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tmcs@inf.unideb.hu
http://tmcs.math.klte.hu

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The Mathematics Education Traditions of Europe (METE) Project

Judit Török

Abstract. This study is based on the work of the METE (Mathematics Education Traditions of Europe Project) team. Following a short introduction of the project, its theoretical background, methods and research design are presented in the next three sections. In the 4^{th} section the tools developed by the METE team for qualitative and quantitative analysis of the collected data are discussed in details. The 5^{th} section contains some personal remarks about using these tools. The 6^{th} section presents the main results of the project, followed by a summary of the project’s educational and theoretical significance.

Key words and phrases: comparative studies on mathematics education in different countries .

ZDM Subject Classification: D10.

1. Introduction

METE was a two year (2003–2004) project founded by the European Union with participants from five European countries: England (Paul Andrews, Gillian Hatch, Judy Sayers), Belgium (Eric De Corte, Fien Depaepe, Peter Op’t Eynde, Lieven Verschaffel), Finland (George Malaty, Tuomas Sorvali), Spain (Nuria Climent, José Carillo) and Hungary (Katalin Fried, Sári Pálfalvi, Éva Szeredi, Judit Török). These five countries represent a sort of different mathematics teaching traditions of Europe and a variation of students’ attainment in other international comparisons like TIMMS and PISA.

The aim of the project was a comparative study of the different ways in which mathematics is conceptualized and the different methods teachers use to present mathematics to learners of the age group 10–14. We collected data from live observations and from series of videotaped lessons on four standard topics and developed instruments for both qualitative and quantitative analysis ([1], [2]).

2. Theoretical framework of the project

Comparative researches are generally either large-scale and quantitative or small-scale and qualitative ([5]). Recently the processes of comparative studies have been classified into three typologies. Type I studies are large scale, quantitative and focused on performance or systematic analysis. Type II studies are intended to be policy-informing and attempt to implement one country's practices into another country's practices. Such studies can use both qualitative and quantitative approaches. Type III studies attempt to deepen our understanding of processes of education in the broadest sense and do not intend to influence explicitly the practices of a country. Our project falls in this third category.

We adopted a socio-constructivist approach on learning and teaching, acknowledging that learning is a social construction mediated by teaching and that understanding learners' performances requires an analysis of the socio-cultural setting in which learning takes place. We considered classrooms as complex social settings in which all actors play a part in the construction of meaning ([7]) and acknowledged that teachers' didactic perspectives are culturally determined ([8], [6]).

3. Research design and methods

At the beginning of the project the representatives of each country introduced their school system, mathematics curriculum and textbooks to the others. Watching video clips of mathematics lessons from each country we began to develop a common understanding of the nature of teaching and learning mathematics and a common language for further work.

The research group agreed on focusing our study on the age groups 10–14 because that is the age when some basic mathematical concepts are formed and the transition happens from collecting experimental pieces of knowledge to more abstract and structured mathematical knowledge. Besides as other studies show the mathematics curricula of higher classes in different countries are very divergent so comparison would have been difficult.

We decided to videotape four series of about five consecutive lessons in each of the five countries to make us able to observe not only interactions between teacher and students but methods of linking lessons together within certain teaching units. The four topics we chose were the following: percentages for class 5 or 6, polygons for class 5 or 6, linear equations for class 7 or 8 and polygons for class 7 or 8. We chose these topics because they can be found and considered standard topics in the curricula of each of the participating countries. We also decided to transcribe (in English) two lessons of each series to enable each other for a deeper analysis.

The research group agreed on selecting teachers for the project who represent each countries "good teaching practice" or considered such by the professional community (by mentor teachers, supervisors, school managers, university teacher trainers etc.). However in some cases it was difficult to find volunteers amongst the best teachers with whom the research group had contact. Organizing the video recordings within an accessible distance also meant some restraint.

During the first year of the project the METE team visited for one week all of the five participating countries and by observing lessons and discussing them we gradually developed our tools for comparison. We also began preparing and transcribing the videos.

In the second year we shared the work. The English team undertook the elaboration of the scientific background of both teaching equations and a general comparison and also the analysis of the videotaped lessons on equations and the analysis of the overall data.

The Belgian team worked on the topic of teaching percentages, the Spanish team on primary polygons, the Hungarian on secondary polygons. Each national team prepared their report on their topic and during consecutive thematic meetings we discussed them.

4. Our tools

For quantitative analysis of the lessons we developed a *coding scheme* (CS) with four basic categories: *mathematical focus* which relates to the underlying objectives of a teacher's actions and decision making; *mathematical context* which relates to the conception of mathematics underlying the tasks posed in the lesson, considering on one hand whether the task is related explicitly to the real world or a plausible representation of it and on the other hand the genuineness of the data or entities on which the task is based; *didactics* which relates to the different didactic strategies used by the teacher; *materials used by the teacher and the students*.

Creating subcategories and definitions for them was a long process. Eventually we agreed on the following subcategories:

The subcategories of *mathematical focus* are conceptual, derivational, structural, procedural, efficiency, problem solving and reasoning. The subcategories of *mathematical context*: real world fabricated data, not real world fabricated data, real world genuine data, not real world genuine data. The subcategories of *didactics*: activating prior knowledge, exercising prior knowledge, explaining, sharing, exploring, coaching, assessing/evaluating, motivating, questioning, differentiation. We divided the lessons into episodes and coded each subcategory episode by episode with 1 if it was present in the episode and with 0 if it was not present in the episode. We considered an episode as a period of the lesson while the teacher's didactic intentions are constant and distinguished two types of episodes: plenary (whole class work) and seatwork (individual work).

The definitions for the subcategories of *mathematical focus*:

Conceptual	The teacher is seen to emphasize or encourage the conceptual development of his or her students.
Derivational	The teacher is seen to emphasize or encourage the process of developing new mathematical entities from existing knowledge.
Structural	The teacher is seen to emphasize or encourage the links or connections between different mathematical entities, concepts, properties etc.
Procedural	The teacher is seen to emphasize or encourage the acquisition of skills, procedures, techniques or algorithms.
Efficiency	The teacher is seen to emphasize or encourage learners' understanding or acquisition of processes or techniques that develop flexibility, elegance or critical comparison of working.
Problem solving	The teacher is seen to emphasize or encourage learners' engagement with the solution of non-trivial or non-routine tasks.

Reasoning	The teacher is seen to emphasize or encourage learners' development and articulation of justification and argumentation.

The definitions for *mathematical context*:

An activity or task is **related explicitly to the real world** if it is draws from and feeds back into the real world or a plausible representation of the real world. Thus, even if the data on which the task draws is fabricated, if the task is rooted in the real world or a plausibly real world then we argue that it is explicitly related to the real world. For example, if a task concerns the cost of wall papering either a real or a hypothetical room then it draws from or feeds back into some sense of real world.

An activity or task is **not related explicitly to the real world** if it does not draw from and feed back into a real, or plausibly real, world. For example, a task counting and generalizing the number of dots in a systematically growing pattern is not related to the real world. Importantly, if a task draws from the real world but does feed back into it then it is not related explicitly to the real world. For example, a task in which learners are invited to measure the length of a desk for no purpose other than to measure the length of the desk is not explicitly related to the real world problem because it does not feed back into a real, or plausibly real. In such an instance, the real world has been used to provide a background context to the task but is not an explicit component of it. However, should the same task be embedded in a problem concerning, say, the manufacture of desks then it feeds back into the real world and is related explicitly to the real world.

A task is said to **draw on genuine entities** if the data on which it is based are genuine, in the sense that they are true, or derived from students' own actions. A genuine entity is derived from the real world or is the result of students' own activity. Thus, a problem in which students work on, say, an equation given to them by their teacher is not derived from a genuine entity because the entity (the equation) is a fabrication of the teacher's or the writer's of a the text they use. However, should the students be invited to explore a relationship from an equation of their choice, then it is genuine as it derives from them. A task in which learners are asked to explore the relationship between the angles of triangles they have chosen draws on genuine entities.

A task is **said not to draw on genuine entities** if the data on which it is based is fabricated. This would apply to most problems in most text books as they are not derived from genuine data or the activities of learners. Thus, an invitation for learners to calculate the solutions to a series of sums provided in a text would not draw on genuine data but a task on which they had to explore a result based on their choice of sums would.

The definitions for *didactics*:

Activating prior knowledge	The teacher explicitly focuses learners' attention on mathematical content covered earlier in their careers in the form of a period of revision as preparation for activities to follow.
Exercising prior knowledge	The teacher explicitly focuses learners' attention on mathematical content covered earlier in their careers in the form of a period of revision unrelated to any activities that follow.
Explaining	The teacher explicitly explains an idea or solution. This may include demonstration, explicit telling or the pedagogic modeling of higher level thinking. In such instances the teacher is the informer with little or no student input.
Sharing	The teacher explicitly engages learners in a process of public sharing of ideas, solutions or answers. This may include whole-class discussions in which the teacher's role is one of manager rather than explicit informer.
Exploring	The teacher explicitly engages learners in an activity, which is not teacher directed, from which a new mathematical idea is explicitly intended to emerge. Tipically this activity could be an investigation or a sequence of structured problems, but in all cases learners are expected to articulate their findings.
Coaching	The teacher explicitly offers hints, prompts or feedback to facilitate their understanding of or abilities to undertake tasks or to correct errors or misunderstandings.
Assessing or evaluating	The teacher explicitly assesses or evaluates learners' responses to determine the overall attainment of the class.

Motivating	The teacher, through actions beyond those of mere personality, explicitly addresses learners' attitudes, beliefs or emotional responses toward mathematics.
Questioning	The teacher explicitly uses a sequence of questions, perhaps Socratic, which lead pupils to build up new mathematical ideas or clarify or refine existing ones.
Differentiation	The teacher explicitly attempts to treat students differently in terms of the kind of tasks or activities, the kind of materials provided, and/or the kind of expected outcome in order to make instruction optimally adapted to the learners' characteristics and needs.

For qualitative analysis we used a *lesson synthesis sheet* (LS) based on the so called Reusser sheet. This sheet summarizes a lesson in one page. Beside a photo from the lesson and some details (country, school, class, teacher, date, topic and focus of the lesson) it contains two timelines to be colourcoded minute by minute according to the flow of the lesson. One timeline is for the pedagogic activities of the lesson (theory or conceptual development, working on problems or tasks, reporting solutions to problems or tasks, introducing a problem or activity, homework-related activities, task-related management, non task-related management), the other timeline indicates the social activities (whole class activity, individual activity, paired activity, group activity).

The lesson synthesis sheet also contains a brief description of the lesson identifying episode by episode the observed pedagogic/social and other activities concerning the mathematical focus, context and didactic marked on the quantitative coding sheet.

The definitions for the different pedagogic activities of the lesson synthesis sheet are the following:

Theory or conceptual development	The teacher introduces new concepts or theories or activates prior knowledge, perhaps in interaction with the students.
Working on problems or tasks	The students are working on tasks of any kind individually, in pairs or groups or in a plenary.

Reporting solutions for problems or tasks	The teacher involves the students in reporting back what they have achieved on the tasks – this may involve communicating solutions, procedures, ideas, difficulties etc.
Homework related activities	The teacher distributes homework to the students, gives them instructions about is or gives or discusses solutions to it.
Introducing a problem or activity	The teacher introduces a problem or activity by clarifying the concept or the constraints, possibly explains the use of equipment or reminds them of a technique that will be needed.
Task-related activity	Distribution of worksheets or equipment or manipulatives etc. Also the formation of groups/pairs etc. for working.
Non task-related activity	Discipline, introduction at the start, register, notices.

5. Using the equipment – from a personal point of view

During the discussions following the school visits in the different countries our tools changed a lot. In some cases we added new categories to the coding scheme, in other cases deleted earlier ones, modified definitions. Some of these changes were suggested by the Hungarian team or the result of the school visits in Budapest. For example while watching Hungarian lessons we could observe several episodes when the teacher asked a series of questions and by these questions she led the students to new mathematical ideas. We felt that neither of the existing categories reflected to the importance of this type of questions. The Hungarian suggestion to create a new category named “Questioning” was accepted by the team. We also suggested using a double (or triple) timeline for the pedagogic activities of the lesson synthesis sheet which was accepted later.

When me and my colleague Éva Szeredi started to use the final version of the coding scheme for coding the Hungarian lessons we worked together. At the beginning we had in some cases difficulties with deciding whether the mathematical focus of an observed episode was procedural or problem solving, the mathematical context of a word problem was real world or not, the teacher used the didactic tool of questioning, or she just coached the students with helping questions. We also had some problems with the categories “Motivating” and “Differentiating”. In some cases of kind words it was difficult to find the border between “mere personality” and “explicitly addressing learners’ attitudes”, while differentiation was sometime present but not observable (for example I knew that after the lesson the teacher gave different homework to different students but the video did not show that).

After coding a couple of lessons together and negotiating all the problematic cases we became quite confident that we use the coding scheme the same way. Indeed, when we coded the same three lessons independently the Kohen Kappa was in all cases above 0.8 (0.89; 0.87 and 0.89).

While using the lesson synthesis sheet my main concern was that even the double timeline for the pedagogic activities was not enough. Our categories were not exclusive, and Hungarian teachers quite often try to build new concepts or theories through a series of quick questions and answers. In these cases I coded “working on problems” and “reporting solutions” simultaneously but had to abandon “theory or conceptual development”.

When I worked on the analysis of the secondary polygon lessons of the other countries I also met some problems. In the case of the English lessons I compared the given codes with my own coding and the overall Kohen Kappa was above 0.6 but did not even reach 0.7 (0.67). The main source of difference was in the didactics categories “Questioning” and “Coaching”. The lessons were recorded in a low ability class and the host coder judged the teachers questions as coaching while in many cases I considered them as questioning. I had another problem with the Spanish codes in the category “Motivating”. According to the host coder motivating occurred in all the episodes of the lessons because they worked with computers. I think that the computer can be a tool of motivation but not all along through four lessons. After watching the problematic parts together with the METE team both of my points were accepted, but this experience shows that the uniform usage of the coding scheme was not guaranteed even within the research group.

6. Outcomes of the project

Our analysis of the Finnish data has not been completed yet but we found significant differences between the other four participating countries ([2], [3]).

In respect of mathematical focus we found that Belgian lessons are the closest to the average. They differ significantly only on one focus: there were fewer episodes per lesson with a problem solving emphasis. The English and the Spanish lessons differed on two foci. English lessons contained significantly more conceptual and significantly fewer structural episodes while Spanish lessons contained significantly more problem solving and fewer reasoning episodes than the average. Hungarian lessons differed most from the average with four foci: there were significantly less conceptual and procedural episodes while significantly more episodes with the emphasis on structure and efficiency. We have not found significant difference between the countries in respect of the proportion of derivational episodes.

As for the mathematical context we found that in all the four countries there were very few episodes with tasks connected to real world or with genuine data. The only significant difference from the average was the more frequent English use of not real world genuine data.

From the point of view of didactics all the four countries showed several significant differences in the means. Belgium and Spain differed significantly from the average on three categories, England on four while Hungary on seven. Belgian lessons contained more episodes with exploring (however scores for exploring were very low in all countries) and fewer episodes with coaching and motivating. Spanish lessons contained higher number of episodes with motivating and lower with sharing and assessing. English teachers put much more emphasis than the others on exercising prior knowledge and there were also a higher proportion of episodes with differentiation and coaching while fewer episodes with questioning. Hungarian lesson contained more episodes with sharing, assessing, motivating and questioning; fewer with coaching, differentiating and exploring (in fact no episodes with exploring at all, but the proportion of such episodes were very low in other countries too).

The analysis of the lesson synthesis sheets also showed many significant differences between the countries.

In respect of the observed pedagogic activities Spanish lesson differed significantly on two, Belgian and English lessons on three while Hungarian lessons on six of the seven categories.

Spanish teachers spent more time on homework-related activities and less time on reporting solutions. Belgian teachers spent more time on task-related management and less time on on introducing tasks and on homework-related activities. During the English lessons more time was spent on reporting solutions and on non task-related management while less time on homework-related activities. In Hungary there were more reporting, introducing tasks and homework while teachers spent less time on both task-related and non task-related management and on working on problems or tasks. There was no significant difference between the countries concerning the time spent on theory or conceptual development.

As for social activities most of the times the whole class worked together and there were very few occasions of group work in all the countries. Belgian lessons contained significantly more paired activity than the average while Hungarian lessons less individual and paired work.

7. Summary

In accordance to other studies (e.g. [9], [4]) our investigation shows that small-scale video based comparative studies which emphasize the importance of the socio-cultural context of learning allow us to gain a deeper insight in different nations’ classroom practices. Our analysis confirms the existence of nationallydefined perspectives on mathematics and its teaching and it is indicative of possible pedagogic flows.

From a methodological point of view the project is an example for a process when a multinational team developed descriptive instruments for analyzing classroom practices, and the use of these instruments shows their relevance and applicability.

The educational significance of the project lies in the nearly hundred videotaped lessons which show different approaches of important mathematic concepts and their teaching. Our findings can lead to an international exchange of classroom practices and are relevant for curriculum and textbook developers as well as for teacher trainers while contributing to our understanding of the nature of an effective learning environment for mathematics.

In my work of teacher training I can make good use of both the videotaped lessons and the instruments. My students are very interested in other countries’ teaching practices and find very useful to watch some video clips on the same topic from different countries and discussing the differences. Besides the categories of the coding scheme and the lesson synthesis sheet help them to be more aware of the different aspects of mathematics teaching.

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EÖTVÖS LORÁNT UNIVERSITY, DEPARTMENT OF MATHEMATICS TEACHER TRAINING
H-1117, BUDAPEST, PÁZMÁNY PÉTER SÉTÁNY 1/C
HUNGARY

E-mail: t.jutka@gmail.com

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