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tmcs@math.klte.hu
http://tmcs.math.klte.hu
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Writing a textbook – As we do it

István Czeglédy and András Kovács

Abstract. Recent surveys studying mathematics teaching show that there is a great variety in the level of mathematics teaching in Hungary. To increase efficiency (and decrease differences between schools) it is essential to create textbooks with new attitudes. The experiment we started after the PISA survey of 2000, produced a textbook that is new, in some sense even unusual in its attitude and methods. This paper presents the experiences we gained in the course of this work.

Key words and phrases: textbook, system theory, problem-solving, survey.

ZDM Subject Classification: U23.

Motivations for writing a mathematics textbook for secondary school

The National Institute for Public Education (Országos Közoktatási Intézet) operating in Hungary, has started the survey project *MONITOR* in the middle 1980s, which measured several thousands of students in each year (in years 4, 6, 8, 10 and 12) on six occasions (in 1986, then every second year between 1991 and 2001). These surveys measured students' real-life skills, the ability to acquire information and apply knowledge in society. In the case of 14–15 years old (year 8) students results showed that students' “attainment decreased considerably” [10], moreover “the variation in attainment increased” [12]. Our own experiences from public education and higher education, and a local study [7] also underlined these facts. In 2000, a survey called *PISA* was carried out in 32 countries, which measured 15-year-olds' real-life skills. (The interpretation of real-life skills in this study was very similar to that used by the MONITOR survey. It includes *posing mathematics problems, mathematical thinking, interpreting mathematical situations, modelling, problem-solving, deriving proofs, and communicating in mathematical language.*) Besides giving a useful international comparison, the PISA survey reinforced the most significant finding of MONITOR. Namely, that "one of the essential components of improving the attainment of Hungarian students is ... moderating differences between schools, that have been increasing for decades" [11]. The survey howed that in most cases "the greatest difficulty was not caused by computations, but by matching textual and graphical information" [11]. Hence we conclude that "mathematical achievement greatly depends on reading and comprehension skills" [11]. Moreover, it was found that students' achievement is science is in stronger correlation with their reading comprehension skills, than with their mathematical skills.

Hence Hungarian students achieved lower than the average on problems where "the solution involved understanding and comparing a graph and a table, a graph and a text, or two graphs, and where a text needed to be contrasted with graphical information" [11]. Although this phenomenon can be explained in several different ways, each of these partly successful, there is no theory that fully answers this question. Nevertheless, it is an interesting fact, and maybe not a mere coincidence, that at the end of the 1970s and at the beginning of the 1980s new methods were introduced for the teaching of reading, that shortened the time devoted to the learning of reading and writing. As a result of the new curriculum introduced in 1978, many primary teachers started to use the method of global, or "word as image" reading instead of the specially Hungarian method of loud syllabifying. However, in Hungarian there are many affixes, and therefore word forms are long and inconstant. Beginner readers are unable to decode these. For them words need to be divided into smaller units, syllables. According to certain experts, giving up this principle lead to *reading comprehension problems* [1] that could affect the teaching of mathematics through the comprehension of mathematics texts. The Hungarian MONITOR studies, as the PISA survey, indicate that the third of year 8 students have such difficulties with reading comprehension that can hinder them from progress. Before, when the syllabifying method was taught, nearly every student learned how to read. Nowadays a growing proportion (often as much as 30%) of students are diagnosed with dyslexia, and so their teachers and parents give up on their development. According to the findings of the PISA-survey, achievement in mathematics correlates with reading comprehension. So, as a matter of fact, it is natural that our mathematics education, which was considered outstanding at the beginning of the 1980s according to studies of that time, has sunk below average by now. Yet the textbooks used then are still in use, and a great part of teachers from that time teach still. And presumably, the problem-solving ability of students has not changed considerably from genetic causes, either. Changes in society have certainly had consequences, but this process of regress started before the change of regime in 1989. However, the analysis of this question is beyond our capacity here, and it is not our goal. We only wanted to point out that probably we cannot expect to improve mathematics results solely by reforms of mathematics teaching. We do not have the opportunity to engage in disputes over reading and change the situation. In the present circumstances, along with many colleagues, we would simply like to improve the quality of mathematics teaching. Having written several primary and secondary textbooks before, it became clear that our possibility to contribute to solving the problems shown in the survey results was to write a mathematics textbook that helps students catch up with the best students in the area of apprehensive, analysing reading and in real-life skills (as well). Throughout this transformation, however, we needed to preserve methods that carry on positive results and traditions of mathematics teaching.

> "Hungarian students achieved best on a problem that fitted into the practice of the Hungarian primary school: they needed to match plane figures to detailed geometric descriptions. Not only did they solve this question well, but they were the best among the students of every other participating country" [11].

Two years ago, in consequence of the shocking results of the PISA survey, we decided to write textbooks based on the principles outlined before. Such that might better develop the components of real-life skills described above. In other words, such that makes it possible for students to acquire knowledge that is practical and can be applied easily. It is not possible to judge the success of our aims after such a short time. So our project could be thought of as an experiment whose justification, modification, or rejection can only be realised on grounds of later feedback. In this article our goal is to give an account of our experiences so far. Although we are making this study in a *Hungarian context*, we hope that writing a textbook in a specific country can have *inferences* useful elsewhere. Hence we would like to present our work at a professional forum.

We started by working on the secondary textbook. (As we both have experience with this age group. Besides, we were able to ground our work, as we had joined in the development of one of the most popular sets of Hungarian textbooks from year 1 to year 8 earlier. This set of textbooks is considered as the continuation of that one.) In Hungary, secondary school lasts from year 9 to year 12 for comprehensive and vocational schools ("gimnázium" and "szakközépiskola"). In 2001, at the beginning of our study, the three major sets of textbooks available for students were the same ones as in the middle 1980s. In our opinion, the altered social and educational environment does not only permit, but expressly encourages attempts as this one. (Presently, two other sets of textbooks are being prepared with a conception different from that of traditional ones, and previous ones are being rewritten.)

Description of terminology

In Hungarian terminology an *exercise* has to be solved by the student, and in case of an *example* we show the solution to him. A problem can mean either an example or an exercise. We use the term *problem* for a mathematical situation where the route to the target to be reached is unknown to the student. Literature uses the term problem-solving for direct translational strategy based on a superficial approach on the one hand, and modelling strategy based on understanding on the other hand. By *problem-solving* we principally mean the latter method.

A set of Hungarian textbooks usually includes extra material such as problem books, worksheets and teacher's books. We use the word *textbook* for the central piece, i.e. the student's book.

The aims of our Mathematics teaching

According to the definition we use "*the aim of our mathematics teaching* is to develop and improve mental characteristics that are indispensable to fit into and to participate in society" [4]. This really means developing and improving a certain system of mental processes, results, characteristics, experiences, concepts, knowledge, operations, planning of actions, abilities, expertise, skills, attitudes, and so on, that are directly connected to an actual mathematical content.

According to the views above, the aim of mathematics teaching cannot be the process of students learning exact definitions and theorems simply for their own sake. It is rather learning and applying these pieces of knowledge in order to develop and improve the greatest number possible among the mental characteristics listed in the previous paragraph. For instance, the fact that students know the basic notions of percentage and are able to calculate an unknown from given data, is useless unless they are able to apply this knowledge for a real-life problem such as calculating Value Added Tax.

Besides the determining role of the teacher, the textbook is a significant factor in the development of the characteristics in question. It is the important function the textbook has in the learning process that urged us to try and write a secondary school textbook that builds on the textbooks we had written to lower years (years 5–8) in content and form, and helps best in reaching the aims listed before. 242 students from 10 schools, one class in each, have been participating in our study. Our experiment is not representative. As teachers who try our texts get a modest remuneration from the publisher, we were not able to involve a multitude of classes into the study. We used the following principles for selection:

(1) teachers with good professional and educational background;

(2) a range of schools from the best to the weaker ones; from big cities to smaller towns;

(3) both types of secondary schools that prepare for the secondary final examination ("érettségi"), that is, comprehensive ("gimnázium") and vocational ("szakközépiskola") schools.

(The study is still going on, after finishing the year 9 and year 10 material, we are working on the year 11 one.) Schools get the drafts of the textbook from the publisher in form of booklets with 40–80 pages, depending on the subject matter, and after trying them, teachers and students evaluate the material with the aid of questionnaires. Then we process this information with the help of a mathematician (a university professors having also taught in secondary school), mathematics teacher and linguistics adviser who we have invited, revise our previous ideas if needed, and bring the book into a ready-to-print form in collaboration with the publisher's copy editor. In Hungary it is the Textbook Office of the Ministry of Education that accredits books as official textbooks. The independent readers they invite examine the subject matter, teachability and conformity to the Frame Curriculum. It is the interest of the publishers that a book would be accredited as a textbook. Teachers are permitted to teach from other textbooks, but accreditation makes a book trustworthy in their eyes. Moreover, there is less tax on these books, so we can offer them at a lower price to schools, who can choose from the official list of textbooks distributed yearly by the Ministry of Education. (In Hungary most students buy new textbooks every year, and only a few of them use second-hand ones. Hence the purchase of books is often a serious expense in the household budget.)

Questionnaires compiled for assessment refer to intelligibility, learnability, teachability, accuracy of subject matter, practical applicability, language, layout, visuality, levels of difficulty, the potentials for practice and drilling, the way topics are built on each other, progressivity, the correctness of problems, and reader-friendliness. Besides evaluating questionnaires, we also take a look at teachers' own copies, in which they make notes for us. They draw colour-coded lines on the margin indicating the level of difficulty, they correct errors, or suggest alternative ways of presentation that they prefer to the one in the text. It is the copy editor, the representative of the publisher, who decides in matters of dispute. It has happened that a worked example was replaced with one suggested by a teacher. Such cases encourage teachers to give their opinions decidedly.

Besides the ways of co-operation described above, we hold a one day meeting with teachers and the copy editor every year. Here we discuss experiences and work out the course of action that follows.

Feedback helps us answer such questions as what a good textbook is like (expectations) and to what extent our set of textbooks meets these expectations (elaboration). These questions comply with ideas János Karlovitz outlined concerning a textbook's functions [6].

Main functions of textbooks, sets of textbooks

(1) *Source of information*
Characteristics: collecting, storing, conveying, arranging information; presenting and explaining matters appropriately; following the newest results in science.

(2) *Transformational role*
Characteristics: raising problems, solving exercises and problems, practical application, practice, developing and improving abilities and skills.

(3) *Pedagogical role*
Characteristics: exploiting educational, pedagogical and teaching aims underlying the curriculum; motivational basis; logical ordering, simplicity, expedience and usefulness.

(4) *Guiding function, co-ordinating role*
Characteristics: "leads" and "guides" the teaching and learning process; often "substitutes" the curriculum; ensures building and progressivity; excludes unnecessary overlaps and repetitions, shows the possibilities for connection within and outside mathematics.

(5) *Role as a motivator for independent learning*
Characteristics: appropriate examples, thorough analysis of subject matter, emphasis of important elements, filtering perturbance, appropriate quantity and quality of exercises.

(6) *Educational document*
Characteristics: "equalizing" valve, conveyer of uniform requirements, a uniform system of assessment. (The latter statement is true even if schools use different textbooks.)

Realizing the main functions of textbooks in our work

Below we show the treatment of *number theory* in the textbook, including the process of acquisition, and the accomplishment of requirements set for textbooks. We also show how the functions described above are realized in the textbooks we wrote. Preparation for the system of knowledge starts at primary school, and its acquisition lasts until the end of year 12, bearing building and progressivity in mind throughout. When processing other topics in the textbook, we use the same system, and pay utmost attention to previous curriculum, too.

As being a *source of information* is concerned, let us examine the system of knowledge of the basics of number theory found in the year 9 textbook. Students need to acquire the following concepts, skills, and their connections:

- Euclidean division
- Divisor and multiple
- Division as an operation; divisor as a relation; properties
- Divisors and multiples of particular numbers
- Finding every divisor, divisor pairs
- Common divisor, common multiple, Greatest Common Divisor (GCD), Lowest Common Multiple (LCM)
- Primes and composite numbers

- Prime factoring of composite numbers, finding the GCD and LCM from prime factoring
- Relative primes, twin primes
- The number of all divisors and prime factoring
- Theorem on the divisibility of sums
- Theorem on the divisibility of products
- Divisibility rules in the base 10 number system
- Calculations with remainders, congruences
- Interesting divisibility rules (by 7, 11, 13, etc.)
- Perfect, abundant and deficient numbers
- Number of primes
- The Main Theorem of Number Theory
- The Euclidean Algorithm, GCD
- Diophantic equations
- Number systems of different bases
- Divisibility rules in number systems of different bases, generalization

The concepts listed span the material of several years. For instance, in year 5 we show the relationship between Euclidean division and divisor through specific examples, later, still through specific examples, we get to the concept of divisor and multiple, divisibility rules, and to the notion of prime and composite numbers. In year 9, the same concepts reappear in higher level, we discuss connection generally, and we prove several basic theorems (taking the requirements of the Frame Curriculum into consideration). Hence, a such system includes the material of 4–6 years.

The order of topics is governed by the *guiding function, co-ordinating role*. To establish this order we used the method described in the works of Frigyes F. Gyaraki [5], who had built on the ideas of I. B. Morgunov, and which we had tried before ourselves. In this method the curriculum is structured with the aid of system graphs and relation matrices. (Connections between topics are represented by a directed graph, which is written into a matrix. Then the matrix is transformed by an ordering algorithm to get the logically correct order of topics.) The order obtained is differed from the traditional one in several instances, hence we needed to alter the wording of definitions and theorems, and change proofs. This was the case not only for number theory, but for other topics, as well. For instance, we did not prove the converse of Thales' Theorem in the traditional way, using reflection, but we reversed the original theorem. So we did not need to arrange it in a way another new textbook did, where the proof of the original theorem is on page 141, while the proof of its converse is on page 217. Let us look at another example, special lines in a triangle. Hungarian books traditionally teach medians in year 10, while other special lines in year 9. We teach special lines in year 9 and give a different proof for the theorem concerning medians. The third example, also from plane geometry is that of mid-lines. Every Hungarian mathematics textbook starts from the median of the parallelogram, that is, a quadrilateral, than they infer to the mid-lines of the triangle and the trapezium. In our book, we start from the triangle, and from here we get to the mid-lines of the other two shapes. (Every other book teaches triangles before quadrilaterals, but they treat the mid-lines of triangles only with quadrilaterals.) In our experience, Gyaraki's structure helps develop systems of knowledge. This might not be the most modern method, but it can be taught simply, and applied fairly easily. We show it to our student-teachers, too, so that they would be able to re-structure the curriculum on their own, if needed.

The above list includes both *compulsory material* that is essential to proceed into the next year, and *optional material* that is recommended for more able students, but is not obligatory. Optional material is indicated by a grey stripe on the margin. At the end of each chapter there is a *chapter check* conforming to the two-level final examination. These give students feedback on the quality of acquisition. As in the two-level final examination to be introduced in 2005, these, to the request of the teachers trying our textbooks, include optional exercises. In each worksheet there are compulsory problems, and there are some specially intended for the core and the extended levels of the exam.

The equalizing valve is present in the book from the beginning. Uniquely among Hungarian textbooks, the year 9 book [3] starts with a systematic revision of 54 pages, with the aim of bringing all students to nearly the same level. (The success of this is also measured by a chapter check.) We would like to help synchronize the preparation for the final examination, by making it possible for teachers to compare their results with others, on demand. Thus the textbook can functions as an *educational document.* This function is also enabled as the textbook reflects the requirements of the Frame Curriculum precisely. We flag compulsory exercises, easy drills, and average or more difficult problems with the initials of the Hungarian words *Fontos* (important), *Könnyű* (easy) and *Átlagos* (average), respectively. Furthermore, a grey stripe on the margin marks material that exceeds average requirements (needed for the extended level final examination). This layout is of great help for teachers, students and their parents in finding what material to study for which target.

There are many worked examples in the textbook. In the final version of the year 9 book with 336 pages there are 163 examples, in the year 10 book with 552 pages there are 233 examples with solutions. (The great number of worked examples also makes independent learning possible.) Besides, in the year 9 book there are 548, in the year 10 book there are 552 exercises classified into three different levels of difficulty. With the great number of exercises our target is that students wouldn't have to buy separate problem books. (Most year 9 and year 10 students have to use six different problem books.) Besides examples and problems, fairly long readings (occasionally several pages) on the history of mathematics have an important role. We try to embed these motivation building sections into the learning process by issuing appropriate problems and library or internet research. This way we hope to *encourage independent learning*.

The method we use for deducing, practicing, revising and assessing knowledge belongs partly to the *transforming role*, and partly to the *pedagogical role*. Our principle even in the higher years of secondary school is Skemp's hypothesis, that "we cannot transmit to students notions of higher order than what they already know simply by a definition, but only by giving them a multitude of appropriate examples" [9].

In our books this principle is realized by the following procedure. Students *gain experience* concerning divisibility from particular examples, then *word conjectures*, state these in special cases, *prove* them, then *generalize* their knowledge. This way we achieve that students take an active part in acquisition, rather than being passive recipients. We strive to accomplish this on all levels, throughout primary and secondary school. (Our endeavour to consequently carry inductive and heuristic treatment through *secondary school* material is unusual in Hungarian mathematics textbooks. "But this is like a primary textbook!" a teacher expressed genuine surprise when she first saw our year 9 textbook.) Of course, a textbook is not omnipotent, the role of the teacher is indisputable in education.

Treatment of the basics of number theory in the textbook

To demonstrate principles described above, we shall now follow how the concept of divisibility rules are developed in the textbook

(1) What can we see from the last digit?
(Divisibility by 2, 5 and 10.)

(2) What can we see from the last two digits?
(Divisibility by 4, 25, 100, and the divisors of 100.)

(3) What can we see from the last three digits?
(Divisibility by 8, 125, 1000 and the divisors of 1000.)
And so on, till the generalization for the divisors of the powers of 10 in secondary school.

In year 5 we treat division, in years 6 and 7 the divisibility of products and sums through specific numbers, and in years 8 and 9 we prove general theorems. Students realize through examples that $10^0; 10^1; 10^2; 10^3; \ldots$ divided by 2 give a remainder $1; 0; 0; 0; \ldots$ respectively, so dividing each term of the number $A = a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + \ldots + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0$ gives a remainder $0+0+\ldots+0+0+a_0$. Hence it is the nature of the powers of 10 that determines divisibility by 2. (The same stands for divisibility by 5, 10, 4, 25, 100, 8, 125, 1000, etc.)

(4) What can we see from the digit sum?
(Divisibility by 3 and 9.)

We show students that the rule which differs from the previous one can be traced back to it. For example, the powers of 10 divided by 9 give a remainder 1, so it is the sum $a_n + a_{n-1} + \ldots + a_2 + a_1 + a_0$ that gives the remainder of the number A.

The year 9 textbook also treats division by 11. Dividing $10^0; 10^1; 10^2; 10^3; \ldots$ by 11 gives the sequence $+1; -1; +1; -1; \ldots$ as a remainder. Hence we can deduce the well-known rule that the $(-1)^n \cdot a_n + (-1)^{n-1} \cdot a_{n-1} + \ldots + a_2 - a_1 + a_0$ remainder determines divisibility. In order to make students aware of the general nature of the procedure, and facilitate its application, we examine the case of 7 and 13, as well. Dividing the powers of 10 we get the periodic sequence $1; 3; 2; -1; -3; -2; 1; \ldots$ in the first case, and $1; -3; -4; -1; 3; 4; 1; \ldots$ in the second. In each case, we start the procedure with an example, and after developing the rule, we check it on one or two specific numbers. We do not mention that the above, general method cannot always be applied so simply. We do not show the rule for divisibility by 17, for instance, but in schools where an appropriate computer program is available (DERIVE, MAPLE etc) we recommend examining this sequence consisting of 16 different terms. So students realize, that it is possible to design a divisibility rule for any prime number. (Thus they no longer look at different divisibility rules as isolated cases, but as interconnected parts of a greater system.)

Our inductive method implies further generalization. First we examine in what cases a specific number in the base 6 number system can be divided by 2 and 3, then we analyse the situation generally. This involves a base n number system, and divisibility by the divisors of n. Similarly, we can determine if a number is divisible by $n-1$, and the divisors of $n-1$. As in the case of other topics, phases of acquisition follow the procedure described after point 3). Building on previous knowledge, and not re-teaching it, we proceed from the concrete to the abstract, from the specific to the general.

As a result of the above treatment, students think in schemes (systems of knowledge), hence they need to remember less information, they can recall these more easily, and they are better able to apply them to other topics, even in non-mathematical context. In general, we strive to structure the material in a way that helps students see the system of different subtopics rather than seeing them as separate, independent phenomena.

In our experience, this teaching method is greatly motivating, thus students were keen on looking for further divisibility rules. They also found calculations in different number systems amusing. That is why we included the story how Babylonians, who were using the base 60 number system, approximated $\sqrt{2}$. Adapting scientific sources [13] for students enables them to learn more about history and old ages. On the other hand, we also show them the most modern results. Not only do we give account of the recent proof of Fermat's theorem, but we also give them a proof for the irrationality of $\sqrt{2}$ that connects certain parts of algebra and geometry [2].

We try not teach the basics of number theory for their own sake, simply for the verbal intake of knowledge, but we also show its practical applications, and raise students' attention to connections within and outside mathematics.

As we mentioned before, the system of mathematical knowledge is not taught linearly, but spirally in Hungary. The same concept or skill can and should be reintroduced on a more advanced level in a higher year, when students' background knowledge is more advanced and different. (Simpler divisibility rules, for instance, are already present in the year 6 book.) This way, concepts seemingly far from each other can be organized into a system, and brought close to each other. Besides considerations of content, this is also important because it realizes one of the essential didactical principles, the principle of constant revision.

Finally, let us examine if using our textbook really helps develop students' real-life mathematics skills, in other words, if we can reach our aims set in chapter 1.

(1) We try to set problems in a *real life context*, and in a form students would meet them in reality:
"How can you interpret the following statement of the evening weather forecast? Tomorrow we can expect some sort of precipitation with a chance of 25% in the country.

 (a) It will rain in one-quarter part of the country's land, and it won't rain in three-quarters part.

 (b) We should expect 6 hours of rain in the country, and 18 hours without rain" [3].

 This way students do not get the description and modelling of the problem ready-made, but they have to interpret the problem themselves.

(2) We try to make problem solving a conscious, rather than a spontaneous process. That is why we *teach, and not only use* Pólya's method [8], and we show reasonings that are inaccurate or lead into a dead end, than we show how to gain experience and resolve these situations.

(3) It is the task of our mathematician advisor to check the *validity of mathematical communication* and proofs. He is the leading educational professional of our surroundings, well-known for his preciseness, who would not let a vector to be loosely defined as a directed segment, as it is done traditionally. He is also the one insisting that Jensen-convexity cannot be listed simply as convexity among characteristics of functions, even if every other Hungarian textbook, inaccurately, uses the latter term.

We need to translate the preciseness of mathematics to students' language. The way we can be assured that we are on the right track is asking them, our prospective users. For this reason, we ask each student at the end of each year about our textbooks.

The following statements are typical of student feedback about the textbook:

(1) "I like the book because figures help me see the point." [G.K.]

(2) "I like worked examples, because I am able to solve harder problems following these." [A.B.]

(3) "It's good that the text that we have to learn is highlighted." [J.A.]

(4) "Chapter checks tell me the most important things that I have to learn." [T.L.]

(5) "I really like stories about mathematicians." [D.D.]

Students participating in the study didn't know other secondary textbooks, and teachers trying them are obviously partial. So even though we are flattered by these opinions, we need detached outsiders to give us a more realistic picture. Thus we gave out our books to 10 colleagues who had actively taken part in our advanced level, 2-year-long extension training programme, and who have taught in secondary school for a long time, and asked them to compare them to the ones they were using. We quote from these, matched to the ones above:

(1) "The book lacks those coloured figures that could grab one's imagination. Children today have a different world, it's much more difficult to hold their attention, and somewhat more difficult to captivate them." [L.K.]

(2) "The great number of worked examples makes the book suitable for independent work. I think that after missing a few hours, an average ability student is able to catch up with the class with teacher guidance." [T.L.]

(3) "Marking core and extended level parts with two different colours helps both students and the teacher in finding their way about parts with different levels of difficulty." [Gy.K.]

(4) "Chapter checks contain problems of various difficulty, so we can find problems for students with different abilities." [Z.L.]

(5) "I think that only good ability students are able to understand the curiosities from the history of mathematics, but it is a special treat for those interested in mathematics." [O.A.]

We could quote many other opinions. The ones listed above convince us that we are basically on the right track, but we will need to make minor modifications in certain respects. For instance, we used to think that a too colourful book draws away students' attention from learning, so we insisted on two colour print. Now, based on teacher feedback, we think that it is worth reconsidering this view.

Our plans also include an area that is only in loose connection with textbook writing, and that we have not had the time to develop yet. We would be pleased if the geometrical figures and graphs of functions in the book could be put on the homepage of the publisher. For instance, it would make a significant difference if students could visually follow the way the centre of the circumcircle of a triangle follows the change of its largest angle. Or if they could see that making a certain transformation of a function really modifies the graph in the way they have learnt it. Moreover, an on-line forum connecting the writers and the users of the textbook would certainly improve their relations.

In sum, we see our experiment as a *novelty* in the following respects:

- In a year's time students *from year 1 to year 12* can study mathematics from *textbooks with a uniform conception and identical structure* (the copy editor of the whole series is the same person, Sándor Hajdu).

- We used *considerations of system theory* to establish the order of topics *in the 12 books* (sometimes these lead to new treatment of topics, as in the case of divisibility and geometry).

- *Every part of secondary textbooks* (every topic in every chapter of every book) introduces knowledge in an *inductive, explorative way*, contrary to the theories of Piaget and other psychologists, but adjusting to special circumstances shown by Hungarian and international surveys, namely to students' fairly low level of learning and understanding.

- The books intend to teach learning based on Pólya's guidance: we present conscious planning, the main steps of Pólya's problem solving procedure *in practice, through examples.* There are many books that use Pólya's principles, but we *do not only use his problem-solving method, but also teach it.* In worked examples we do not simply show the solution that gets to the target in a straightforward way, but we also give suggestions for situations where the student does not immediately see what to do. We also emphasis the importance of checking answers, we give examples for solving a problem in different ways, as well as for the dimension test. (This method is not often used in Hungary. For instance, the final examination requirements for algebraic operations include neither the analysis of conditions nor checking answers, which, we believe, is methodically inadequate.) In our opinion, this method helps developing consciousness in student's way of problem solving.

- *Separating core and extended level parts* (corresponding to the two levels of the final examination), and grouping problems according to difficulty from the beginning, supports enrichment and differentiation in our textbooks. Other textbooks teach concepts preparing for the extended level examination only in years 11 and 12, but present the same material to every student in years 9 and 10. In our opinion, talented students need to be enriched every year. (This structure is present in primary textbooks, too, and it has proved to work well there. We wrote two versions of each textbook, core and extended. We found that almost every primary school ordered the version they felt more suitable for differentiated learning.)

The fact that we chose an inductive way of acquisition, learning by discovery, and to use system theory rather then being deductive, and we provided an appropriate number of exercises for developing and improving abilities and skills while paying attention to building and progressivity, and we enabled differentiation, we intended to create a learnable, teachable and reader-friendly book that hopefully makes students like mathematics, and prepares them properly to both higher education and participation in life. Yet, it is possible that our experiment will fail, as

(1) After completing each topic we give students a test compatible with the final examination requirements. (Teachers regularly inform us of the results of these.) However, we will learn only next year if we succeeded in preparing students for an exam that will be introduced only after we will have completed our experiment.

(2) The system of the two-level final examination, that the Ministry of Education have been designing for years, and that we have also built on, seems to be doomed to failure. The reason is that universities and colleges content themselves with the core level, in order to have enough students.

Therefore, we do not and cannot think that we succeeded in creating the ideal textbook for the Hungarian setting, but we believe that exchanging experience about textbook writing contributes to the improvement of mathematics textbooks, and that of our book in particular.

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ISTVÁN CZEGLÉDY
INSTITUTE OF MATHEMATICS AND INFORMATICS
COLLEGE OF NYÍREGYHÁZA
SÓSTÓI ÚT 31/B
H-4400 NYÍREGYHÁZA
HUNGARY

E-mail: `czegledy@zeus.nyf.hu`

ANDRÁS KOVÁCS
INSTITUTE OF MATHEMATICS
UNIVERSITY OF DEBRECEN
H-4010 DEBRECEN, P.O. BOX 12
HUNGARY

E-mail: `kovacsa@math.klte.hu`

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