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Teaching
Mathematics and
Computer Science

Proof without words: Knopp series for π

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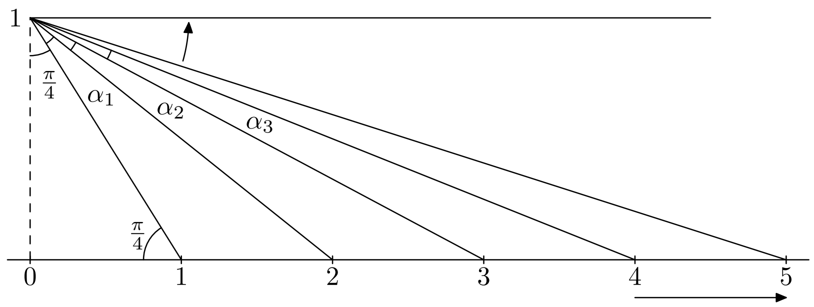
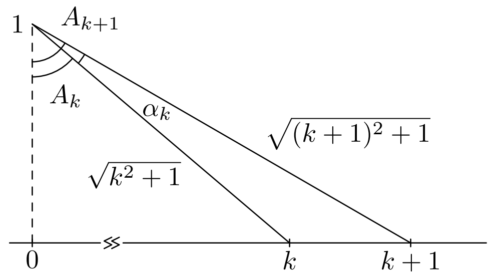
There are many expressions for number π as infinite series or infinite product (see for example [1, 2, 3]). In [1] the following series for number π is attributed to K. Knopp:

$$\frac{\pi}{4} = \sum_{k=1}^{\infty} \arctan\left(\frac{1}{k^2+k+1}\right)$$

Note that in this formula, transcendent number π is represented as the infinite sum of transcendent numbers. However a simple visual proof is provided here.



$$\frac{\pi}{4} = \sum_{k=1}^{\infty} \alpha_k$$



$$\alpha_k = A_{k+1} - A_k \;\Rightarrow\; \tan\alpha_k = \frac{k+1-k}{1+(k+1)k} = \frac{1}{k^2+k+1}$$

References

[1] X. Gourdon, P. Sebah, *Collection of Series for* π, `http://numbers.computation.free.fr/Constants/Pi/piSeries.html`.

[2] K. Knopp, *Theory and Application of Infinite Series*, Dover, New York, 1990, 214–215.

[3] E. W. Weisstein, *Pi Formulas*, MathWorld – A Wolfram Web Resource, `http://mathworld.wolfram.com/PiFormulas.html`.

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