holder of the Alexander von Humboldt scholarship she spent 1964/65 school year at the University of Frankfurt/Main. The following six years she spent as a lecturer professor at Imperial College in London (University of London), where she was also engaged in her research work. She spent in Novi Sad almost every Christmas holiday of these years, giving a series of lectures at the Faculty of Sciences. In 1970 she was a visiting professor at the University of Perugia (Italy). From 1971 to 1978 she taught mathematics at the universities in Tübingen and Mainz. During this period she took part in three major conferences – International Colloquio on Combinatorial Theory held in Rome, Combinatorial Geometry and Applications held at the University of Perugia and the International Conference on Projective Planes held at the Washington State, which hosted all the important mathematicians of that time whose field of work involved Projective Planes. All the lectures were published in the Conference Proceedings.⁸ She was advisor of the Ph.D. thesis *Teilfaserungen und Parallelismen in endlichdimensionalen projektiven Räumen* defended in 1976 at Gutenberg–Universität Mainz by her student Albrecht Beutelspacher, later very prolific author with more than 30 students and descendants and now professor at University of Giessen.

⁸Colloquio Internazionale sulle Teorie Combinatorie. Tomo II. (Italian) Tenuto a Roma, 3–15 settembre 1973. Atti dei Convegni Lincei, No. 17. Accademia Nazionale dei Lincei, Rome, 1976. pp. 526; Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Università degli Studi di Perugia, Perugia, 11–17 settembre 1970), Istituto di Matematica, Università degli Studi di Perugia, Perugia, 1971. pp. 432; Proceedings of the International Conference on Projective Planes (Washington State University, Pullman, Wash., April 25–28, 1973). Edited by M. J. Kallaher and T. G. Ostrom. Dedicated to the memory of Peter Dembowski. Washington State University Press, Pullman, Wash., 1973. pp. 287.

From 1978 till 1993 she worked at the state Putney High School in London and began to interest in teaching and methodology of mathematics. The academic year 1985–86 she stayed at St Hilda’s College, Oxford, where she enjoyed a teacher fellowship in Trinity term. She spent these years professionally absolutely dedicated to the pedagogical-methodological work with teachers of mathematics and talented students and her activities in this field were numerous. We shall mention some of them. She taught on several advanced masterclasses held at the City of London School, together with teachers Terry Heard and Martin Perkins. In 1987 she was the member of training staff of British Olympic team at 28th International Mathematical Olympiad held on Cuba.⁹ In 1993 she participated at Second German meeting of the European Woman in Mathematics in Tübingen with a talk *On the role of problem solving in math classes* and at 2nd Gauss Symposium in Munich with a talk *Interplay of ideas in teaching mathematics*. She collaborated with several foundations and associations (Sir John Cass Foundation, Advanced Royal Institution Mathematical Classes, Association of Gifted Children in Great Britain). She was organiser, and active participant and lecturer of several inspirational International summer camps for young mathematicians. The number of people that attended the maths camps were also coached by Judita in preparation for the Maths Olympiads (Jeroen Nijhof, Anders Bjorn, Alex Selby for example). She was the editor of journal *Hypotenuse* whose contents was in relation with her seminars and these camps. She also gave regular seminars in Germany for teachers of mathematics and students preparing to become teachers, as well as lectures within didactic seminars (Didaktik der Mathematik – Seminar der Universität Freiburg)¹⁰.

In 1984 Judita started an intensive collaboration with associations and methodological centres dedicated to teaching in the Hungarian language in Vojvodina. She gave a number of teacher training lectures, but she also worked with

⁹Report by Mr. Robert Lyness, leader of the British team at 28th International Mathematical Olympiad, Cuba 1987: Our team was selected by means of the National Mathematics Contest and the British Mathematical Olympiad, followed by some postal tuition and a residential selection/training session which included a further test. This session was held at the Ship Hotel, Reading from Friday 8th May to Sunday 10th May 1987. It was staffed by Judita Cofman, David Cundy, Terry Heard, John Hersee, Paul Woodruff, and myself. The training programme consisted of short lectures and tutorial periods during which the participants had opportunities to expound their own solutions to problems. It proved extremely helpful. All these activities are the responsibility of the Mathematical Association’s “National Committee for Mathematical Contests”.

¹⁰“Mathe mal anders”, *Freizeitaktivitäten für Schüler und Studenten*, lecture held on 05.12.1995.

elementary and high school students in schools of Serbian cities Novi Sad, Subotica and Zrenjanin. Her work is known in Hungary as well. She maintained a close contact with the universities in Budapest, Szeged and Debrecen, and took part in the work of camps for young talents as well as future teachers of mathematics.

From 1993 till her retirement in august 2001 she worked as the professor of didactic of mathematics at University of Erlangen, Nürnberg. In her spare time she conducts “Maths–Workshops” for 13–19 years old youngsters. About her experience gained during these Workshops it could be heard and seen from the video being recorded at the FAU College Alexandrinum (Collegium Alexandrinum) as a part of Projekt Uni-TV. Judita Cofman gave the talk *Mathematik macht Spass! – Über Workshops für Gymnasialschülerinnen und -schüler* am Mathematischen Institut on 24th of June 1999.¹¹ In september 2001 she was invited to participate in the work of the Postgraduate Studies Department for Mathematics Teaching Methods and appointed a professor at the Faculty of Natural Sciences at University of Debrecen (Kossuth Lajos Tudományegyetem), Hungary, where she passed away on 19th December of the same year.

3. Scientific work

Judita Cofman’s Ph.D. thesis as well as her scientific work till the end of 70’s belong to the theory of finite projective planes, Möbius planes and Sperner’s spaces. Her results within these topics were published in prestigious international mathematical journals (Mathematische Zeitschrift, Archives Mathematica, Canadian Journal of Mathematics, for example) and were presented at high ranking conferences devoted to these field of projective geometry. Many of her results and theorems that she posed and/or proved were called Cofman’s Theorem.¹² The scientific work from 1980’s till 2001 was orientated towards the problems of teaching and didactic of mathematics.

3.1. Work in the field of Projective geometry

The beginning of theory of finite projective planes as a result of investigations of foundations of projective geometry is in the thirtieth years of 20th century. At

¹¹See the Web page <http://www.university-tv.de/ca.html> or directly the video of her talk http://giga.rrze.uni-erlangen.de/movies/collegium_alexandrinum/ss99/19990624.mpg

¹²See, for example, H. Lüneburg [1980], N. L. Johnson [2000] and [1988], V. Jha, N. L. Johnson, [1987], P. Sziklai [1997], M. Biliotti, A. Montinaro [2009].

the end of 19th and beginning of 20th century on seek intensively for the models which satisfy one of systems of axioms of geometry. The projective plane, real as well as complex, gave many possibilities for such constructions. The results were reached through algebraic-geometric methods of importing coordinates in a given projective plane, while the theorems, which were geometric in character, were proved on the basis of algebraic structure of the coordinate domain. The divergement of certain branches within the general theory of projective planes yielded the idea of finite projective planes, i.e. the planes consisting of a finite number of points and a finite number of lines. In this case any line is incident with finite number of points, and any point is incident with finite number of lines.¹³ Special topic in theory of finite planes is the geometry of Non-Desarguesian planes, opposite to the geometry of Desarguesian planes depending on validness of Desargues theorem in projective plane.

The title of Judita Cofman’s Ph.D. thesis is *On finite non-Desarguesian projective planes generated by quadrangle*. At the time when J. Cofman prepared her Ph.D, not all of possibly classes of projective planes were known. Even in the case of known planes, there did not exist the general way to find if the plane can or can not be generated by the quadrangle. R. B. Killgrove has conjectured that every finite non-Desarguesian projective plane is singly-generated, i.e. it can be generated by quadrangle.¹⁴

Judita Cofman proved in her Ph.D. thesis that Killgrove conjecture is valid for

- a) all finite Hall planes;
- b) André translation plane of order p^2 (p is a prime number);
- c) translation planes over distributive Hughes quasifield of order p^2 (p is a prime number).

The proofs are based on the fact that the theory of finite projective planes is close connected with theory of finite groups. Simple groups have been studied at least since early Galois theory, where Évariste Galois (1811–1832) realized that the fact that the alternating groups on five or more points was simple (and hence not solvable), which he proved in 1831, was the reason that one could not solve the quintic in radicals. Galois also constructed the projective special linear

¹³For basic definitions and results on the subject of projective planes see [Hall, 1943], [Dem-bowski, 1968], [Hughes and Piper, 1973], [Beutelspacher, 1995], [Coxeter, 1993].

¹⁴Killgrove [1964, p. 68]. On property singly-generatedness, explicit in Hall [1954] and Wagner [1956] and implicate in Hughes [1960], see Section 5 in Killgrove [1964, p. 67] where it is defined.

groups over prime finite fields, $\text{PSL}(2, p)$, which are the next example of finite simple groups. The next discoveries were by Camille Jordan (1838–1922) in 1870. Jordan had found 4 families of simple matrix groups over finite fields of prime order, which are now known as the classical groups.¹⁵

One of the problem investigated both by algebraists and geometers is: Let us consider a simple group G . Which geometrical structure allow the group G as one of their automorphism? And inversely: if we have a finite projective structure J , which simple groups can act on J as its groups of isomorphism? In papers [1969], [1970a] and [1971a], J. Cofman gave the answer to the second question for Möbius planes of even order. Using involutory automorphisms of finite Möbius planes, she gave full classification of non-Abelian simple groups acting on Möbius plane.

Paper Cofman [1966a] is devoted to the Lenz–Barlotti planes class I 6. The Lenz–Barlotti classification for projective planes refines the Lenz classification of projective planes by considering transitive groups of homologies and it is useful in discussing properties of projective planes. The question of the existence of a plane in each of the classes has been studied by many geometers. The projective plane Π is of class I 6 if and only if it contains a point-line pair (B, b) , with B on b , such that

- (i) Π is $X - \Theta(X)$ transitive for each $X \neq B$ on b , where Θ is a one-one correspondence between the points $\neq B$ on b and the lines $\neq b$ on B
- (ii) Π is not $P - l$ transitive for any other pair (P, l) , [Barlotti, 1957].

Strong but inconclusive non-existence results for Lenz–Barlotti class I 6 have been obtained, for the general case by Jónsson [1963], who has used coordinate methods, and for the finite case by Lüneburg [1964], and Cofman [1965], whose proofs are primarily lean on deep results in finite group theory.

T. G. Ostrom and A. Wagner in [1959, p. 186] proved the theorem (Theorem B, Ostrom and Wagner [1959] and Theorem 4, Wagner [1965]) which states that a finite affine plane \mathcal{A} of order $n = 2k + 1$ not a square, admitting a collineation group Δ doubly transitive on the affine points of \mathcal{A} , is a translation plane and Δ contains the translation group of \mathcal{A} . This result has been generalized by Hall [1943] and Hughes [1957a, 1957b], to include the case where n is even and not a square. Judita Cofman also noticed these theorems and in [1967a] she considered the following problem: What can we say about finite affine planes \mathcal{A} of order n possessing collineation groups which are doubly transitive on a subset \mathcal{D} of affine points in \mathcal{A} ? She gave an answer to the above question in the case

¹⁵For theory of groups and projective planes see Hall [1959].

when a finite affine plane \mathcal{A} is of order n , the number of the affine points in set \mathcal{D} is $k > n + 1$ and Δ does not contain planar involutions.¹⁶ For affine planes of non-square order, Cofman’s result represents a generalization of the mentioned theorem obtained by Ostrom and Wagner.

After Cofman’s paper, conclusive results have been obtained for example when \mathcal{A} is a projective plane and \mathcal{D} is an oval in Korchmaros [1978] or a unital in Kantor [1971]. Relevant results are also been achieved when \mathcal{D} consists of a line in Korchmaros [1981] or a line minus a point in Hiramane [1993]. In a recent paper Ganley, Jha and Johnson [2000] essentially classified the line-sized sets \mathcal{D} in a translation plane for Δ non-solvable. Also the case when \mathcal{D} is the line at infinity of a translation plane is now completely settled in Kallaher [1995]. About the role of Ostrom and Wagner works on double transivities on planes from the hystoric point of view see Johnson [1983].

Let π be a projective plane of odd order, Q an oval of π and G a collineation group of π leaving Q invariant. In [1967d] Cofman showed that if

- (i) G acts doubly transitively on Q ,
- (ii) all involutions in G are homologies,

then π is desarguesian of order q , Q is a conic and G contains $\text{PSL}(2, q)$. Afterwards, Kántor [1971] proved that condition (ii) can be weakened by assuming only that G contains involutorial homologies, and Korchmaros [1978] showed that condition (ii) is quite unnecessary, as already conjectured by Dembowski in [1968]. For the further life of Cofman’s results in [1967a] and [1967d], see for example papers by the group of Italian geometars – Maschietti [2006], Biliotti and Montinaro [2005], Biliotti and Francot [2005], Montinaro [2007], Biliotti and Korchmaros [1986], and specially see Johnson, Jha, Biliotti [2007, p. 111, 515, 643, 674].

The Cofman’s papers [1970b], [1972a], [1972b], [1973b] and [1975] are devoted to the subplanes, especialy to the Baer subplanes of finite projective planes, and the papers [1974] and [1976b] to the Sperner spaces. These papers written in the period from 1972 to 1976 are the last Judita’s papers devoted to projective geometry.

Her first result inspired by Baers papers is in Cofman [1970b] where she considered finite projective planes with Baer subplanes admitting Baer involutions. According to Baer [1947] an involution of a finite projective plane of order n is

¹⁶For the similar theorem which is proved by the method with three cases used in [1967a] see McLean [1975].

either a perspective or it fixes a subplane of order \sqrt{n} (in this case n is necessary a square number). In the last case it is called a Baer-involution. It is known (see Hall [1959]) that in a finite Desarguesian projective plane of square order, the vertices of every quadrangle are fixed by exactly one Baer involution. Judita proved the converse i.e. that a finite projective plane in which the vertices of every quadrangle are fixed by exactly one Baer-involution are necessarily Desarguesian.

In [1972a] she investigated Baer Subplanes in finite planes too, but without considering collineations and in [1972b] these results are extended to the planes of infinite order.¹⁷

In the 1960’s Ostrom invented the process of derivation in finite affine planes. Almost all affine planes are derivable due to a choice of their coordinate structures.¹⁸ Cofman in [1975] investigated arbitrary derivable affine planes and established an affine space associated with the derivable plane. This structure is used to show that the Baer subplanes involved in the derivation process are always Desarguesian, thus extending Prohaska [1972] result for finite derivable affine planes. (See Johnson [1990].)

If a projective plane π is said to be derivable i.e. if it contains set \mathcal{D} of points on a line l such that for any two distinct points $A, B \notin l$, with $AB \cap l \in \mathcal{D}$, there exists a Baer subplane π_o through A, B meeting l exactly in the points of \mathcal{D} , in [Cofman, 1975] it is proved that any such subplane is Desarguesian. Extending these results, Johnson [1988] has shown that every derivable affine plane admits a natural embedding into a 3-dimensional projective space.

In 1959 E. Sperner initiated the possibility of constructing weak affine spaces in which in general the axiom of Veblen and the theorem of Desargues are not valid and which generalize the notion of affine spaces. A finite Sperner space is an incident structure $S \equiv (\mathcal{P}, \mathcal{L}, \mathcal{I})$, where the elements of the set \mathcal{P} are called points, these of \mathcal{L} are called (straight) lines, \mathcal{I} is the incident relation, and which possesses an equivalence relation, parallelism, defined on \mathcal{I} such that:

- i) any two distinct points are incident with exactly one line;
- ii) every line is incident with some number of points;
- iii) for any point P and line m , there is exactly one line incident with P and parallel to m .¹⁹

¹⁷An useful survey of contemporary results on Baer subplanes is given in Salzmann [2003].

¹⁸See Ostrom [1960, 1964, 1965], Albert [1966], Johnson [1972].

¹⁹See Samardžiski [1974], Barlotti and Nicolletti [1976], Boros, Szönyi, and Wettl [1987].

An automorphism of S fixing each equivalence class of parallel lines but no points, is said to be translation. S is then called a translation space if translations plus the identity automorphism form a group transitive on \mathcal{P} . In [Cofman, 1974], translation spaces are deduced from spreads of finite projective spaces, while in [Cofman, 1976b] a class of Sperner spaces which can be characterized in terms of affine Baer subplanes is deduced and it is shown that any t -spread of a vector space produces a Sperner space. Cofman’s results on Sperner spaces, especially in the joint paper with A. Barlotti ([1974]) are frequently mentioned and cited in different contexts even nowadays after more than 30 years (for example see [Johnson, 2008], [Mazzocca, Polverino and Storme, 2007], [Ball, Ebert and Lavrauw, 2007], [Lunardon, 1999], [Ebert and Mellinger, 2007], [Blunck, 2002], [Giuzzi and Sonnino, 2009], [Lunardon, 2006], [Mavron, McDonough, and Tonchev, 2008].)

And finally, we mention the Cofman’s subplane problem in Johnson, Jha and Biliotti [2007, p. 111]. Judita Cofman posed the question where there exist finite affine planes that contain more subplanes than does a Desarguesian plane? It turns out that any cyclic semifield plane has more affine subplanes than does the corresponding Desarguesian plane of the same order. Hence, the Cofman problem of finding all orders of planes for which such affine planes exist is completely determined by the theorem of Jha and Johnson [1989, 1987].

3.2. Work in the Field of Mathematical Education

The second period of Judit Cofman’s life and scientific work was marked by theory and practice in the field of pedagogy and didactic of mathematics and was dedicated to students, young mathematicians and their teachers. It was a completely different type of scientific engagement the achievements of which were also totally different. There were no momentary ascents of mind and creation of new systems, no new theories nor proving theorems or conjectures; that what mattered was an overall understanding of teaching mathematics process, approach to students and different teaching methods through a well-balanced proportion between theory and problems designed to motivate students to think and work independently on solving them. Judita Cofman possessed all the preconditions to be successful in the methods of teaching mathematics – she was a mature personality, proven mathematician, and most importantly, she cherished deep and true love for children.

In her pedagogical work, Judita Cofman had the ability to raise simple mathematical truths onto a higher level and turn the elementary into a science. She knew historical genesis of each problem, where it originated from and how it was

solved throughout history. She thought that an important reason for teaching mathematics in schools was to promote independent pupils’ thinking processes and powers of observation.²⁰ Throughout their schooling, pupils should be made aware of the links between various phenomena and they should be given the opportunity to discover these links on their own whenever this is possible. Moreover, pupils should be motivated to search for interdependence between seemingly unrelated topics. How can this be achieved? The key answer to the above question and generally to teaching of mathematics she gave in several papers and five books dedicated to mathematics teaching methods, which represent an outstanding approach to solving non-standard mathematical problems. Her historical approach to science and mathematical problems was the focus of her books which feature problems based on famous topics from the history of mathematics and a selection of elementary problems treated by eminent twentieth-century mathematicians.

The central problem of her engagement in mathematical methodology was how to motivate pupils to think and work independently on their solutions even at an early age.²¹ Judita Cofman thought that one way of introducing youngsters to independent study was to get them involved in work on projects. One such example were her International Camps for young mathematicians held in England and Germany.²² The contents and organization of her well-known book *What to solve? - problems and suggestions for young mathematicians*, as a compilation of problems and solutions discussed during seminars and sessions on problem solving in these camps. The organization of the text, selecting and grouping of questions, comments, references to related mathematical topics, and instructions on teaching process represent the core of Judita’s ideas of teaching mathematics. In the aim of problem solving the campers-pupils (and readers of her book) were led gradually step by step through encouraging independent investigation in finding an answer to a question and demonstrating different approaches to problem solving.²³

The atmosphere at the camp held in the summer of 1984, near Chelmsford in Essex is described well by Heather Cordell, at the time a fifteen-year-old girl from London.²⁴ Here it is her account:

²⁰Cofman, J. [1998, 1].

²¹Cofman, J. [2000].

²²Cofman, J. [1986], Gardiner, T., and Jones, L. [1985].

²³See preface of her book *What to solve?- problems and suggestions for young mathematicians*.

²⁴Heather Cordell is today Professor of Statistical Genetics and a Wellcome Senior Research Fellow in the Institute of Human Genetics at Newcastle University, UK. When she was once asked about persons who contributed to her becoming a scientist, among others, mostly professors

Like its predecessors, the camp in 1984 proved to be a resounding success. This was in no small way helped by the fact that its participants came from all over Europe, from nine different countries altogether, each bringing his/her own way of looking at mathematics and tackling mathematical problems.

An average working day at the camp would start at 9:00 a.m. with a demonstration of problem-solving techniques. These problems varied from day to day in both standard and topic, so that a wide variety of interest and ability could be catered for. Examples included a proof of the existence of an infinite number of primes, ways of solving problems by the “pigeon hole principle”, and many others. Next came a session of project work. The projects were stimulated by particular problems (from a list provided) or were chosen by the students themselves. A practical interest in bell-ringing inspired one participant to investigate the various permutations possible on a certain set of bells. Many of the projects were a result of combined efforts; a more advanced student could often use his/her knowledge to work with a less advanced but equally dedicated student. In the second week, the students themselves led discussions about their results and difficulties with their projects.

The first afternoon session lasted from 3:00 p.m. to 4:00 p.m., and consisted of a lecture a guest speaker or one of the tutors. The topics again varied, but usually involved some less traditional subjects such as codes and ciphers, non-conventional geometries, topology and ways to win “Nim”. The lecture contents would be expanded in the second afternoon session, from 5:00 p.m. to 6:00 p.m., to give more insight for the advanced participants. The fourth session was not compulsory – but many younger students did attend and enjoy what they could understand.

Of course, there were many other opportunities for both relaxation and study. These included a visit to London (alternatively a country walk for those who preferred), an invitation watch the local bell-ringing and day spent in Cambridge, with two excellent lectures on “Convex sets and their applications in Economics” and “Algorithms”. All in all, everyone had a most enjoyable fortnight and is looking forward to participating again next year.

*In her article *On the Role of Geometry in Contemporary Mathematical High School Education* ([1996b]), Judita Cofman gave her remarks and ideas concerned primarily to the teaching of geometry, based on her experiences with high school*

and colleagues from University, she mentioned Judita Cofman as her high school mathematics teacher. (See the article *Moving from Promise to Proficiency*, *Scientists*, 17, 8, p. 56.)

students. But these thoughts relating to the teaching of geometry could be easily apply to Judita’s conception of entire mathematical education. She wrote:

In the contemporary education, with a curriculum overburdened with details from various fields of mathematics, there is a danger that the study of mathematics can stray into memorising facts and a mechanical learning of algorithms. On the contrary to this, the efforts should be directed at pupils’ understanding the existing links between the phenomena they encounter in different fields of mathematical study. Geometry can play a certain role in such efforts, because the mathematical disciplines taught in high school are rich in details for which there are geometrical illustrations appropriate to the pupils’ age. The application of such illustrations, on the one hand, facilitates the process of understanding of the totality of teaching material, and on the other, presents geometry as a science of an actual importance. Teaching of geometry can also play a useful role in illustrating the achievements in the most current fields of mathematics. The knowledge gained in the study of geometry can contribute to a better understanding of the phenomena from different fields of natural sciences.

The importance of Euclidean geometry in teaching is supported by the fact that the shapes of this geometry are encountered in our living environment. The study of space is particularly facilitated by the study of solid geometry, which is, unfortunately, often neglected in syllabus and curriculum. There is an important fact in respect to geometrical features of space which is often forgotten; the majority of children possess a lot of elementary knowledge about objects, such as the cube or the sphere from the earliest age. This elementary knowledge can be extremely useful for introducing notions such as: defined and undefined elements, axioms and theorems, necessary and sufficient conditions, etc. All these notions are important for the field of mathematics – while the familiarity with space can be used to make pupils grasp the essence starting from concrete examples.

Mathematics is one of the earliest scientific disciplines, an important segment of human cultural heritage. This fact must be reflected on the teaching of mathematics: it is advisable to draw pupils’ attention, whenever an opportunity arises, to their historical background. The history of geometry is an important part of the history of mathematics, not only because geometry is one of the oldest branches of mathematics. The importance of geometry mostly lies in the fact that there were several major problems in this field, starting from the Ancient Greek age, which could finally be solved only in the 19th century. The solutions to these problems had been sought for for ages; the attempts led to a series of new discoveries and contributed to a further development of the entire science of

mathematics. One of the famous problems of geometry was the so called Delian problem of doubling the cube.

For teaching of geometrics to be successful, teaching personnel must have a solid knowledge of this subject. However, not only at schools, but also in university courses and other pedagogical institutions for training future mathematics teachers, there is a tendency of neglecting the study of geometry. This fact can lead to a drastic deterioration in the level of geometry teaching at schools. What is needed is an effort at elevating the respectability of geometry with the students of mathematics.

4. Conclusion

In the period from 1964 to 1977 Judita Cofman worked in the field of projective geometry – a very up-to-date and lively mathematical field, closely related to algebra and group theory. Her results complemented the results of many great geometricians of the early 20th century on the one hand, and on the other, the active follow-up and advancement of certain subfields of projective geometry rest upon her results. In the next period of 20 years, from 1980 till 2001, Judita wrote no papers on projective geometry. During this period she was completely orientated to the mathematical education and didactic of mathematics and improvements in teaching process, especially working with gifted teenage mathematicians.

Judita Cofman collaborated with a number of universities around the Europe, which prepare future mathematical educators, and she also had an intensive cooperation with institutes for mathematics and associations of mathematicians. She constantly emphasized the importance of the quality of teaching of mathematics, from the lower grades of elementary school to university level. She was known for being an exceptionally good teacher, who had a very responsible attitude towards this profession, which was the result of her great respect for her audiences and science, and prepared thoroughly for her lectures.

Cofman worked with most prominent world’s scientists mathematicians, but she equally collaborated with elementary and high school teachers, who she treated as her peers. She would speak with an elementary school pupil with a great appreciation of his person and opinion. Moreover, everyone who had an opportunity to hear her, had a chance to learn the true meaning of being humane and modest, and how to appreciate other people’s work and person. She so wholeheartedly thanked for each collaborative effort, as if she had not been the one who contributed more to the joint work and shared knowledge.

It is not easy to judge which of the results of two scientific period of Judita Cofman have greater importance. Splendor of results related to the projective geometry from the first period of her scientific activity, does not fade over time. They are still, after more than 30 years after their creation, cited in the articles and monographs of many geometers. But in the wider mathematical world, Judita Cofman is perhaps better known for his pedagogical work and for ideas related to the teaching of mathematics.

5. Published papers by Judita Cofman

1. 1964a. The validity of certain configuration-theorems in the Hall planes of order p^{2n} . *Rendiconti Lincei – Matematica e Applicazioni* (5) **23**, 22–27.
2. 1964b. Perspectivity of finite projective planes. O perspektivitetima konačnih projektivnih ravni. (*Serbian*) *Matematički Vesnik* **1** (16), 141–147.
3. 1965. Homologies of finite projective planes. *Archiv der Mathematik (Basel)* **16**, 476–479.
4. 1966a. On the nonexistence of finite projective planes of Lenz–Barlotti type I_6 . *Journal of Algebra* **4** (1), 64–70.
5. 1966b. On a characterization of finite desarguesian projective planes. *Archiv der Mathematik (Basel)* **17** (3), 200–205.
6. 1967a. Double transitivity in finite affine planes. I. *Mathematische Zeitschrift* **101**, 335–342.
7. 1967b. On a conjecture of Hughes. *Mathematical Proceedings of the Cambridge Philosophical Society* **63**, 647–652.
8. 1967c. Triple transitivity in finite Möbius planes. Atti della Accademia Nazionale dei Lincei. *Rendiconti. Classe di Scienze Fisiche, Matematiche e Naturali* (8) **42**, 616–620.
9. 1967d. Double transitivity in finite affine and projective planes. Atti della Accademia Nazionale dei Lincei. *Rendiconti. Classe di Scienze Fisiche, Matematiche e Naturali* (8) **43**, 317–320; Proceedings of Projective geometry conference (University of Illinois, Chicago 1967), 16–19.
10. 1968. Transitivity on triangles in finite projective planes. *Proceedings of the London Mathematical Society* (3) **18**, 607–620.

11. 1969. Translations in finite Möbius planes. *Archiv der Mathematik (Basel)* **19** (6), 664–667.
12. 1970a. Inversions in finite Möbius planes of even order. *Mathematische Zeitschrift* **116**, 1–7.
13. 1970b. On Baer involutions of finite projective planes. *Canadian Journal of Mathematics – Journal Canadien de Mathematiques* **22**, 878–880.
14. 1971a. J., Simple groups and Möbius planes of even order. *Mathematische Zeitschrift* **120**, 299–306.
15. 1971b. Triple transitivity in projective planes of even order. *Archiv der Mathematik (Basel)* **22**, 556–560.
16. 1971c. On automorphism groups of finite geometries. *Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Università di Perugia, Perugia, 1970)*, 173–174. Istituto di Matematica, Università degli Studi di Perugia, Perugia, iii+432.
17. 1972a. Baer subplanes in finite projective and affine planes. *Canadian Journal of Mathematics – Journal Canadien de Mathematiques* **24**, 90–97.
18. 1972b. Baer subplanes of affine and projective planes. *Mathematische Zeitschrift* **126**, 339–344.
19. 1973a. On combinatorics of finite projective spaces. *Proceedings of the International Conference Projective Planes (Washington State Univ., Pullman, Wash., 1973)*, 59–70. Washington State University Press, Pullman, Wash.
20. 1973b. On subplanes of affine and projective planes. *Geometriae Dedicata* **2** (2), 195–199.
21. 1973c. The significance of t -schemes in finite geometry. (Hungarian: Trendszerék jelentősége a véges geometriákban) *A Magyar Tudományos Akadémia Matematikai és Fizikai Tudományok Osztályának Közleményei* **21**, 399–407.
22. 1974. (with Barlotti, A.) Finite Sperner spaces constructed from projective and affine spaces. *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **40**, 231–241.
23. 1975. Baer subplanes and Baer collineations of derivable projective planes. *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **44**, 187–192.

24. 1976a. Configurational propositions in projective spaces. Foundations of geometry. Proceedings of the Conference, University of Toronto, Toronto, Ontario, 1974, University Toronto Press, 16–53.
25. 1976b. On combinatorics of finite Sperner spaces. Colloquio Internazionale sulle Teorie Combinatorie (Rome, 1973), Tomo II, 55–56. *Atti dei Convegni Lincei*, No. 17, L’Accademia Nazionale dei Lincei, Rome.
26. 1977. On ternary rings of derivable planes. *Mathematische Zeitschrift* **154** (2), 157–158.
27. 1981. Operations with Negative Numbers. *Mathematics Teaching* **94**, 18–20.
28. 1983. Mathematical Activities for Motivated Pupils. *Gifted Education International*, **2** (1), 42–44.
29. 1986. Thoughts around an international camp for young mathematicians. *The Mathematical Intelligencer* **8** (1), 57–58.
30. 1992. Verallgemeinerung der Fibonacci-Folge. Projekte für Schuelerzirkel. *Praxis der Mathematik* **34** (4), 157–160.
31. 1993. From Fibonacci numbers to fractals (Hungarian: Fibonacci-féle számtól a fraktálokig). *Möhelysarok, Polygon*, **3**, No. 1, 91–102
32. 1994a. Šestougaonik ili kocka? Ispitivanje trodimenzionalnih objekata u osnovnoj školi (Serbian), [Hexagon or square? Investigation of three-dimensional objects in the primary school]. *Nastava matematike XXXIX* 1–4, 1–5.
33. 1994b. Čas ponavljanja u jednoj školi u engleskoj sa jedanestogodišnjim učenicima (Serbian), [Class reps in a school in England with eighteen-year students]. *Nastava matematike XXXIX* 1–4, 41–43.
34. 1994c. Zahlen- und Figurenmuster. *Didaktik der Mathematik*, 245–254
35. 1995a. Interplay of ideas in teaching mathematics. Proceedings of the 2nd Gauss Symposium. Conference A: Mathematics and Theoretical Physics (München, 1993), *Symposim Gaussiana, de Gruyter, Berlin*, 85–95.
36. 1995b. Patterns of shape and numbers, 7th International Geometry Conference, Haifa, Israel, 1995. *Zentralblatt für Didaktik der Mathematik*, **27** (5), 153–156.
37. 1996a. Bemerkungen zu Zuluagas Beweis des Satzes von Pythagoras. *Praxis der Mathematik* **38** (6), 269–270.
38. 1996b. O ulozi geometrije u savremenom matematičkom obrazovanju u srednjoj školi (Serbian), [On the role of geometry in the modern mathematical

- education in high school]. *Metodika i istorija geometrije, Matematički vidici* **7**, 12–25.
39. 1996c. Lösungsmethoden einer Aufgabe über ein Minimum. *Praxis der Mathematik* **38** (4), 180–181.
 40. 1997a. Catalan numbers for the classroom? *Elemente der Mathematik* **52** (3), 108–117.
 41. 1997b. Bestimmung der kleinsten einbeschriebenen Quadrats. *Praxis der Mathematik* **39** (5), 205–207
 42. 1998a. Explorations and discoveries in the classroom. *The Teaching of Mathematics* I–1, 23–30.
 43. 1998b. The role of finite geometries in modern mathematics (Hungarian: A véges geometriák szerepe a modern matematikában.) Plenary lecture on the International Hungarian Mathematics Competition (1998, Subotica) in Kántor Sándorné – Kántor Sándor: International Hungarian Mathematics Competitions (1992–2003) Studium, Debrecen, 2003, 262–264 (Hungarian: Nemzetközi magyar matematikai versenyek (1992–2003))
 44. 1999a. Über Aufgaben mit 'beweglichen Elementen'. *Der Mathematikunterricht* **45** (1), 37–51.
 45. 1999b. La nice [lattice] paths with U -terms and generalized Pascal's triangles. *Octagon Mathematical Magazine* **7** (1), 20–26.
 46. 1999c. Zur Einführung, im: Vergleichen, Ordnen und Klassifizieren, *MU Der Mathematikunterricht* **45** Heft 5, 3–4.
 47. 1999d. Number Series and coordinates (Hungarian: Sz' amsorok és koordináták (Egy középiskolai szakkör munkájából) Műhelysarok, Polygon IX. No. 1. 45–54
 48. 2001a. (with Merkel, C.) Geometric models for handling with. (Modelle zum Anfassen im Mathematikunterricht) (German). *Mathematikunterricht (Seelze)* **47** (2), 4–29.
 49. 2001b. Properties of convex n -gons. (Eigenschaften konvexer n -Ecke) (German). *Mathematik Lehren* **105**, 49–53.
 50. 2002. How to motivate 10–18 years old pupils to work independently on solving mathematical problems. *The Teaching of Mathematics*, Vol. III-2, 2000, 83–94. Plenarno predavanje Kako motivisati 10–18-godišnje učenike da u toku nastave samostalno rešavaju razne matematičke zadatke, 10. kongres

matematičara Jugoslavije, Beograd 21–24. januar 2001. Nastava matematike XLVII-1–2, 2002, 11–23.

Books by Judita Cofman

1. Problems for young mathematicians, *Pullen (Knebworth)*, 1981, 66.
2. What to solve? Problems and suggestions for young mathematicians. Oxford Science Publications. *The Clarendon Press, Oxford University Press, New York*, 1990, xiv+250.
3. Numbers and shapes revisited. More problems for young mathematicians. *The Clarendon Press, Oxford University Press, New York*, 1995, xii+308 pp.
4. Insight into the history of mathematics I. Problems and materials for lower secondary and teacher education. (Einblicke in die Geschichte der Mathematik I. *Aufgaben und Materialien für die Sekundarstufe I (German) Spektrum Akad. Verl., Heidelberg*, 1999, 326.
5. Insight into the history of mathematics II. Problems and materials for the upper secondary and teacher education. (Einblicke in die Geschichte der Mathematik II. *Aufgaben und Materialien für die Sekundarstufe II und das Lehramtsstudium.*) (German) *Spektrum Akad. Verl., Heidelberg*, 2001, 426.
6. (with Pejić, S.) Matematički projekti 4–5, uputstva za rešavanje zadataka sa napomenama i predlozima za nastavnike (Serbian), [Instructions for solving tasks with notes and suggestions for teachers], *Eduka*, Beograd, 2002, 14.

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References

- [1] A. Albert, The finite planes of Ostrom, *Boletín de la Sociedad Matemática Mexicana* **11**, no. 2, 1966, 1–13.
- [2] J. André, Über Perspektivitäten in endlichen projektiven Ebenen, *Archiv der Mathematik (Basel)* **6**, 1954, 29–32.
- [3] J. André, Über nicht-Desarguesche Ebenen mit transitiver Translationsgruppe, *Mathematische Zeitschrift* **60**, 1954, 156–186.
- [4] R. Baer, Projectivities of finite projective planes, *American Journal of Mathematics* **69**, no. 4, 1947, 653–684.
- [5] R. Baer, Projectivities with fixed points on every line of the plane, *Bulletin of the American Mathematical Society* **52**, 1946, 273–286.
- [6] S. Ball, G. Ebert and M. Lavrauw, A geometric construction of finite semifields, *Journal of Algebra* **311**, no. 1, 2007, 117–129.
- [7] A. Barlotti, Le possibili configurazioni del sistema delle coppie punto-retta (A, a) per cui un piano grafico risulta A - a transitivo, *Bollettino della Unione Matematica Italiana* **12**, 1957, 212–226.
- [8] A. Barlotti and G. Nicolletti, On a geometric procedure for the construction of Spener spaces, *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **45**, no. 1, 1976, 251–255.
- [9] A. Beutelspacher, *Projective planes*, in Handbook of Incidence Geometry, (F. Buekenhout, ed.), North-Holland, 1995, 107–136.
- [10] M. Biliotti and G. Korchmaros, Collineation groups which are primitive on an oval of a projective plane of odd order, *Journal of the London Mathematical Society (2)* **33**, 1986, 525–534.
- [11] M. Biliotti and E. Francot, Two-transitive orbits in finite projective planes, *Journal of Geometry* **82**, no. 1–2, 2005, 1–24.
- [12] M. Biliotti and A. Montinaro, Finite projective planes of order n with a 2-transitive orbit of length $n - 3$, *Advances in Geometry* **6**, no. 1, 2006, 15–37.
- [13] M. Biliotti and A. Montinaro, Variations on a theme of Cofman, *Note di Matematica* **29**, suppl., no. 1, 2009, 11–22.
- [14] A. Blunck, A new approach to derivation, *Forum Mathematicum* **14**, 2002, 831–845.
- [15] E. Boros, T. Szönyi and F. Wetzl, Spener extensions of affine spaces, *Geometriae Dedicata* **22**, no. 2, 1987, 163–172.
- [16] H. S. M. Coxeter, *The Real Projective Plane*, 3rd ed. Cambridge, England: Cambridge University Press, 1993.
- [17] G. L. Ebert and K. E. Mellinger, Mixed partitions and related designs, *Designs, Codes and Cryptography* **44**, no. 1–3, 2007, 15–23.
- [18] M. J. Ganley, V. Jha and N. L. Johnson, The translation planes admitting a non-solvable doubly transitive line-sized orbit, *Journal of Algebra* **69**, 2000, 88–109.
- [19] T. Gardiner and L. Jones, Saturday morning maths, *Mathematics in School* **14**, no. 2, 1985, 35–37.

- [20] L. Giuzzi and A. Sonnino, LDPC codes from Singer cycles, Vol. 157, *Discrete Applied Mathematics*, 1723–1728.
- [21] A. M. Gleason, Finite Fano planes, *American Journal of Mathematics* **78**, 1956, 797–807.
- [22] D. Gorenstein, The classification of finite groups with dihedral Sylow 2-subgroups, *Symposium on Group Theory*, 1963, 10–15, (Harvard, 1963).
- [23] M. Hall, Jr., Correction to “Uniqueness of the projective plane with 57 points”, *Proceedings of the American Mathematical Society* **5**, 1954, 45–52.
- [24] M. Hall, Projective planes, *Transactions of the American Mathematical Society* **54**, no. 2, 1943, 229–277.
- [25] M. Hall, Jr., *The theory of groups*, , The MacMillan Company, New York, 434, 1959.
- [26] Y. Hiramane, On finite affine planes with a 2-transitive orbit on l_∞ , *Journal of Algebra* **162**, 1993, 392–409.
- [27] D. R. Hughes, Generalized incidence matrices over group algebras, *Illinois Journal of Mathematics* **1**, 1957a, 545–551.
- [28] D. R. Hughes, Collineations and generalized incidence matrices, *Transactions of the American Mathematical Society* **86**, no. 2, 1957b, 284–296.
- [29] V. Jha and N. L. Johnson, *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **57**, 1987, 127–137.
- [30] V. Jha and N. L. Johnson, An analog of the Albert–Knuth theorem on the orders of finite semifields, and a complete solution to Cofman’s subplane problem, *Algebras Groups Geometry* **6**, 1989, 1–35.
- [31] N. L. Johnson, The geometry of Ostrom, *Finite geometries: proceedings of a conference in honor of T. G. Ostrom* By Theodore Gleason Ostrom, Norman Lloyd Johnson, M. J. Kallaher, Calvin T. Long. *Lecture notes in pure and applied mathematics* **82**, 454, 1983.
- [32] N. L. Johnson, Derivable nets and 3-dimensional projective spaces, *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **58**, 1988, 245–253.
- [33] N. L. Johnson, Derivation by coordinates, *Note di Matematica* **10**, no. 1, 1990, 89–96.
- [34] N. L. Johnson, *Sub-plane Covered Nets*, Vol. 222, *Monographs and Textbooks in Pure and Applied Mathematics*, Marcel Dekker Inc., New York, 2000.
- [35] N. L. Johnson, W. Jha and M. Biliotti, *Handbook of Finite Translation Planes*, *Chapman & Hall/CRC, Taylor and Francis Group*, 861, 2007.
- [36] N. L. Johnson, Extended André Sperner spaces, *Note di Matematica* **28**, no. 1, 2008, 141–162.
- [37] W. J. Jónsson, Transitivität und Homogenität projektiver Ebenen, *Mathematische Zeitschrift* **80**, 1963, 269–292.
- [38] M. Kallaher, *Translation Planes*, *Handbook of incidence geometry*, (F. Buekenhout, ed.), Elsevier, 1995, 137–192.
- [39] W. M. Kantor, On unitary polarities of finite projective planes, *Canadian Journal of Mathematics – Journal Canadien de Mathématiques* **23**, 1971, 1060–1077.

- [40] R. B. Killgrove, Completions of Quadrangles in projective planes, *Canadian Journal of Mathematics – Journal Canadien de Mathematiques* **16**, 1964, 63–76.
- [41] G. Korchmaros, Una proprieta gruppale delle involuzioni planari che mutano in se un’ovale di un piano proiettivo finito, *Annali di Matematica Pura ed Applicata* **116**, no. 4, 1978, 189–205.
- [42] G. Korchmaros, Collineation groups doubly transitive on the points at infinity in an affine plane of order 2^r , *Archiv der Mathematik (Basel)* **37**, 1981, 572–576.
- [43] G. Lunardon, Normal Spreads, *Geometriae Dedicata* **75**, 1999, 245–261.
- [44] G. Lunardon, Blocking sets and semifields, *Journal of Combinatorial Theory, Series A* **113**, 2006, 1172–1188.
- [45] H. Lüneburg, Endliche projektive Ebenen von Lenz–Barlotti Typ I 6, *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **27**, 1964, 75–79.
- [46] H. Lüneburg, *Translation Planes*, Springer–Verlag, Berlin, 1980.
- [47] A. Maschietti, Two transitive ovals, *Advances in Geometry* **6**, no. 2, 2006, 323–332.
- [48] V. C. Mavron, T. P. McDonough and V. D. Tonchev, On affine designs and Hadamard designs with line spreads. *Discrete Mathematics*, Vol. 308, no. 13, 2008, 2742–2750.
- [49] F. Mazzocca, O. Polverino and L. Storme, Blocking Sets in $PG(r, qn)$, *Designs, Codes and Cryptography* **44**, no. 1–3, 2007, 97–113.
- [50] J. T. McLean, On Double Transitivity in Finite Affine Planes, *Mathematische Zeitschrift* **144**, 1975, 277–282.
- [51] A. Montinaro, Large doubly transitive orbits on a line, *Journal of the Australian Mathematical Society* **83**, no. 2, 2007, 227–269.
- [52] T. G. Ostrom, Doubly transitivity in finite projective planes, *Canadian Journal of Mathematics – Journal Canadien de Mathematiques* **8**, 1956, 563–567.
- [53] T. G. Ostrom, Transitivity in projective planes, *Canadian Journal of Mathematics – Journal Canadien de Mathematiques* **9**, 1957, 389–399.
- [54] T. G. Ostrom and A. Wagner, On projective and affine planes with transitive collineation groups, *Mathematische Zeitschrift* **71**, 1959, 186–199.
- [55] T. G. Ostrom, Translation planes and configurations in Desarguesian planes, *Archiv der Mathematik (Basel)* **11**, 1960, 457–464.
- [56] T. G. Ostrom, Semi-translation planes, *Transactions of the American Mathematical Society* **111**, no. 1, 1964, 1–18.
- [57] T. G. Ostrom, Collineation groups of semi-translation planes, *Pacific Journal of Mathematics* **15**, 1965, 273–279.
- [58] G. Pickert, *Projektive Ebenen*, Springer, Berlin, 1955, 2nd edition, 1975.
- [59] O. Prohaska, Endliche Ableitbare Affine Ebenen, *Geometriae Dedicata* **1**, 1972, 6–17.
- [60] H. Salzmann, Baer subplanes, *Illinois Journal of Mathematics* **47**, no. 1–2, 2003, 485–513.

- [61] A. Samardžiski, A class of finite Sperner spaces, *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg* **42**, no. 1, 1974, 205–211.
- [62] P. Sziklai, *Mind the Gap*, Ph.D. Thesis, 1997.
- [63] R. Taton, *Introduction biographique*, In Desargues, *L’oeuvre mathématique*, (Paris), 1951, 1–67.
- [64] A. Wagner, On finite non-desarguesian planes generated by 4 points, *Archiv der Mathematik (Basel)* **7**, 1956, 23–27.
- [65] A. Wagner, On perspectivities of finite projective planes, *Mathematische Zeitschrift* **71**, 1959, 113–123.
- [66] A. Wagner, On finite affine line transitive planes, *Mathematische Zeitschrift* **87**, 1965, 1–11.

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