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Teaching Mathematics and Computer Science

"Why do we complicate the solution of the problem?"

Reflection of Finnish students and teachers on a mathematical summer camp

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Abstract. This paper deals with reactions and reflections of Finnish secondary school students and teachers on Hungarian mathematics teaching culture. The experiences were collected at a mathematics summer camp in Hungary.

Key words and phrases: problem solving, heuristics strategies, representations.

ZDM Subject Classification: D54.

1. Introduction

We have organised for some years mathematics summer camps for Finnish secondary school students in Balatonberény, Hungary. We found the bicultural setting an ideal opportunity to carry out some research on mathematics teaching.

Our main research question was: *How do Finnish pupils and their teachers react to the Hungarian mathematics education culture?*

More specifically, our main interest was to see how Finnish pupils and their teachers react to the characteristics of Hungarian mathematics teaching, which is different from their own: *The Hungarian view considers mathematics as a unified subjcet – this is manifested for example in the specific application of problem-solving methods: The Hungarian approach emphasizes finding several different* *solutions to the same problem, employing methods from for ex. geometry and combinatorics to solve problems that seem to have an algebraic nature.*

Of course, we can solve very often a problem in a very short manner, but our aim when giving several different solutions is to connect different topics of mathematics: functions, means, analytic geometry, geometry, differential calculus. When stimulating paper-folding activities our aim is to activate the students´ constructive and visual capacities. In our opinion it is important to confront the students with different problem solving strategies, so that pupils can compare them, find their advantages and disadvantages. Then they can learn to choose the most effective method for each situation.

As an example of this approach to mathematics teaching we present below one specific problem and eight different solutions.

2. Literature review and some theoretical background

Looking at the Hungarian mathematics teaching culture from a Finnish point of view has also some broader interest: Hungarians have been open to the strong Eastern mathematics education tradition with Vygotsky and Rubinstein, but have also had interest in integrating Western ideas from the schools of Piagét, Skemp, Pólya and Freudenthal, while Finns have mostly followed the Western ways, see also Malaty (1998).

The idea of mathematics as a whole is one of the cornerstones of Hungarian mathematics teaching culture. “One of the main disadvantages of traditional school mathematics is its piecemeal character; on this there is a general agreement.” Servais–Varga (1971).

Hungarians have made a very strong contribution to the technique of problem solving, the classical master in the field being Pólya (1945). Later on many contributions to the field have been published, for example by Schoenfeld (1985).

Recent discoveries in cognitive psychology, among others Lakoff (2000) and Tall (2004), have enriched our understanding of education and have given theoretical support to the Hungarian problem solving approach, which tries to give many solutions to the same problem. For example, the use of different representations plays a dominant role in effective problem solving.

“Enactive representation is at the root of human activity and leads naturally to mathematical thinking through using basic sensory information and action. Iconic representations provide ’summarising images’ to give an overall picture, often sacrificing detail to represent the global relationships. Proceptual representations use symbols for computation and manipulation, to carry out procedures in a succession of manageable chunks and to represent concepts that can be manipulated mentally or on paper. As the individual builds up cognitive links between these differing forms of representation, they can fruitfully be used to focus on properties that arise and lead to the formal representation based on definitions and deduction."

"The mutual support from different representational forms is usually enormously beneficial. The psychologist Paivio developed a 'dual-coding' theory relating verbal and non-verbal (including visual) representations. He showed that remembering a number of items through both visual and verbal coding was more efficient than either of them alone." Tall (2004), Paivio (1986).

We apply these statements just quoted above by trying to show pupils several algebraic and geometrical solutions in a parallel way, to make learning more secure, since we are aware of the limits of single representations.

3. Methodology

In 2005 the Finnish group participating in the mathematics summer camp consisted of 15 pupils and two teachers. The pupils were from Helsinki area, selected by a "natural selection"; those who had interest could participate. The two teachers were mathematics teachers with major in mathematics and long experience in teaching at a mathematics oriented secondary school.

At the end of the course the Finnish pupils and their teachers were asked to answer some questions and give their general opinions, i.e. we used a questionnaire and also let Finnish students and teachers write freely on their impressions about the mathematical teaching culture of the camp. Below we present the results.

4. Results

The group consisted of 15 pupils, 9 boys and 6 girls.

(1) Their overall impressions of the camp is given, after choosing corresponding options, in the following form:

The camp was ...

disappointing	0	
neutral	0	
enjoyable	6	(4 boys, 2 girls)
very much enjoyable	7	(4 boys, 3 girls)
expremely enjoyable	2	(1 boy, 1 girl)

(2) Which topics did you like? (Pupils could write freely and mention also several choices.)

Everything	5
Algebra	1
Sum problems	4
Geometry	3
Combinatorics	4
Number theory	1
I do not know	2

Here "sum problems" means for example "calculating the sum of first n natural numbers".

(3) What was unusual in the methods to solve problems? (Pupils could write freely and mention also several topics.)

Not to use derivative	6
Use of geometry	8
Many solutions	8
Nothing	1

(4) Did you notice some difference in the Hungarian teaching style compared with the Finnish? (Pupils could write freely and mention also several choices.)

Use of geometry:	14
Many ways to solve the same problem:	8
Not to use the derivative in extremal problems:	5

(5) How did you find the level of problems?

a bit too difficult:	1
good to have challenging problems:	6

(6) Opinions, freely given by Finnish students and teachers:

(a) Students' opinions
One or several solutions:
One student was satisfied with one way of solving the problem: "Why do we complicate the solution of the problem? I can give a very short solution for this problem using the derivative."
The major part (or most participants) of the group found several solutions interesting and useful: "Solving the same problem in many different ways was interesting. Drawing with squares was something new that I learned to use at this camp, and seemed to be a very good way to find solutions to many kinds of problems." "Using unit squares is useful and I don't know why, but I used the squares to solve another mathematical problem a few days ago."
"The Hungarian teachers use geometric ways to solve problems more often than Finnish teachers who use algebraic ways to solve problems."
"The Hungarian teaching methods are much more geometric, which isn't so much in the spotlight in Finland so it's good to get new viewpoints and knowledge of geometry."
"It was unusual for me to see things from a geometric point of view and that there are many possible ways to solve a problem."

(b) The Finnish teachers' opinions on some general questions:

  (i) What was interesting at the math camp?
  Our talented students are interested in mathematics. They are eager to learn more mathematics than that content which is presented in their courses in the school and they are curious to learn it in different ways. For example they learn new problem solving techniques. Visual solutions (for example in factorising or in sums of certain finite series) are new for Finnish students. Of course participating in the mathematics summer camp is a nice opportunity to get abroad with friends and schoolmates.

  (ii) Differences between the mathematics lessons at the Hungarian summer camp and in Finnish schools
  At the summer camp studying is more convenient because there is no hurry and no strict curriculum. There is time to discuss problems and find solutions together. Also the groups are smaller at the camp (only half of the usual Finnish math class). This small size of the group makes it easy for everybody to participate in solving the problems. If the pupils attack the problem in various

ways, there is time to investigate these various approaches. We also noticed that the Hungarian teachers were not used to the Finnish pupils' practise of solving problems as a joint effort instead of individual work.

(iii) Effect of the camp on the teaching of Finnish teachers participating in the camp
We have received many useful problems and methods to solve them. We have a special course called Seminar, where the pupils who were in Hungary present these problems and methods to their classmates.
We are convinced that to teach less but more profoundly whenever possible is better and more motivating than following a strict curriculum. This means continuous balancing between the time available and the official curriculum.

5. Example of a problem with several solutions

A farmer has an adjustable electric fence that is 120 m. He uses this fence to enclose a rectangular grazing area. Find the maximum area he can enclose.

Below we present eight solutions to this problem, without describing in detail how could we manage with Finnish students them. Our aim is to call the attention for the rich variety of different solution methods.

(1) Systematic trial (lower grades). Let a be the length of one side and let b be the length of the other side. So the area is $A = a \cdot b$.

a	1	10	20	25	29	30	31	40	50
b	59	50	40	35	31	30	29	20	10
A	59	500	800	875	899	900	899	800	500

Using calculator it is possible to investigate the cases 29.9, 30.1, 29.99 and 30.01 to get a feeling, that the square might have a maximum area and where.

(2) Let a be the length of one side and let b be the length of the other side. It follows from the condition $2a + 2b = 120$, so $a + b = 60$.

$$A = a \cdot b = a(60 - a) = 60a - a^2 = -(a - 30)^2 + 900.$$

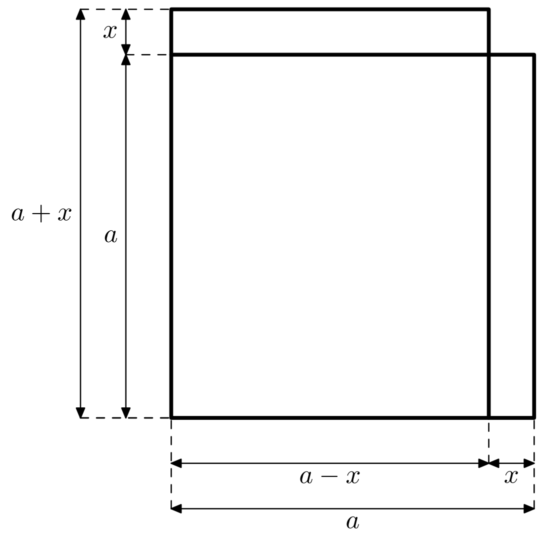
It follows from the characteristics of quadratic functions, that the area has a maximum at $a = 30$ with value 900 area units.

(3) Using the inequality between arithmetic and geometric mean:

$$A = a \cdot b \leq [0.5(a+b)]^2 = 900, \text{ because } a + b = 60.$$

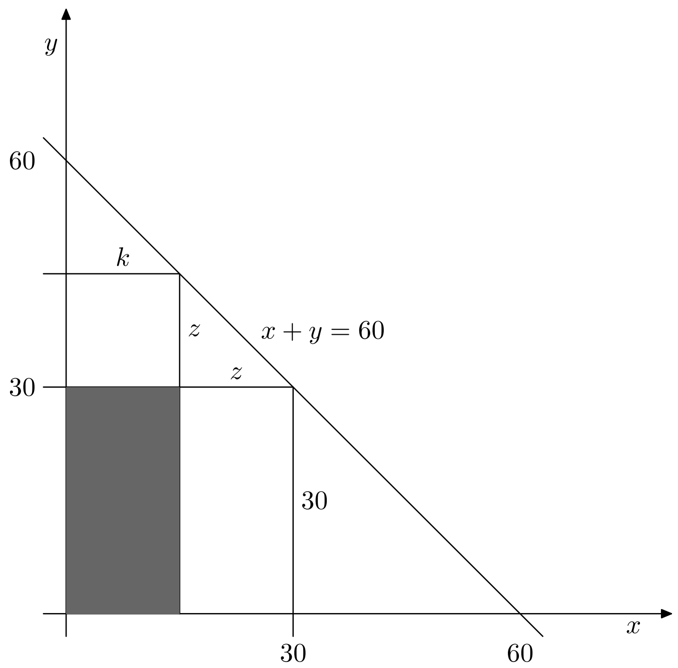
The equality is obtained in the case $a = b = 30$, hence the square has the maximal area.

(4) Using the table above, we may have an idea and conjecture that the maximum value will be obtained for the rectangle in the case of a square. Starting with the square with length of its side 30 m, it is possible to show that the rectangles with the same perimeter have smaller area:



$A = (a-x)(a+x) = a^2 - x^2$. At $x = 0$ the area will be largest.

(5) The coordinates of the points of the straight line $x + y = 60$ are the lengths of the sides of possible rectangles. If we compare the area of the square with length of sides 30 with the area of an arbitrary rectangle of the type mentioned before, we can see in the figure the marked common part.

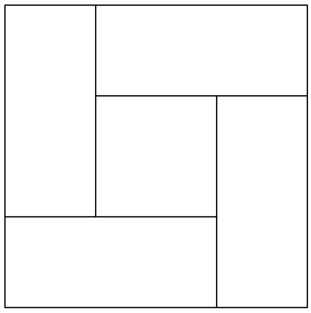


The remaining part of the square is larger, because its sides are 30 and z, while the rectangle has sides z and k, $k < 30$. Hence the area of the square is larger.

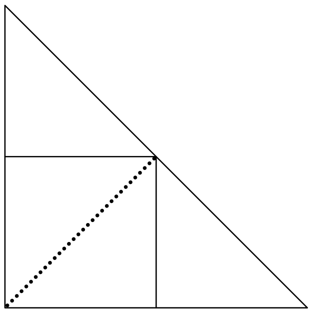
(6) We put together four congruent rectangles with sides a resp. b as seen in the figure, so we get a square with side $a+b$, inside of it a small square with sides $a-b$.

$$4A = 4ab = (a+b)^2 - (a-b)^2$$

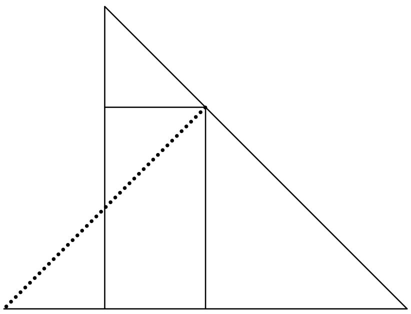
The area of the rectangle with sides a, b will have a maximum if $a = b$. In this case it will be a square and the inside square disappears. (We used that $a + b$ is constant.)



(7) We cut off an isosceles rightangled triangle from a sheet of paper. (We may choose the sides belonging to the right angle as 60 units, for example 60 mm.) (See solution 5!)



If we fold the small rightangled triangles on the square – see figure! – they will cover up the square. (The points on the sides are the midpoints of them.) Also the area of the square is half of the area of the original rightangled triangle.



Folding the rightangled triangles on the rectangle, we will get a part outside, that means that the area of the rectangle is smaller than the half of the original triangle. The square has the maximum area.

(8) It is possible to solve the problem with help of derivate. This topic will be taught in Hungary only at grade 11 (17 years old students).

6. Discussion and conclusion

Our focus was to get answers to the questions: *How do Finnish students and their teachers react to a quite different mathematics education culture? Can the different concepts of mathematics itself and its teaching be captured by the pupils? How do Finnish pupils and their teachers react to the characteristics of Hungarian mathematics teaching, i.e. the general view of mathematics as a unified subject which means for example trying to find several, different solutions to the same problem, employing methods from for ex. geometry and combinatorics?*

The outcomes were as follows:

(1) New methods were enjoyable

The result of the study was that both pupils and teachers enjoyed being introduced to new topics of mathematics.

(2) Different solutions were found useful and enjoyable

The pupils reported that seeing different solutions to the same problem, especially employing geometric methods was new and enjoyable to them.

(3) Challenge was appreciated

The result, that so many Finnish pupils enjoyed getting challenging problems, leads us to question the mathematics teaching at the present Finnish comprehensive school with least mathematics teaching lessons in Europe (UNESCO 1986). In addition, in recent international comparisons Finland comes up each time as a country with exceptionally small deviation. One explanation could be that adequate challenges are not offered to stimulate the best students. On the other hand, the weak ones are given special support.

(4) Boys and girls

As for boys and girls, the average percentage in TIMSS 99 of students belonging to the highest group in positive attitudes for mathematics showed in Finland statistically significant difference between boys and girls, the strongest after Japan. In Hungary this difference was not found. A parallel phenomenon was observed in PISA-study. Girls showed in Finland significantly more anxiety with respect to mathematics than boys but this did not come out in Hungary. At the camp the participating girls showed slightly more positive attitudes than the boys, but of course the group was small and selected by interest in mathematics.

(5) Teachers appreciated having time to teach more profoundly

In general, teachers agreed with the pupils. Also the small group was found convenient to guarantee the participation of the pupils.

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