7/2 (2009), 187–199
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Teaching Mathematics and Computer Science

Designing a 'modern' abacus for early childhood mathematics

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Abstract. In this paper, the design of a multi-material, the 'modern' abacus ('modabacus'), for developing early childhood mathematics, is proposed. Presenting the main theories for the design of educational materials as well as similar materials and their educational use, it appears that a new material is needed. The 'modabacus' would be an apparatus which could serve as a multi-material for acting out mathematical tasks as well as a material that could hopefully overcome the limits and restrictions of traditional abacuses and counting boards.

Key words and phrases: 'modabacus', early childhood mathematics, educational materials, top-down process of design, bottom-up process of design, abacus, counting board, AL abacus, arithmetic rack.

ZDM Subject Classification: B10, F30, U60.

Introduction

The fundamental role of tools in general and of educational materials in particular for mathematics teaching and learning is accepted in theoretical discussions [16]. Piaget suggests that children do not have the mental maturity to grasp abstract mathematical concepts presented in words or symbols alone and need many experiences with concrete materials and drawings for learning to occur. Dienes mentioned that 'multiple embodiments' are necessary to support pupils' understanding. Bruner quoted that children demonstrate their understanding in three stages of representation: enactive, through the handling of physical objects; iconic, through pictures and images of objects; and symbolic. Finally, Vygotsky referred to tools as mediators of acting and thinking, according to socio-cultural construction of knowledge.

The use of educational materials is also accepted in today’s mathematics classrooms at all educational levels [16]. According to research studies they are useful to the extent that they encourage pupils to think in problem solving ways [12], they play an important role in the discovery and expression of mathematical relationships [1], and they also engage pupils in mathematical activities [8] and argumentation [13]. They give equal opportunities to all pupils to develop and understand the concepts, the procedures, and other aspects of mathematics and to set new ideas into practice [4], [8]. They can actually generate and help explore new mathematical ideas [18].

Educational materials developed over the last several years have shown that they have been designed to represent explicitly and concretely a variety of abstract mathematical ideas [16] and that they have been created to cover a large range of learning outcomes [3]. They are also, at times, especially created to overcome the limits of common tools and to create opportunities to reveal misconceptions [21].

The effective use of educational materials engages pupils in meaningful experiences that promote mathematical understanding [16]. But oftentimes, in the teaching-learning process, educational materials are chosen and used in a rote manner, with little or no learning of the mathematical concepts by the children, who are then unable to connect their actions with abstract mathematical ideas [16]. As it was mentioned by Pimm [18] “children may end up just manipulating the materials”.

To counteract the usual practice of choosing ready-made educational materials whereby children manipulate them by rote, it is important to consider a process of great significance - that of designing and developing educational multi-materials. With this in mind we propose in this paper the design of a ‘modern’ abacus (‘modabacus’) for early childhood mathematics. In the next sections we will present the two main theories for educational materials’ design and we will examine the characteristics of some materials commonly in use and the mathematical procedures supported by them in order to support our proposal and contrast them with the ‘modabacus’.

Designing educational materials

"To decide whether the design of a new product is 'good', the researcher must answer three questions: (a) Is there a need for the product? (b) Is there a reasonable probability that the product being considered will fulfill that need? and (c) Among other products, what priority does this product have?" [19]. Following are some answers to the first two questions.

For many years, the predominant view in mathematics education was the classical instructional approach in which it was thought that educational materials should portray mathematical structures transparently. They were designed so as to represent mathematical meanings or concepts in a readily apprehensible form. The design of materials with this approach follows a top-down process, that is to say, analysis and simplification of mathematical content aiming at its incorporation in the material. This is a highly technocratic viewpoint in which the designers develop the materials from static models which "are derived from formal crystallized expert mathematical knowledge" [11].

The adoption of this top – down approach led to the development of varied educational materials. Examples of such materials are those of Audemars and Lafendel, Comenius, Cuisenaire, Dienes, Froebel, Gattegno, Montessori, Pauli, Pestalozzi and Walter. Teachers selected and focused on these materials when planning mathematical activities for their pupils. They considered that by engaging pupils in mathematical activities using these materials, the pupils somehow miraculously developed mathematical knowledge [8].

However, this approach to design and use of manipulatives began to create problems in pupils' understanding and proficiency. The materials involved are thought to be 'transparent' in the sense that the pupils are expected to see the mathematical concept that they represent which are developmentally more advanced than their current understanding [10].

The most fundamental critique to the classical instructional approach came from the constructivist perspective and the broader socio-cultural theory [6]. In constructivism the meaning of external representations is dependent on the knowledge and understanding of the interpreter [10]. Similarly socio-cultural theories maintain that pupils will be able to appropriate cultural tools by participating in social-cultural situations under the guidance of the teacher. The pupils are not expected to copy the teacher but to appropriate and internalize cultural tools for use in their own activity [10].

Nowadays, constructivism and the broader socio-cultural theory – and combinations of the two, constitute the main background theories of educational materials' design and use. According to these approaches educational materials should be developed in a 'bottom-up' manner; the material is designed and developed [11], by considering the experiences and the informal knowledge of pupils. Materials do not have a 'magic property' which promotes the automatic acquisition of knowledge. The materials constitute a base, a starting point to begin an activity where the child, through multiple interpretations of material and his/her interaction with this material can manufacture new significances and relations thereby constructing his/her knowledge.

Abacuses and counting boards and their educational use

There are various types of abacuses and counting boards which are used in classrooms today. The main difference between the abacus and the counting board is the orientation of the beads; abacuses have a vertical orientation (Figure 1) and counting boards have a horizontal orientation (Figure 2).

Figure 1. Abacus

Abacuses consist of vertical rods either of the same or of varying lengths. In the former there are 10 beads of the same colour or different colour on each rod. In the latter there is 1 bead on the first rod, 2 on the second etc and 10 on the tenth rod of the same or different colours. Counting boards consist of 10 horizontal rows with 10 beads on each row. These beads, depending on the type of counting board, can be:

– all the same colour or
– alternating colours on each row or
– alternating colours on the first 5 rows and the same on the next 5 rows or

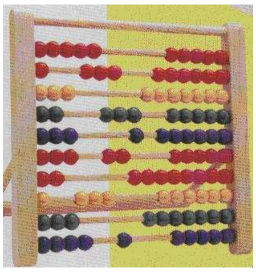

Figure 2. Counting board

– a grouping-of-five structure on each row.

The abacus is a cultural instrument which follows the theoretical development of mathematics and which is very commonly found in early childhood classes because it focuses on the polynomial representation of natural numbers [2], [21]. "With computing on an abacus the movements are actual; physical substitute objects can be grasped, they have both tangible qualities and visible components . . . an abacus offers physical counterparts for numbers . . . it uses single beads as holders of value, but also as representative units . . . the particular wire they are on takes care of the different powers of ten, without a need for this to be marked on the beads themselves or to have differentiated beads" [18].

Abacuses and counting boards can be used to reach early childhood mathematics' goals. These goals are related to fundamental skills like counting on from any number, locating numbers and jumping towards numbers which are considered important and necessary for the later learning of operations like addition, subtraction, multiplication and division.

Abacuses and counting boards enhance young children's abilities to imagine or visualize a quantity. Children must be able to subitize that is to recognize the number of a quantity immediately without counting. Subitizing has long been recognized as an important skill for developing number sense [5]. People cannot, however, recognize and visualize quantities more than six without some type of grouping. Visualization can be emphasized through the use of a counting board and an abacus with a grouping-of-five structure.

To understand that our number system is based on tens, children, according to the Japanese Council of Mathematics Education [7], must experience the pattern of trading: 10 ones for 1 ten, 10 tens for 1 hundred etc. something that can be done with the use of the abacus.

Many studies have been done concerning the educational use of abacuses and counting boards. For example, in one study [2], the abacus was used to describe the shift from the 'concrete' instrument to the 'mental' one in pupils from the first and second grade of primary school. The abacus was handled by the children of the first grade to count and to reckon with under the teacher's guidance. They were asked to represent the abacus with a typical sketch, where the drawing of the rods of the beads was accompanied by the representation of symbols for units, tens and so on. In the second grade the teaching experiment included an initial individual problem, a discussion about the problem, a meta-discussion with individual tasks and a final individual problem. The observations of the experiment were that the abacus was used not only for counting and reckoning but also to support the understanding of the polynomial representation of numbers. The pupils finally produced a mental tool that could be applied to situations out of reach of real experience.

In another study [20] the use of the counting board by 5 year olds was examined. The activities were related to (a) enumeration, (b) conservation, (c) subitizing and matching quantities; recognition of number symbols (1–6), (d) simple addition and subtraction word problems as well as (e) construction of number partners. It was found that the counting board helped kindergartners deal with such activities. But the observation and recording of the manner in which kindergartners handled the counting board, showed the following particularities in its use:

– The free space, in which they slide the beads, in a row, appeared the least. The beads of calculation were confused with the other beads.

– In activities where it was not essential to use all the rows the children used mainly the fourth and fifth rows (which were in the middle) and not the first two.

Other kinds of counting boards are the AL abacus and the arithmetic rack. The AL abacus [7] is a kind of counting board which consists of 10 horizontal rows with 10 beads on each row, grouped in fives in contrasting colours on each of 10 rows, but the colours are reversed after five rows. Its peculiarity is that along one edge of the reverse side is a label that designates two wires each for the thousands, hundreds, tens and ones; the third and seventh wires are not used.

Cotter [7] used the AL abacus (Figure 3) in a study involving two first-grade classrooms. The first class (experimental) used AL Abacuses and the second used a traditional workbook. After the experimental lessons the classroom teacher remarked that the children had advanced much more than she had expected and that children in particular who had learning difficulties, learned much more than they would have achieved with the traditional program. The experimental group visualized a concept and answered mentally more easily than the control group did.

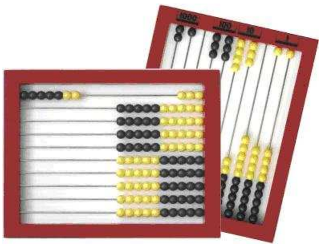

Figure 3. AL abacus

According to Cotter [7] the abacuses with rods of varying lengths and colours have several disadvantages. Some of them are the following:

– The ten different colours used in abacuses cannot be distinguished by eight percent of the population because of varying degrees of colour blindness. Also, the ten different colours do not help children in subitizing quantities.
– The small separated pieces (beads) which can easily be moved to their places in an abacus are difficult to manage by young children because they roll in all directions.
– The different length of each rod in an abacus can lead children to regard each rod as a single unit.

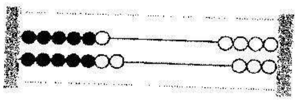

Figure 4. Arithmetic rack (from [22])

The arithmetic rack sequence (Figure 4) was designed by Treffers in 1991 [9] and in its design, research results of pupils' informal solution strategies were taken into account. It consists of 2 parallel rows with 10 beads in each row. Beads are grouped in fives in contrasting colours (red and white). Pupils use it by sliding beads to the left. Just like children's fingers, these bars consist of 10 beads with a grouping-of-five structure.

This device was created to support flexible quantitative reasoning with numbers up to 20 with the general intention of acting as an emerged model. Children can interact with the device by touching and moving the beads, just by looking at the beads or just by thinking of the beads when the transfer is made on a mental level. It is used for number operations–completing to 10, splitting numbers up to 10, and jumps of 10 [14]. The arithmetic rack involves the transition from acting as a 'model of' a phenomenologically appropriate scenario to acting as a 'model for' more sophisticated mathematical reasoning [9]. The main purpose of the use of the arithmetic rack is for pupils to have the experience of directly perceiving relationships as they interpret and solve arithmetical problem situations. The use of the arithmetic rack is based on research indicating that using this device by grouping beads rather than by counting beads one by one is itself a developmental achievement for young children. This device seemed to be very useful "in supporting the shift from counting to grouping solutions" [9].

In all the above studies the materials were used in a rote manner. The educational materials (abacus, or counting board or AL abacus or arithmetic rack) were given to the children so they could manipulate it and develop their own solution for the specific task. None of the above studies gave children the freedom to choose the most convenient material (from an assortment of materials or a multi-material) for them.

The 'modern' abacus ('modabacus')

The 'modabacus' which we propose would be a combination of the abacus and the counting board and would consist of 10 horizontal rows with 10 cubes on each row grouped in fives in contrasting colours (white and black) (Figure 5[1]).

The grouping-of-five structure of this material would support the development of part-whole relations in early number sense, because five is an amount that can be subitized as it is easily perceived as a whole. The length of each row would be double the length of that occupied by the cubes so that the child could comfortably move the cubes without unintentional mixing; something that happens often with traditional counting boards.

The 'modabacus' would be split into sections. The sections into which it would be split would be by row, depending on the objective of the task (Figure 6). For example, a single row (10 cubes, in two different colours) could be used by

[1]Figures 5, 6, 7, 8, 9 are sketches of how 'modabacus' might look like.

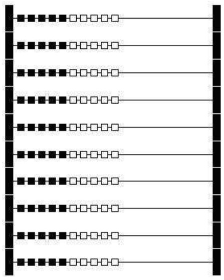

Figure 5

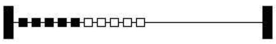

Figure 6

the children, for enumeration and solving simple problems. The use of two rows (Figure 7) would formulate the arithmetic rack and could be used for flexible quantitative reasoning with numbers up to 20 [9]. Similarly the use of three, four... and ten rows would serve for more advanced mathematical needs.

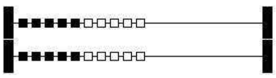

Figure 7

The sections would also form a horizontal union of rows therefore would create a line of 10 to 100 cubes, as a chain of cubes. The cubes in the line, of two different colours, would be arranged in several different ways (for example-one white, one black, one white, one black... etc; two white, two black... etc; five white, five black... etc; ten white, ten black... etc), depending on the purpose of the activity. This chain of cubes (Figure 8) could be used as a forerunner of the number line.

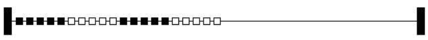

Figure 8

The sections would create a vertical ‘modabacus’ too (Figure 9). Placing the rows vertically rods would be created which would help with the segregation of the units, the tens, the hundreds, the thousands etc. Each rod, of the vertical ‘modabacus’, could have ten cubes of the same colour or of two contrasting colours (white and black). An alternation of colours per five in the vertical ‘modabacus’ (something not existing in traditional abacuses) could function as an optical cue for reading the number of cubes which appear in the columns. Young children would also have the freedom to remove and reposition the cubes (we use cubes instead of beads; beads roll and are less functional), thus learning to enter quantities by grouping on the abacus without counting.

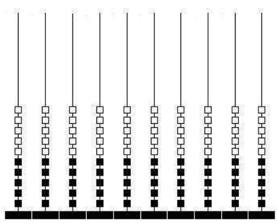

Figure 9

The way ‘modabacus’ could be constructed and used depends on the user. According to Gellert [8] neither do all pupils use materials in ways that their teachers foresee, nor do the teachers employ materials to the full satisfaction of the designers of these educational materials. ‘Modabacus’ would constitute an educational multi-material where pupils choose the type of material they want to construct each time they consider a specific task. From that perspective pupils are viewed as active learners that can participate increasingly in the mathematical practices and who must reflect on their actions with the ‘modabacus’ in order to build their meaning [15]. This process could offer pupils the freedom to describe the different ways in which they perceive things and to develop their mathematical thinking in their own way.

Discussion

According to Burkhardt [3], high quality design can help in various ways; “good design can make something more widely available and easy to use, can lead to continuous improvement of familiar things, can enlarge the space of possibilities for practice, and can provide existence proofs" (p. 22–23).

But materials are never designed in a final form; they gradually develop as a result of reflection guided by epistemological or even aesthetic values [17]. Educational materials become more efficient, relevant and transparent through a systematic process of development through trials [3] and through their use in specific activities, in the context of specific types of social interactions, and in relation to the transformations that they undergo in the hands of users [1], [2], [12].

After studying the main theories for the design of educational materials as well as similar materials and their educational use, it appears that a new material is needed. With the above in mind, we propose the design of the 'modabacus' for developing early childhood mathematics. The 'modabacus' would be an apparatus which could serve as a multi-material for acting out mathematical tasks as well as a material that could hopefully overcome the limits and restrictions of traditional abacuses and counting boards. The 'modabacus' could represent an attempt to link tradition with modernity, and indigenous knowledge with scientific knowledge. However, its manufacture remains to be realised and its use to be investigated.

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(Received July, 2008)