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Teaching
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Interactive web portals in mathematics

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Abstract. Many of the recent problems in higher education (less contact seminars, the heterogeneity and the increasing number of our students) call for new instructional methods. At University of Szeged we have developed a mathematical web portal which can offer a solution for such problems among the changing circumstances. This freely available, easy-to-use web-surface supports interactive mathematical problem-solving and student self assessment. Our computer program cooperates with a lot of free software (computer algebra systems, formula parsers, converters, word processors). WebMathematics Interactive has been available for the public since June 2002 on its web page `http://wmi.math.u-szeged.hu`.

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ZDM Subject Classification: U55, U75.

"Technology is a keystone for innovative global teaching of mathematics at universities in a world-wide situation." [1]

1. Introduction

It has to be recognized that due to the changing circumstances in education there is a big call for new technological tools to achieve our didactical aims. Basic (applied) mathematics should be mediated to a wide range of students with different mathematical skills and precognition at the universities [2]. The education of the basic courses for masses legitimate the search for invention in tools and methods [3]. Today's generation, the age of 14–21 has grown up using computers as a natural tool – not only for playing in their childhood, but for general tasks like word processing or using Internet Information Services which are required by modern society. Without any doubt, future of science education should be strongly based on computers. That was the reason why we chose the Internet as the interface between student and mathematics.

There are several mathematical web applications in the net. Free mathematical web applications however mostly deal only with particular topics, but give no complete solution for the problems of the basic courses. Other kwown progressive solutions (e.g. Wolfram's *Mathematica*) are expensive and have no open source code. Our free software intend to give a comprehensive answer to the problems mentioned above by providing a web-surface that supports interactive mathematical problem-solving and student self assessment. Integrating the power of the computer algebra systems and self-developed packages, a wide range of mathematical exercises and tutorial items serve the learning and teaching process.

The concepts and algorithms considered on the basic mathematical courses can be worked up intensively and self-containedly with the help of the services in *WebMathematics Interactive* (WMI). In praxis, the original reason for the development of this software is to counterbalance the decrease of practical contact seminars in a semester and increase the time which is spent with solving mathematical problems (inter)actively by students (which is particularly important in distant/remote learning). To exploit the services offered by WMI, the user only needs to have an Internet browser (e.g. Opera 7.x, Mozilla 1.x or Internet Explorer 6.x.) and the minimal skills to use it.

We emphasize that WMI is free software, available for anyone, via the Internet or directly (downloading it and using at home). WMI consists of about 20 open source computer programs and a central controlling system which communicates to all other parts in a well-organized, new structure. Statistics and theoretical results show that software development based on open standards can improve effectiveness and may decrease costs in the long term [4], [5]. This seems to be a good basis for WMI development in the future. The authors hope that this new integration provides sufficient conditions for a new didactical approach and computer aided interactive modules [6] will be available in mathematics for everyone soon.

2. Expectations

The system we develop should make possible to construct new, problem- and task-oriented mathematical knowledge modules, which improve activity, provide multi-functional representations (textual, symbolic, numeric, graphical)[1] for all levels of learning, and are available to everyone. There are two ways of describing the application area of computer algebra systems: they can be used

- only as a super-calculator to speed up solving large numerical problems,
- also for authentic educational/research purposes.

While many mathematicians still support the first point of view, our opinion is the second one. Using a suitable system, the developer/teacher has the possibility to help the student by offering an interactive shell as a higher communication level instead of the traditional command driven CAS interfaces. This can improve computer aided education with artificial intelligence and CAS-methods. This shell offers, after submitting a web query, not only the pure solution of a mathematical problem – which was generated in most cases by calling a CAS –, but e.g. the way to find it step by step, including graphical items and other explanations. In addition, the user interface should be so simple that the student would not need to know anything about the CAS, neither the syntax nor the programming knowledge and other details. This might increase student motivation as well – extending the usual "blackboard and chalk" method –, and it should also produce a framework for dealing with mathematics very practically and actively. A suitable system should also offer:

- A new way of assessment with its question forms.
- The complete set of exercises (basic objects and their relationships, properties, algorithms) to basic courses (linear algebra, discrete mathematics, calculus).
- Possibilities to create modules for unique needs, configurable navigation between themes.
- Support for self-checking knowledge at the students' side (ensuring the existence of many reasonable exercises, defining good learning strategies etc.).
- Support for checking of knowledge at the teachers' side (automatic test paper generating option with auto-generated solutions).

[1]Rule of Four: "(re)present every mathematical topic numerically, graphically, algebraically (analytically) and descriptively" [7].

- Modules for self-learning to help interested students to find connections between parts of mathematics and continue self-study in given directions.
- Internationalization support for different language students to share the same educational material.
- Easy development and integration of tests and other material for teachers who have basic programming experience.
- Concurrent access for many users.

We intended to satisfy all these expectations during the implementation of our WMI system.

Taking these expectations into consideration, we developed the WMI system that support local basic courses at University of Szeged both on the students' and the teachers' side. Helping the teacher, modules can be put together including a dynamic set of exercises extending existing lecture notes using the same structure, notation and phrases as the printed book does[2]. This original concept covering the local problems grew into a global application. Flexibility and configurability of the WMI system made it possible to answer global challenges in education as well.

There have been already attempts to use WMI in computer labs for educating small groups of students; however, statistically significant results are not yet available to us to determine to what extent WMI contributes to the effectiveness of education in mathematics. Our plans include measuring and evaluating the output of such experiments, too.

3. Features of WMI

Structurally WMI consists of basic services and thematic modules which are highly configurable. These elements can support all levels of learning process (introduction-discovery, practicing, preparing for exam, application). In Chapter 4 we summarize the modules which we have built up from these basic elements in a higher development level corresponding to our didactical considerations.

[2]For example, in the *Linear Algebra* module we used [8].

3.1. Types of the basic services and elements of WMI

3.1.1. Basic services

WMI offers many basic options (solving equations, calculation of limits and derivatives, function plotting etc.). These services are obtained by giving the input object (e.g. a function or an equation) with typing it into an input box and after starting a web query. In the future these basic services can be extended and the existing options may be enhanced with configuration possibilities (finding the roots among integers, real numbers or complex ones; with respect to which variable we antiderivate; what range we plot the graph on etc.).

3.1.2. Thematic modules

Beyond the basic services from the didactical approach the most important possibility in WMI is to create complex mathematical modules. These modules are built up to the following basis items.

- Web notes: This is an external, static document in HTML format which has a loose relationship to the WMI system. It may contain the definitions for basic objects, theorems and motivating parts as well. (This component may be downloaded in PDF or PS format, too.) Many lecturers wrote such tutorials and lecture notes which can be easily linked to WMI (using converters, if needed). These materials may also contain colored figures and animations like in WMI.
- Tutorials: This item supports basically the discovery and the understanding of the core concepts in the appropriate module and contains many animations and full descriptions of prototypes of the examined mathematical concepts.
- Static, closed-ended tests: Interactive tests are a strength of WMI. They may be static ones which are all questionnaires generated from questions stored in a database. This database also contains the answers from which the student must choose a right one. The WMI system then evaluates the answer and it can also comment on it with remarks and hints.
- Dynamic open- and closed-ended tests: A new feature of the dynamic tests is, that by clicking on the "Reload" button of the web browser, the user gets a new questionnaire all the time. That means that constants, parameters are changed or functions are generated from another class of functions. In addition, static questions may be added to questionnaires randomly, but using some given criteria (the same theme, level of difficulty etc.). The user

can enter his/her answer in an input box using the appropriate syntax (however WMI supports a wide variety of syntax including intuitive input as well, e.g. `sinxcosx` instead of `sin(x)*cos(x)`). After machine evaluation the WMI gives remarks or hints. Hints are displayed in a pop-up window which may contain hyperlinks to the theoretical background, other (similar) exercises, solutions of occurred sub-problems. In general, hints may contain static and dynamic elements depending on the parameters of the concrete exercise, and so theory and praxis get as close to each other as possible.

Before showing the details of the thematic modules, we would like to point out some tasks we had to face up, and give a detailed technical description according to the system and its development.

3.2. Tasks

There are many tasks which play important role in the present during the planning of didactical concepts and technical implementation. The following list shows some examples of the various problems which occurred during the development. The first two ones formulate general technical questions that have to be solved by all such web-based systems. The following ones correspond to a higher level of the development, i.e. to the construction of thematic mathematical modules.

(1) What kind of user interface is required to help the most students to find solutions for their e-learning needs (easy-to-use handling, intuitive icons etc.).

(2) How can a computer detect an incorrect formula and convert it to a proper one. A similar problem is to find the exact place of miscalculations if a bad result is given by the student, and to give the appropriate hint towards the solution.

(3) What methods are sufficient to ensure that in the point of view of didactics, significantly different cases (e.g. in the case of rational functions the types of discontinuity or at solving linear equation systems there i) is exactly 1 solution, ii) are infinitely many solutions, iii) is no solution) will be equally frequent among the randomly generated exercises.

(4) Which way should we choose the dynamically generated entries of square-matrices in the theme *Linear algebra* if we want

    (a) the matrix to be invertible,

    (b) the matrix entries should be integers or "easy" rational numbers, and

    (c) the inverse matrix should also fulfill property (b).

Interdisciplinarity (i.e. co-operation of information technology, pure and applied mathematics and didatics) in this field means that the answer to the questions 3 and 4 can be

(1) given based only pure mathematical theory (e.g. in question 4 determinant of Vandermonde-matrices having all distinct parameters will be non-zero),

(2) algorithms taken from computer science (probably a sequential loop with adequate terminating conditions can ensure that random generated parameters own the expected properties in the easiest case),

(3) mixing 1. and 2., and taking statistical measurements into considerations.

3.3. Technical details

WMI can exploit many services of existing computer algebra systems. WMI is able to integrate several CAS as mathematical engines. (Currently most modules are supported using Maxima[3] and Maple; some modules can be run using MuPAD.) The navigating menu system and the mathematical modules are multilingual (currently English, German and Hungarian texts are available). The WMI framework can handle many types of documents: textual (*HTML*), mathematical (*TEX* formulae), static (generated by *gnuplot* and *eukleides*) and dynamic (*Java* applets, CAS outputs) plots. Harnessing the power of colors, styles and hypertextuality, one can create elements, which can be built up to modules being available as compact units of knowledge. Additional software components can be added easily to the WMI system. The HTML web pages are generated dynamically, this means, for example, that the WMI system can auto-choose constants or functions from certain classes etc. (see Chapter 4.1 for details).

From the programmer's perspective, WMI is a set of PHP scripts installed onto a Unix/Linux server. Its kernel is responsible for maintaining requests from the world and optimizing responses using an intelligent caching system via an SQL database system. It communicates with the underlying software which are the following (only those which are not listed above): *formconv*, *ImageMagick*, *Apache*, *PostgreSQL/MySQL*, *awk* and *gettext*. Additional software were also used for generating documentation and solving further tasks (*LYX*, *LATEX2html*, *sed*, *ps*, *grep*, *GNU make*, *GNU textutils*, *GIMP*, *webalizer*, *rcs*, *cvs* and other Unix/Linux tools).

[3]Maxima is the free version of the oldest CAS, Macsyma. Maxima is available on several operating systems. [9]

The main virtue of developing WMI is that it is programmed using a public model. The developers were able to build on existing software bases that has been already written by others, hence authors can concentrate on didactical question during the biggest part of the development. This software engineering process is supported by the SourceForge.net[4] open source development network portal, using CVS. In the recent past, many articles reported the cost effectiveness of the open source model compared to the commercial software model. The central point of these studies lies in the GNU/Linux system [14].

4. Didactical result of development: mathematical content covered by thematic modules

At the present there are four parts of mathematics aided by WMI: 1. *Linear algebra*, 2. *Calculus*, 3. *Discrete mathematics*, 4. *Analytic geometry*. Currently *Linear algebra* and *Calculus* contain the most elaborated modules.

4.1. Modules in the part of Linear algebra

This part consists of the following modules: matrix algebra (multiplying matrices, inversion, mixed operations), determinants (2×2 determinants, parametric 2×2 determinants, expanding 3×3 determinants via a given row/column), linear equation system, linear dependency of set of vectors, transformation matrix in standard base, subspaces.

Example 1. Advantages of the random generated exercises

Operations on the set of matrices are a basic element of teaching linear algebra. Using WMI, the user can start his/her study by multiplying of square (2×2, 3×3) matrices. Matrices to multiply have their elements from a dynamically generated random set. In interactive mode, the student can enter the entries of matrix using commas (or spaces). There is an evaluation and a detailed hint for this element of module Matrix algebra which can be seen after clicking on the "?" button. Concentrating on special problem types, this synthesis supports creating detailed explanations (like in books) with full preciosity using the advantages of interactivity and hypertextuality (Figure 1).

[4][13] is the official web page of the WMI development. The newest version and the source code can be downloaded freely from this web site. System administrators can read more details on installation here.

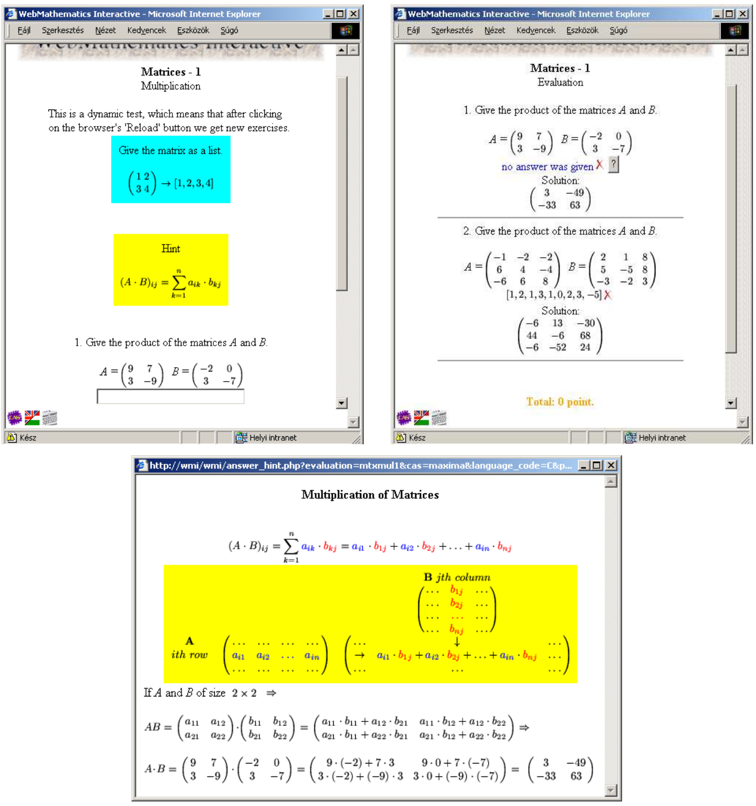
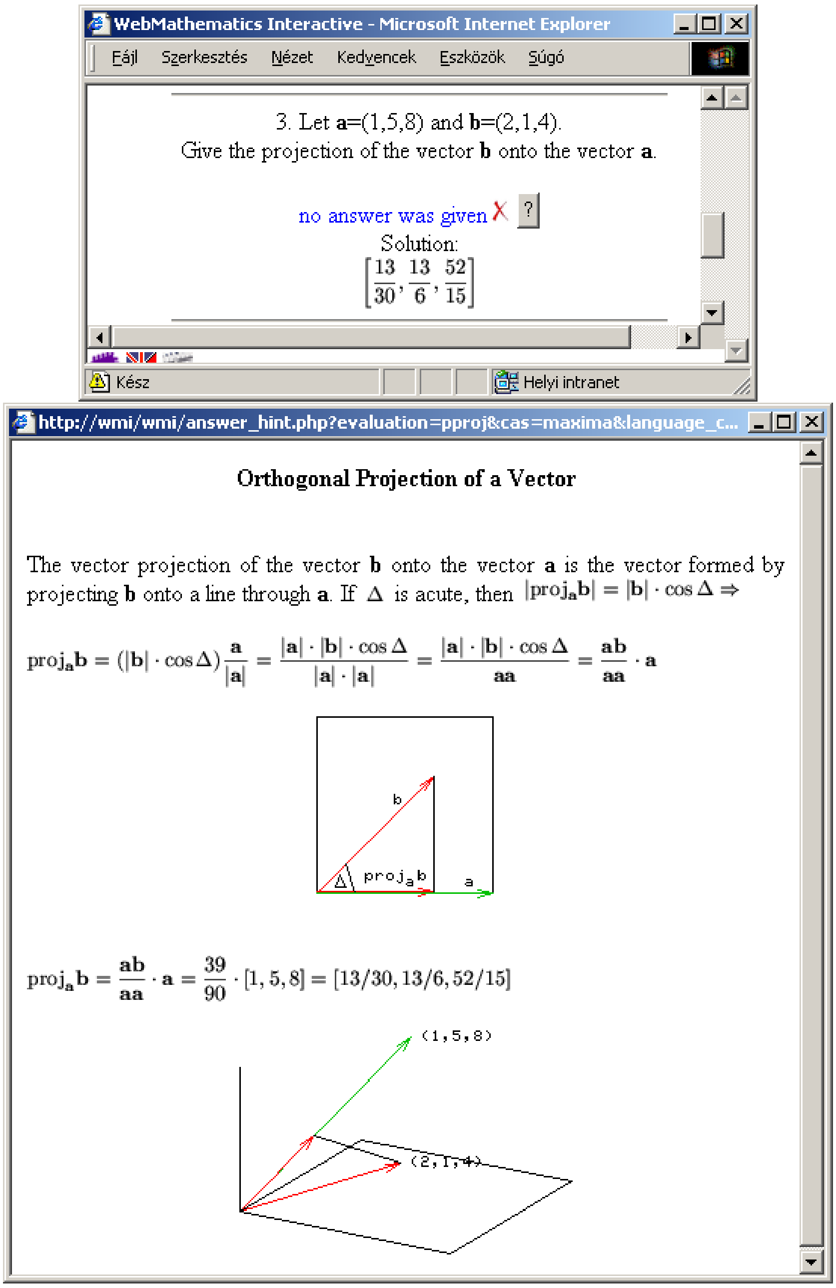


Figure 1

The next parts of the module consist of, for example, multiplication of non-square matrices and inversion. Standing on this base, it is suggested that the student practice the 'mixed matrix operations' element. The connection of the elements demonstrates the model of step-by-step leveling and module hierarchy in the development.

Example 2. Integration of versatile representations

We chose an algebraic exercise for the next example which has a geometrical interpretation in a natural way. The explanation attached to the solution shows the unity of the theoretical material and the practical problem solving in the same textual context. The multiple representation of the mathematical content is also emphasized (Figure 2).



WebMathematics Interactive - Microsoft Internet Explorer
Fájl Szerkesztés Nézet Kedvencek Eszközök Súgó
3. Let **a**=(1,5,8) and **b**=(2,1,4).
Give the projection of the vector **b** onto the vector **a**.
no answer was given
?
Solution:
$\left[\frac{13}{30}, \frac{13}{6}, \frac{52}{15}\right]$
Kész
Helyi intranet
http://wmi/wmi/answer_hint.php?evaluation=pproj&cas=maxima&language_c...
Orthogonal Projection of a Vector
The vector projection of the vector **b** onto the vector **a** is the vector formed by projecting **b** onto a line through **a**. If Δ is acute, then $|\mathrm{proj}_{\mathbf{a}}\mathbf{b}| = |\mathbf{b}| \cdot \cos\Delta \Rightarrow$
$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = (|\mathbf{b}| \cdot \cos\Delta)\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{|\mathbf{a}| \cdot |\mathbf{b}| \cdot \cos\Delta}{|\mathbf{a}| \cdot |\mathbf{a}|} = \frac{|\mathbf{a}| \cdot |\mathbf{b}| \cdot \cos\Delta}{\mathbf{aa}} = \frac{\mathbf{ab}}{\mathbf{aa}} \cdot \mathbf{a}$
b
Δ
proj_a b
a
$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{ab}}{\mathbf{aa}} \cdot \mathbf{a} = \frac{39}{90} \cdot [1,5,8] = [13/30, 13/6, 52/15]$
(1,5,8)
(2,1,4)

Figure 2

4.2. Modules in the part of Calculus

This part consists of the following modules: Linear transformation of basic functions, Limit of sequences and functions, Derivatives, Indefinite and Definite integrals.

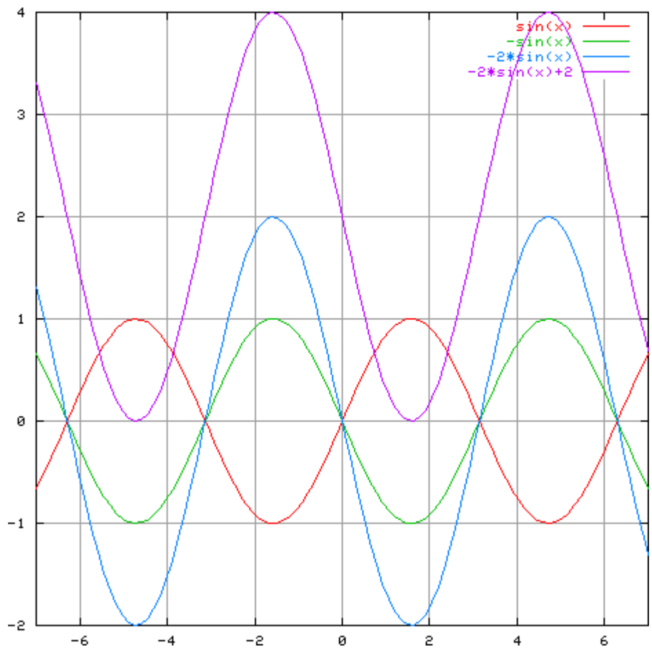
In the next example we show a tutorial item that helps to master the linear transformation of basic function. Figure 3/a shows the result of the web query and the following ones (Figures 3/b-d) present detailed demonstration of the basic transformation steps.

Example 3. Animations and phase portraits of linear transformation of functions

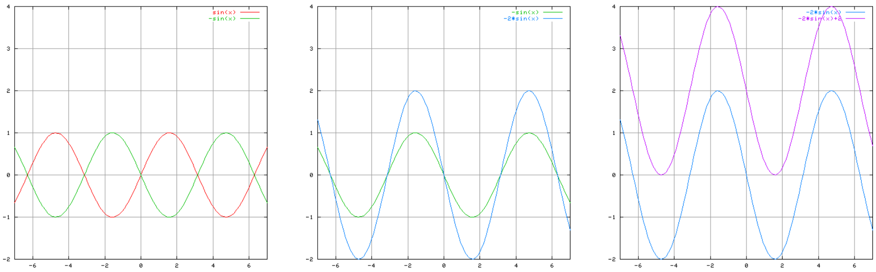
LINEAR TRANSFORMATIONS – 1

Evaluation

EXERCISE 1. Using transformations construct the graph of the function $-2\sin x + 2$ from the basic function $\sin x$.



1. Transformation (Reflection through the x-Axis)	2. Transformation (Vertical Stretching by a Factor of 2 Units)	3. Transformation (Vertical Shifting 2 Units Upwards)



Figures 3/a–d

5. Comparison and conclusion

In this section we present some similar web sites and compare them to the WMI system. It is impossible to enumerate all such projects, however we chose two examples to show them briefly. Internet aided mathematics education is quite a new subtopic of didactics. Standardization and clear criteria are expected to be formulated soon according to the cumulative empirical observations.

MathServ Project (Vanderbilt University) [10]. This web application is developed over a Mathematica Server. There are many basic functions with a wide variety of possibilities of configuration. Evaluation, hints that take didactical viewpoints into consideration, and problem solving analysis are either not well-supported or supported at all. One exception is the Integration-Assistant module which uses artificial intelligence to find antiderivatives and offers a multi-step method teaching integral calculus. However, the output is only textual, so the mathematical formulae do not look very aesthetic.

Mathe Online Project (Universität Wien) [11]. This is a mathematical portal which is designed considering didactical arguments. Its interactive tests were good samples for us while we were developing WMI. Real-time animations and their implied activity, due to its Java applets, puts MatheOnline in advance in that area. This project is supported by the Austrian Ministry of Education (BWK – Bundesministerium für Bildung, Wissenschaft und Kunst) and is developed by several departments of the University of Vienna. A problem for us is that there are too few modules for those service courses (e.g. Calculus) we teach at the

University of Szeged, so actually MatheOnline is not useful for a normal student of ours. There are language limits as well; however, the developers try to make available some modules in English, too[5].

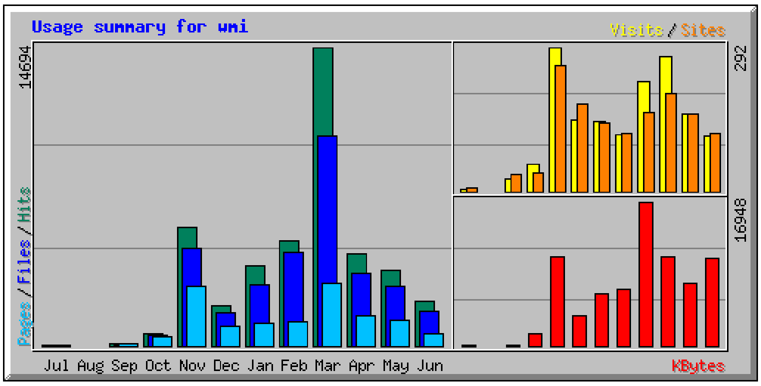


Figure 4

WMI Project (University of Szeged). Here we compare and summarize the main features of WMI. Its strength is offering interactive question forms, their evaluations and explanations with tutorial items. The user can measure his/her solution time during practice drills. Dynamic questionnaires are configurable. Formulae and graphs are of a high typographical quality. Modules can be adapted into several languages. The development is open source based.

As the system logging reports show, WMI was used during 12 months in 30 countries (as of today there are more than 47 000 hits and 14 000 downloaded pages by 1600 visitors not including the test visits of the developers, see Figure 4). There was feedback from France and Taiwan; in the latter country, there is an Internet server which also provides WMI for local users (teachers and students) [12], being also available for the world but mainly serving local needs (clearly e.g. Hungarian language is not supported).

[5]See `http://www.mathe-online.at/literatur.html` subpage for more details on this project.

6. Future plans

Both the WMI itself and its thematic modules are further developed and improved continously. New elements are added and the technology is enhanced. Finally, some concrete objectives only in keywords: real time animations; Java applets; fully intuitive input of complex mathematical structures; configurable WMI elements; logging of user feedback; improvement of the dynamic, intuitive menu system (adding design elements); multi-level tests with branches; developing international contacts; empirical measuring of effectiveness of using WMI; improving development recruiting didactical experts and working teachers.

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