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Teaching Mathematics and Computer Science

CAS-aided Visualization in LaTeX documents for Mathematical Education

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Abstract. We have been developing *KETpic* as a macro package of a CAS for drawing fine LaTeX-pictures, and we use it efficiently in mathematical education. Printed materials for mathematics classes are prepared under several constraints, such as "without animation", "mass printings", "monochrome", and "without halftone shadings". Because of these constraints, visualization in mathematical education tends to be unsatisfactory. Taking full advantages of LaTeX and CAS, KETpic enables us to provide teaching materials with figures which are effective for mathematical education. The effects are summarized as follows:

(1) The plottings of KETpic are accurate due to CAS, and enable students to deduce mathematical laws.

(2) KETpic can provide adequate pictures for students' various interest. For example, when some students who understand a matter try to modify it, KETpic can give them appropriate and experimental figures.

(3) Even though CAS can draw 3D-figures beautifully and automatically, it is expensive for mass printings and the figures are sometimes not easy to understand. Oppositely, 3D-graphics by KETpic are monochrome, but are richly expressive.

In this paper, we give various examples of LaTeX-pictures which we drew by using KETpic. For instance, the picture which is used in order to explain the convergence theorem of Fourier series makes it easier for students to understand the idea that function series converge to another function. Also the picture of skeleton is endowed with clear perspective. KETpic gives us great potential for the teaching of combinatorial mathematics. Through these examples, we claim that KETpic should have great possibilities of rich mathematical expressions under the constraints above mentioned.

Key words and phrases: LaTeX, CAS, accuracy, mathematical expressions, accessories, skeleton.

ZDM Subject Classification: U15, U65.

1. Introduction

When mathematicians or mathematics teachers need to insert pictures (for example, the output of CAS) into their LaTeX documents, many of them will do so with the data of the pictures formatted into the style of EPS file. Then such EPS files are inserted into the LaTeX text file by using command "includegraphics". This procedure also fits in case of various graphic softwares or Gnuplot. Furthermore, the command of Maple and Mathematica which generates LaTeX file follows the above procedure.

However, this method is not satisfactory for us. The reasons are as follows:

(1) The size of EPS file is large. So it is not easy to attach the file to email.

(2) Graphics can not be corrected as we like. For example, the size of the figure is not easily handled.

(3) Mathematical expressions (legends, accessories, hatchings, etc.) are poor.

(4) What we want to do (drawing tangential line to a curve at a specified point, drawing curves defined by implicit functions, etc.) are not supported.

We have developed KETpic so as to overcome these problems in case of inserting the output of CAS. It is a macro package for CAS (Maple, Mathematica, etc). In this paper, we focus on KETpic for Maple [6]. However it can be extended to other CASs, and we will also show an example of KETpic for Mathematica. By using KETpic, graphical outputs of CAS are formatted to generate graphical codes (Tpic specials) and are written on text files. We insert these files into the LaTeX document by the command "input". Remark that the output of KETpic is just a text file, which is much smaller than EPS, JPEG, PDF, etc. Therefore, it is far easier to attach the output to email.

In this paper, we explain the effectiveness of KETpic for mathematical education, using various examples of figures drawn with KETpic.

2. Accurate graphics in LaTeX document

We pick up an example of the convergence theorem of Fourier series. The periodic function with period 2π:

$$f(x) = \begin{cases} 1 & (0 \leq x < \pi) \\ -1 & (-\pi \leq x < 0) \end{cases}$$

has a Fourier series expansion:

$$f(x) = \frac{\pi}{4} \sum_{k=1}^{\infty} \frac{\sin(2k-1)x}{2k-1}.$$

The graph of the 10th term approximation

$$f_{10}(x) = \frac{\pi}{4} \sum_{k=1}^{10} \frac{\sin(2k-1)x}{2k-1}$$

is as follows:

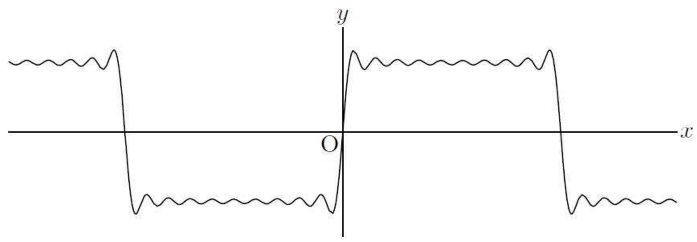


Figure 1. The graph of $f_{10}(x)$

Here we show the step to insert the above figure into LaTeX document.

First, the Maple source file is composed of the following three steps:

(1) Loading KETpic

By the command below, Maple reads KETpic in the folder "c:\\Workfile".

```
> read ‘c:\\Workfile/ketpicw.m‘:
```

Here "ketpicw.m" is the Windows and Unix version of KETpic. There is also Mac version. These are downloadable from web site [5]. Also remark that double backslash means a single backslash in Maple.

(2) Calculation of numerical data

```
> setwindow(-1.5*Pi..1.5*Pi,-1.5..1.5):    #KETpic command
> f:=(4/Pi)*sum(sin((2*k-1)*x)/(2*k-1),k=1..N);
> N:=10:
> g:=plot(f,x=XMIN..XMAX,numpoints=100):
```

(3) Writing down the numerical data onto LaTeX file

In this step, KETpic writes down the numerical data obtained by step (2) onto the LaTeX file named "fig1.tex". These commands are proper to KETpic. Remark that the second line defines the unit length of the picture.

```
> openfile(`c:\\fig1.tex`):
> openpicture("1cm"):
> drwline(g):
> closepicture():
> closefile():
```

Next, the structure of the LaTeX text is very simple as follows:

(4) Insertion of LaTeX file of graphic codes

In preamble part, the following commands are always neccessary to define the length parameter used by KETpic.

```
\newlength{\Width}
\newlength{\Height}
\newlength{\Depth}
```

By the command "input", the file "fig1.tex" is inserted into the LaTeX text.

```
\begin{document}
\input{fig1}
\end{document}
```

Compiling the LaTeX text, we obtain the previous figure in LaTeX document.

If we want to obtain more precise approximation (for example the 30th term approximation), only the substitution of the command "N:=10:" by "N:=30:" will do. In fact, we obtain the following picture:

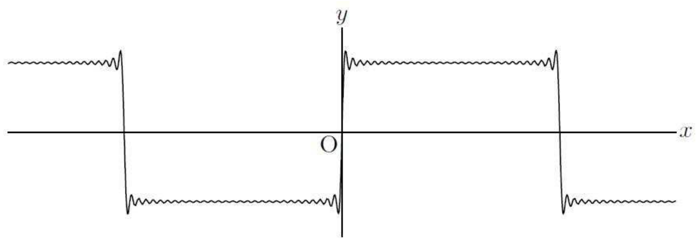


Figure 2. More precise approximation

Also we can change the size of the figure by simply substituting the unit length. If we substitute "1cm" by "7mm", we obtain the following figure:

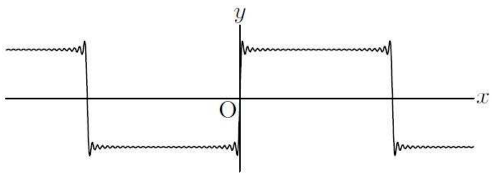


Figure 3. Controllability of size

For students to understand that $f_N(x)$ converges to $f(x)$ as $N \to \infty$, it is desirable that they should see the graphs of $f_N(x)$ for various N's and compare them to that of $f(x)$. Though showing directly the graphics drawn by CAS would serve that purpose, showing them in LATEX printing will be more effective.

Thus, taking full advantages of CAS, KETpic can offer accurate and appropriate graphics in LATEX documents.

3. Mathematical expressions and Various accessories

As seen in the following figure, KETpic enables us to insert mathematical expressions at our favorite position in graphics. Remark that the quality of mathematical expressions in graphics is the same as those in LaTeX text.

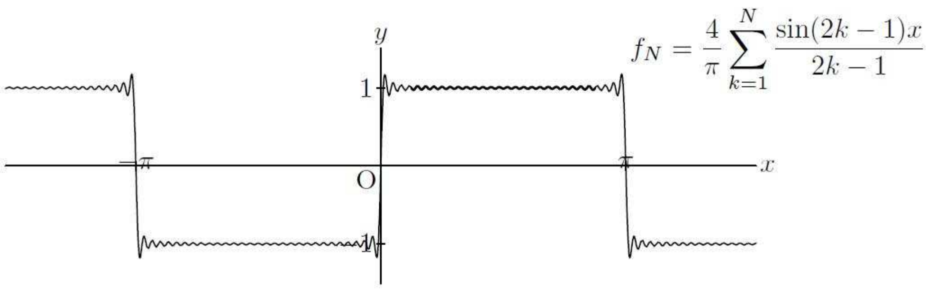


Figure 4. Points of lines and Mathematical expressions

By changing the thickness of curves, we can emphasize the important part of the figure. If we do not like the position of tickmarks, we can easily adjust them as in the following figure:

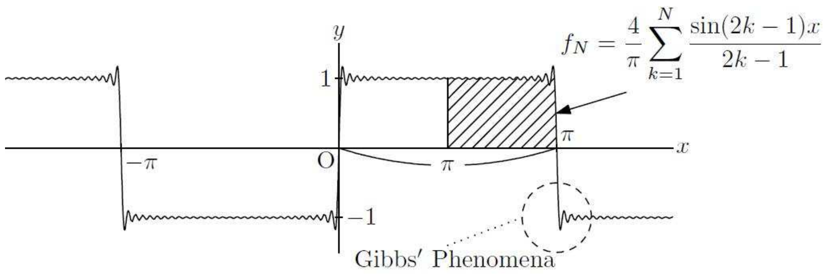


Figure 5. Various accessories

We can also use various accessories such as dashed lines, dotted lines, legends, arrows, bows, bow names, etc., so that we can give students very expressive printed matters.

Various hatchings with lines are also equipped with KETpic. The hatched domain can be specified very easily. They will be particularly useful for the study of integration or inequality.

When we prepare teaching materials in the form of mass printings, we cannot help considering their cost which makes difficult for us to use colors or halftone shadings. Since the direction or width of hatchings are easily handled, the hatchings of KETpic partially make up those deficits. As an example, we show the figure below.

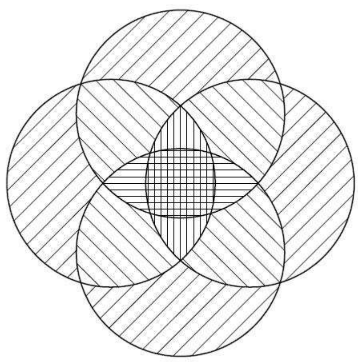


Figure 6. Various hatchings

4. Other examples

In this section, we show some graphics in LaTeX-printings which we actually use in our math class. In the following cases, we make use of the various merits of KETpic.

(1) Accurate plottings

The curves in the next figure are graphs of χ^2 functions with various values of deviation.

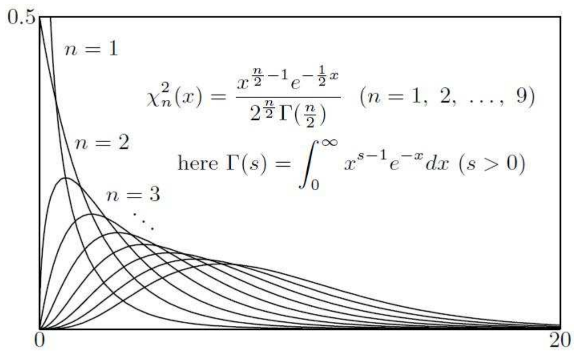


Figure 7. The graph of χ^2 function

The following curves are graphs of functions obtained by integration of the above χ^2 functions. Without using CAS, it would be impossible to draw graphics like this.

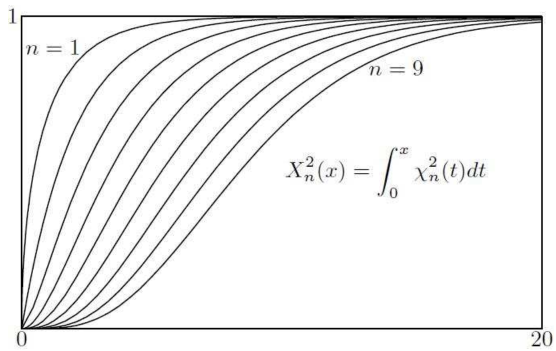


Figure 8. Integration of χ^2 functions

(2) Nine points circle

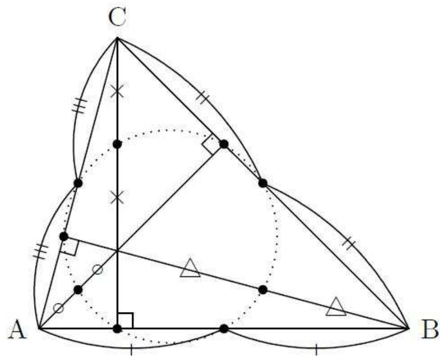


Figure 9. Nine points circle

This is a picture of nine points circle. The dotted circle is drawn by symbolic calculation of Maple.

(3) Polar coordinate
This figure shows an example of the graph of function given by polar coordinate. Here do-loop of Maple is effectively used. Since the unit length is specified to be 1cm, students can measure the exact lengths of radii in this graph.

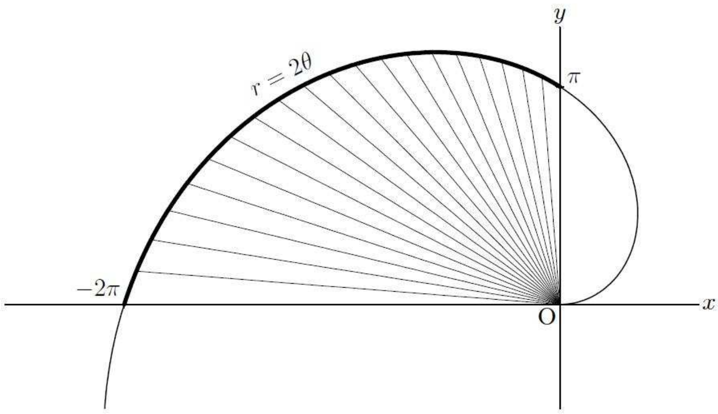


Figure 10. Function given by polar coordinate

(4) Tangential circles
This is a picture of circles which are in contact with each other and with the two lines. Though the reader may not be able to see, 100 tangential circles are drawn in this figure by using automatic calculation of Maple. Troublesome hatchings like this are also done very easily by KETpic.

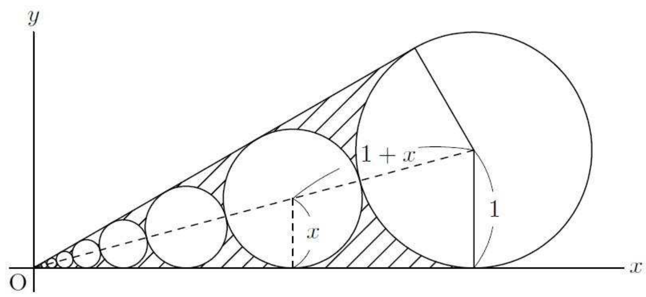


Figure 11. Tangential circles

(5) Economics and Physics
The left of the next figures is used to explain the stability of dynamical systems in economics class. We see that various kinds of lines (real line, dotted line, dashed line,$\cdots$) and arrowhead are very expressive.

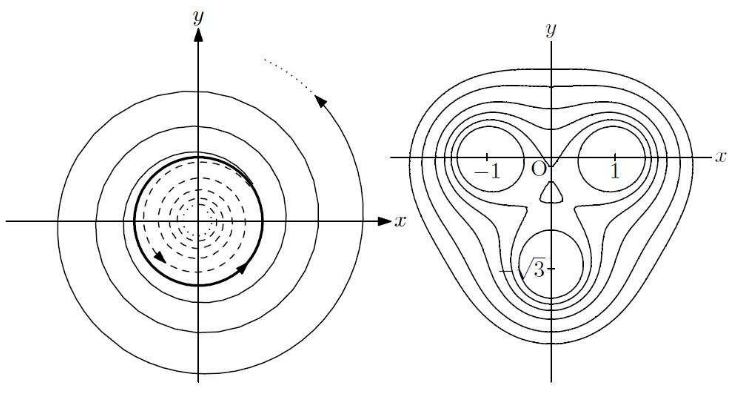


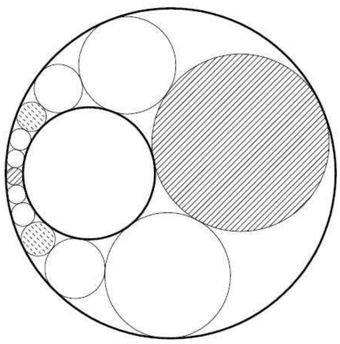
Figure 12. Figures used in Economics and Physics classroom

The right figure is that of contours of the Coulomb potential with three electric charges at $(\pm 1, 0)$ and $(0, -\sqrt{3})$:

$$Z = \frac{1}{\sqrt{(x+1)^2+y^2}} + \frac{1}{\sqrt{(x-1)^2+y^2}} + \frac{1}{\sqrt{x^2+(y+\sqrt{3})^2}}$$

To draw this picture, the command "contourplot" of Maple is used.

(6) Closed chain circle
The next example is the picture of "closed chain circle". This object is studied by Japanese mathematicians from the 18th to the 19th century.



$$\frac{1}{r_i} + \frac{1}{r_{i+6}} = \text{constant}$$

i	r_i	r_{i+6}	$1/r_i + 1/r_{i+6}$
0	1.343486	0.131361	8.356922
1	0.736085	0.142890	8.356922
2	0.366523	0.177665	8.356922
3	0.222881	0.258383	8.356922
4	0.162273	0.455695	8.356922
5	0.136862	0.952112	8.356922

Figure 13. Closed chain circle

Here the two bold circles are given first, then other 12 circles are determined so that they are in contact with the bold ones and with each other. The latter 12 circles are given numbers from 0 to 11 counterclockwise. The data of the table are radii of these circles, and are calculated by Maple. The table is inserted very easily by using the commands "open(close)par" and "letter" of KETpic.

(7) Cycloid and its variations
The following picture of cycloid can be drawn by using do-loop and the commands "rotate" and "translate" of Maple as follows (see [6]):

```
> z1:=point([0,0]):z2:=circle([0,1],1):
> z3:=frmdisp(z1,z2):
> z4:={}: N:=40: dt:=2*Pi/N:
> for i from 1 to N do
> tmp:=rotate(z3,-i*dt,[0,1]):
```

```
> tmp2:=translate(tmp,i*dt,0):
> z4:=z4 union {tmp2}
```

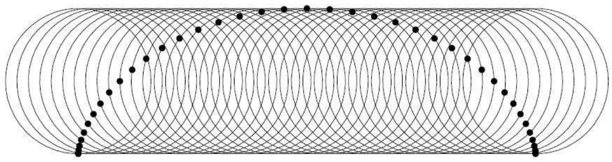


Figure 14. Cycloid

The authors believe that the educational effect of this picture is not inferior to that of animation. Also this picture has the advantage that it can be used in the form of printed matter.

The next picture of trochoid can be drawn by using the same Maple program with only the coordinate of point $z1$ substituted from "$[0,0]$" to "$[0,1/2]$".

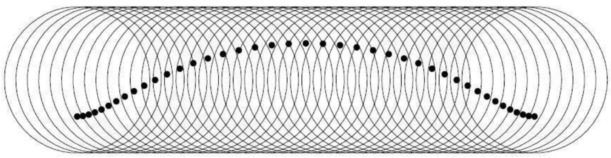


Figure 15. Trochoid

While seeing these pictures, some students may put up a question, "How will the picture become if ellipse rotates?" Then we can give them answer by using KETpic as in the next picture.

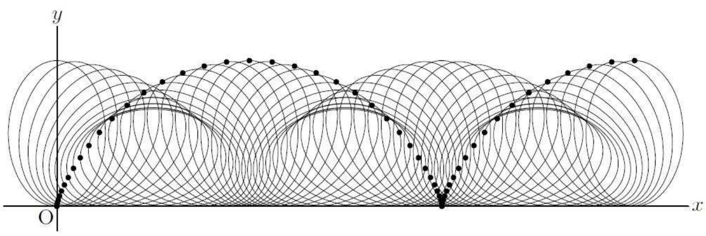


Figure 16. Elliptic cycloid

The reader may think that the appearance of the above figure seems to be a little farfetched. Actually, the trace of focus of ellipse in this case is known as "Delauney curve".

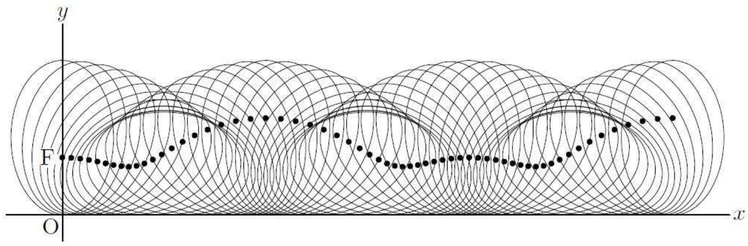


Figure 17. Delauney curve

Using similar programs, we can draw more experimental figures. For example, when a circle rotates around the outer side of an ellipse, the figure becomes as follows:

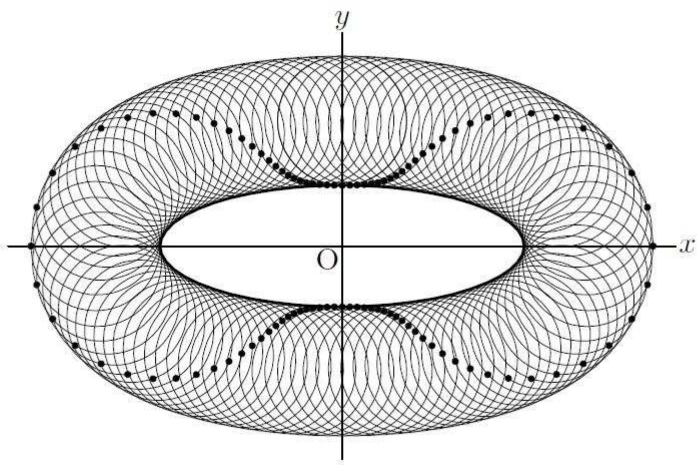


Figure 18. Rotation on the outer side of ellipse

In case of rotation on the inner side of ellipse, the diameter of circle needs to be small to a certain extent so that the rotation will not be interrupted (see the next figure).

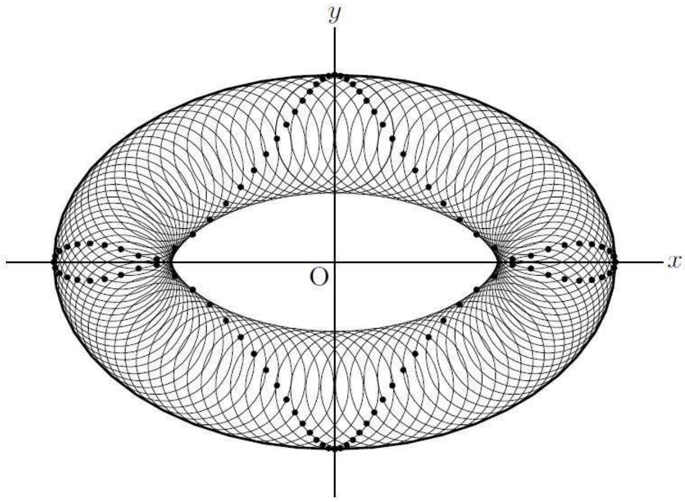


Figure 19. Rotation on the inner side of ellipse (I)

The figure below shows a delicate situation. Whether safe or not safe is easily judged by using the two diameters of ellipse and that of circle. Deduction of this safety condition is a good exercise for high school students.

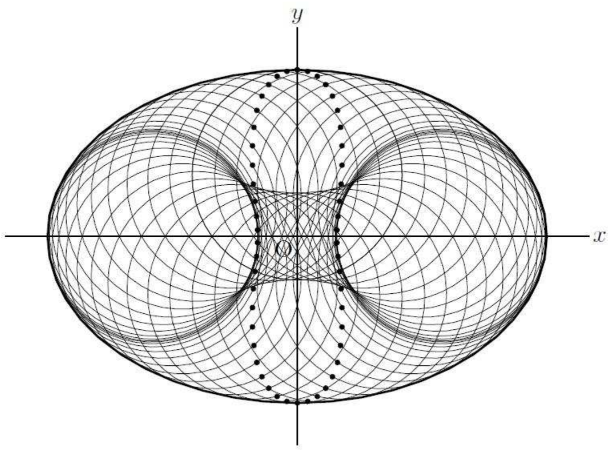


Figure 20. Rotation on the inner side of ellipse (II)

5. Space curves

(1) Projection of Space curves
When we give students the problems concerning space curves, we must draw their images under the projection on a plane. KETpic enables us to draw such a pictuire in LaTeX printing.

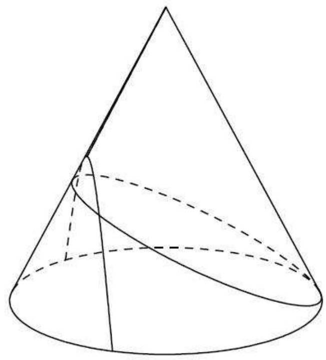


Figure 21. Space curves

Here the command "spacecurve" of Maple is used to calculate the 3D-plotdata of the points in the space curves. Also the command "projP" of KETpic is used to calculate the 2D-plotdata of the image of those points under the projection on a plane.

Though the parts of curves hidden by the surface of this side are represented by dashed lines, such hidden line elimination has not been established generally in KETpic. However, KETpic enables us to eliminate the parts hidden by the other lines. We call the figures drawn by using this technique "skeleton". In the next section, we show examples of skeleton.

(2) Skeleton
For example, the following picture of octahedron will improve students' intuition.

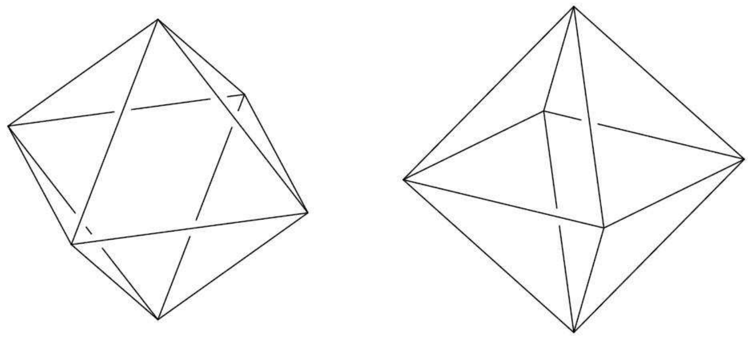


Figure 22. Octahedron

The angle of view is handled by the very simple command "setangle" of KETpic. The commands to draw this picture are very simple as follows:

```
> pA:=[1,0,0]: pB:=[0,1,0]: pC:=[-1,0,0]:
> pD:=[0,-1,0]: pE:=[0,0,1]: pF:=[0,0,-1]:
> d1:=spaceline([pA,pB,pC,pD,pA]):
> d2:=spaceline([pE,pB,pF,pD,pE]):
> d3:=spaceline([pE,pA,pF,pC,pE]):
> ds:=skeletondata([d1,d2,d3],[d1,d2,d3]):
```

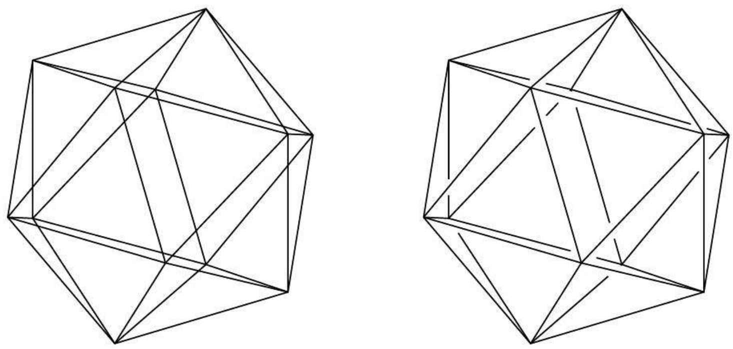


Figure 23. Icosahedron

These are the figures of regular icosahedron. In the right figure, the hidden line elimination of KETpic is effectively operated unlike the left.

6. Mathematica version

The authors believe that the transplantation of KETpic to other CASs is possible. In fact, we have a plan to develop Mathematica version. As an example, we show that the graphs presented in section 2 can be drawn by using "KETpic for Mathematica". The procedure is parallel to the Maple version.

(1) Loading KETpic

```
Get["c:\\Workfile/ketpicmath.m"]
```

(2) Calculation of numerical data

```
setwindow[{-2*Pi,2*Pi},{-1.5,1.5}];
f=(4/Pi)*Sum[Sin[(2*k-1)*x]/(2*k-1),{k=1,N}];
N=10;
g=plotdata[{f,{x,XMIN,XMAX},PlotPoints->100}];
```

(3) Writing down the numerical data onto the LaTeX file

```
openfile["c:\\fig1.tex"];
openpicture["1cm"];
drwline[g];
closepicture[1];
closefile[];
```

The last step of the insertion of LaTeX file of graphic codes into LaTeX text file is just the same as that of the Maple version.

The following figure of chaos is also drawn by using the Mathematica version. To draw it, the iteration

$$\begin{cases} x_n = 1.005y_{n-1} + 0.00025y_{n-1}^3 + f(x_{n-1}) \\ y_n = -x_{n-1} + f(x_n) \end{cases}$$

$$\text{here} \quad f(x) = \frac{-0.495x + 1.01x^2}{1 + x^2}$$

is used.

In fact, 40,000 points are put in this figure. Perhaps it would be the delimitation of LaTeX and DVI.

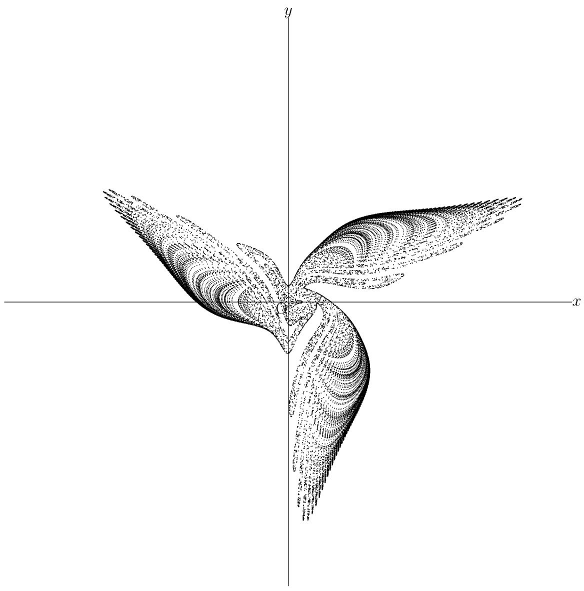


Figure 24. Figure drawn by KETpic for Mathematica

7. Conclusion and Future works

As seen in the examples throughout this paper, KETpic takes full advantages of CAS and LaTeX so that it enables us to insert fine graphics into LaTeX documents with reasonable efforts. KETpic has a number of merits which are effective for mathematical education. Especially, it is indispensable when the outputs are used in math class as printed matter. The authors believe that KETpic can lighten the burden for mathematics teachers to consider how they insert figures into teaching materials. Therefore, it enables us to concentrate on choosing the figures to improve the educational effect.

There remains various future works. For example, transplantation to other CASs has not been accomplished yet. We are developing the Mathematica version now, and have a plan to extend other CASs. The authors hope that KETpic is more wide spread if it is transplanted to free CASs, such as Risa-Asir, Scilab, etc. We need further studies on 3D-expressions. For example, its hidden line elimination is rather incomplete (see Figure 21). In terms of 2D-graphics, however, the authors believe that KETpic serves perfect assistance to our daily classrooms.

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