



Report on

**“The Computer Algebra and  
Dynamical Geometry Systems,  
as the catalysts of the Mathematics  
education”**

**Conference, 6–7 June, 2003, Pécs, Hungary**

COMPILED BY CSABA SÁRVÁRI

*Abstract.* The Department of Mathematics of the University of Pécs, Pollack Mihály Engineering Faculty organized in the year 2003 a conference on the role of CAS and DGS in the Mathematics education. We discuss – based on the authors’ abstracts – the conference’s activities.

*Key words and phrases:* Computer Algebra System (CAS), Dynamical Geometry System (DGS), educational application of CAS and DGS.

*ZDM Subject Classification:* A60.

The informational-communicational technology was essentially renewed at the end of the 20th century. Important elements of the new technology’s tools are represented by the Computer Algebra Systems (CAS) and the Dynamic Geometry Systems (DGS). Their applications open new opportunities in teaching Mathematics and other subjects applying Mathematics. Efficient application of CAS and DGS needs continuous developing works. In addition, their application gives a hard task for didactics of Mathematics to reshape the whole educational process.

The Department of Mathematics of the University of Pécs, Pollack Mihály Engineering Faculty organized a conference from 6 to 7 June, 2003 on the role of CAS and DGS in the Mathematics education. The aim of the conference was to get to know each other's results and to discuss new developments and ideas in these fields.

It is not easy to give an overall picture of the recent developments about the above topics in Hungary. On one hand organizing the conference is good news, since a lot of research was done on educational applications of CAS and DGS in the country. On the other hand, it should be mentioned that these developments are the results of independent educational communities. Introduction of the new tools affects almost each element of the educational process. The educational communities have to work hard to keep the mathematical accuracy in the new environment. This means, that continuous and extensive didactical work has to be done. The second goal of the conference was to coordinate and espouse these research works.

The third goal was to contribute to a quick spread of CAS and DGS in education. It is necessary to apply the new tools also in the secondary schools in the near future. To create the conditions for introduction of CAS and DGS into the accredited further education training courses for teachers of Mathematics could be an important step to achieve this goal.

There were 16 presentations on the conference. In the following, based on the authors' abstracts, we discuss the conference's activities.

### Polyhedra and their duals

Lajos Szilassi

szilassi@jgytf.u-szeged.hu

University of Szeged

The idea of duality is used in several fields of Mathematics sometimes with different meaning such as graphs and their duals on surfaces in the topology and relation among polyhedra in projective planar or spatial geometry. In the talk, we show the relation between the duality concepts in the above fields. We give an algorithm that provides the dual polyhedron of a given one with polarity with respect to a given sphere. We show a Maple implementation of the algorithm that solves an interesting problem in constructive geometry. A particular result of this algorithm was in 1977 to find the dual of the so called "Császár polyhedron".

The role of computer algebraic system in generating  
of curves and surfaces  
with trigonometrically blended spline functions

Terézia Pethőné Vendel  
vendel@witch.pmmf.hu  
PTE PMMFK

In the last ten years research groups have been formed in different countries of the world for working in geometric modelling. Spectacular results were achieved in the field of geometric modelling in present days, but the general theory of geometric modelling for complicated arbitrary 3D objects was only partially elaborated. It means in the practice that the designer's possibilities in the modelling process are limited. For modelling and realistic visualizing of complicated objects that are occurring in our 3D world CAD (Computer Aided Design) systems are using mathematical methods based on spline – technique.

This is a challenge for the university education. The engineers of the future have to know and to apply the newest issues. While learning, a Computer Algebraic System (at our faculty the Maple) can be a great help for the students.

Discover a theorem with a combination of Lénárt Sphere  
and computer animation: totally separable covering of the sphere  
by  $N \leq 16$  congruent circles

Éva Vásárhelyi  
vasar@ludens.elte.hu  
ELTE Budapest

The talk reports an experiment which took place under the project named FWF-Project at the University of Salzburg (by Herber, H.-J., Vásárhelyi, É., Astleitner, H., Hofmann, F., Sams, J.): Adaptive teaching and learning: Inner differentiation and individualization by creating prototypes and analogies under consideration of motivational constraints (taking into account computer based teaching and learning).

The goal of the experiment was to help for pre-service teacher training students to learn the fundamental concepts and basic constructions of spherical geometry using the Lénárt Sphere (transparent plastic ball) and a self-made interactive worksheet with Cabri.

## Applying dynamic methods in different levels and topics of teaching geometry

Tamás Árki

arki@jgytf.u-szeged.hu

Juhász Gyula Teacher Training College, Szeged

The Dynamic Geometry Systems (DGS) could be effectively used in each level of teaching geometry in constructing models (for example the different models of hyperbolic geometry) and concepts (for example the envelope of Simson lines), in motivation or in problem-solving. The most important characteristic attributes of DGS are the following:

### 1. Interactivity

In dynamic geometry we can move basis points and the constructed points follow the changing. With the help of interactivity the teaching significant points, lines and circles of a triangle or other polygon is more suggestive. In elementary school we can use “interactive discussion” in the construction tasks or we can also visualise geometric transformations during the work.

### 2. Visualise trace

We can visualise the trace of a constructed point which depends on a basis point while the basis point is running on a curve which can be a segment, a half-line, a straight line, a circle or other conic. This attribute is very useful for example in solving problems connected with conics. Further examples:

<http://www.sulinet.hu/tart/cikk/am/0/11723/1>

### 3. Animation

This is the most attractive attribute of the DGS. While a basis point is running on a curve the actual phase's figure appears in order on the screen. We can establish new concepts effectively with the use of animation, for example envelope or conjugate diameter of an ellipse and so on.

### 4. Replay construction

It is useful in discussion or finding mistakes in the construction. See details at <http://www.sulinet.hu/tart/cikk/am/0/12532/1>

On the process of mathematical modelling  
in the mathematical education

László Máté

mate@math.bme.hu

Budapest University of Technology and Economics  
Institute of Mathematics

Since the universities became a place of mass education we have to change our strategy and methods in the mathematical education. We have to teach mathematics in the service of science or job activities and hence it is important to teach mathematics in the frame of mathematical modelling.

Teaching mathematics with modelling process provides a more plentiful presentation and mobilizes a larger spectrum of abilities than a presentation only in pure mathematics. Hence the education will be more efficient by this fashion as was experienced during my course on Fractal Geometry since a decade. We show this mathematics with modelling method by an up to date example in Molecular Biology. We show a process developing a DNA chain of amino acid molecules into a visual geometric object.

Using the Cinderella program in teaching geometry

Rita Nagyné Kondor

kondorri@freemail.hu

University of Debrecen

College Faculty of Technology

Department of Industrial Management and Technical Informatics

Computer drawing programs opened up new opportunities in the teaching of geometry: they make it possible to create a multitude of drawings quickly, accurately and with flexibly changing the input data, and thus make the discovery of geometry an easier process. The objective of this paper is to demonstrate the application possibilities of dynamic geometric systems in primary and secondary schools, as well as in distance education. A general characteristic feature of these systems is that they store the steps of the construction, and can also execute those steps after a change is made to the input data. For the demonstration of the applications, we chose the Cinderella program.

The assessment of dynamic geometry and computer algebra systems  
at the University of Michigan

Zsolt Lavicza  
lavicza@umich.edu  
University of Michigan

Dynamic geometry and computer algebra systems have been employed in the undergraduate curriculum for several years at the University of Michigan. The Department of Mathematics has integrated Maple software into its courses and the School of Education offers a course introducing dynamic geometry. However, the effectiveness, the kind of mathematics taught, and the student learning in these courses have not yet been thoroughly examined. An increasing need for evaluation and assessment of these courses arose recently so that researchers want to acquire knowledge about teaching and learning mathematics with computers. In my presentation I described the structure, the substance, and the use of technology in these courses as well as I portrayed the research initiatives for the assessment of the software applications. As a result of these projects researchers anticipate to better understand the teaching and learning take place in courses integrating technology and contribute to the improved applications of mathematical software in K-12 and collegiate education.

Comparison of several Dynamic Geometry Systems (DGS)  
from the aspects of users and developers

István László, Pécs  
laszloistvan@freemail.hu  
University of Pécs  
Pollack Mihály Engineering Faculty  
Department of Mathematics

Dynamic Geometry is one of the new sides of computer-aided research and education of geometry. This article dissects the mathematical model, concept, functionality and the common functions of several DGS programs and answers for special questions of the new software-type. Mentioned softwares are Cabri, Cinderella and Euklides. Each of these softwares are really good implemented, offer many functions, but they are vary in their concepts. The author of the article is also author of Euklides, so he has met the questions and problems in depth. Some of the problems are the question of “point on an object”, dynamic intersection points, the mathematical model, locus of points, arcs, layers, macros.

Conclusions are that Cinderella has the most precise mathematical kernel including the accurate computing with homogeneous complex coordinates. Cinderella is able to handle hyperbolic and elliptic geometries. It is good to demonstrate higher geometry. Euklides has the most functions, consequent concept and user-friendly interface. It is the best for the education of euclidean geometry.

The developers have to merge the advantages of several DGS's in the future.

WebMathematics Interactive –  
interactive universal web portal in mathematics

Róbert Vajda  
vajdar@math.u-szeged.hu  
Department of Analysis  
University of Szeged  
Bolyai Institute

Zoltán Kovács  
kovzol@math.u-szeged.hu  
Department of Analysis  
University of Szeged  
Bolyai Institute

At the University of Szeged we have been working on a mathematical web portal for 1 year which can solve many of the recent problems occurred in the current conditions of education. Any kind of computer user can start an internet connection with our software, WebMathematics Interactive, utilizing a simple web browser. Our computer program uses much free software (computer algebra systems, formula parsers, converters, text processors) and works together with them, coordinating that complex informatical tasks which appear during the implementation of this new type of constructivist didactical method in mathematics. WebMathematics Interactive is available for the public since June 2002 on its web page <http://wmi.math.u-szeged.hu>.

### The role of the IT in the education

Katalin Juhász, Debrecen

nyako@math.klte.hu

University of Debrecen

Institute of Mathematics and Informatics

The efficiency of the education has been decreasing for years in Hungary. Different measurements indicate that the knowledge of the pupils and students – especially in the fields of natural sciences and mathematics – is from bad to worse year by year.

Numbers of pedagogues and researchers try to find the way for stopping or reversing this tendency. One of the possibilities is to modernize the education: to involve the IT tools into the education. The other possibility may be the reforming of teacher training: to teach the future teachers for using IT tools in the education.

Numerous experiments prove that the pupils are more motivated and like to use the IT tools in the schools, at home in doing homework etc. Consequently their approach to learning and scholastic marks are improving.

### Problem-solving in mathematics with the help of computers

András Kovács, Debrecen

kovacs@math.klte.hu

University of Debrecen

Institute of Mathematics and Informatics

One of the most important tasks of the didactics of mathematics is the describing of the process of problem-solving activity and problem-solving thinking. The psychological theories concerning the problem-solving thinking leave the special demand of school subjects out of consideration, and search for connections of universal validity. In our lecture we attempt to connect an abstract theory of psychology concerning problem-solving thinking and a more practical conception of the problem-solving activity of mathematics, which is based on Polya's idea. In this way we can get a structure of problem-solving, which has scientific bases and at the same time it is useful in computer aided learning. Our result is suitable especially for the Hungarian conditions of mathematics teaching.



## Experiences using CAS and multimedia in teaching vectorcalculus

Ildikó Perjési-Hámori  
University of Pécs  
Pollack Mihály Engineering Faculty  
Department of Mathematics

We have performed an educational research for six semesters; the numbers of participants were 161 undergraduate and 57 graduate students. The main goals of the educational research in relation to teaching Mathematics III to mechanical engineer students were to work out a new educational strategy, to develop the needed package of the subject material, to realize the strategy in the practice, to analyze the experience from scientific point of view of the mathematical didactics, to evaluate of the attained results. While developing the method we took into account the opportunities of the modern techniques (Internet and computer algebra systems), and results of the new methodological research (particularly the open learning and the mathematical didactics). In the developed and adapted teaching-learning strategy the teacher is the organizer, designer and the manager of the process. With application of the new strategy, the rate of students who successfully achieved the subject increased to 80 %, while it was only 30 % earlier.

## Assimilation of mathematical knowledge using Maple

Zoltán Kovács  
kovacs@nyf.hu  
University of Debrecen  
Institute of Mathematics  
College of Nyíregyháza

For more than four years I have been teaching a Maple course at University of Debrecen for prospective mathematics teachers. In the end of the course I expect the students to be able to visualize mathematical problems with Maple (or to have some feeling of this ability of Maple). At the last part of the course the student is provided with a problem of geometrical flavor. Within three or four weeks he/she must obtain a solution. In the talk I present and analyze two of student projects: rotation of the hypercube and drawing of complex functions. The concluding remark is that most of the students will profit from using Maple for such type of problems: it helps to assimilate mathematical knowledge.

Models of impulsive phenomena:  
experiences with writing an interactive textbook

János Karsai

karsai@silver.szote.u-szeged.hu

Department of Medical Informatics

University of Szeged

“Take the textbook in computer” – is said quite often by our colleagues. Would it be so easy? No, not at all. If we start such a work we meet a lot of trouble very soon. A book stored on a CD, read on the screen of the computer and containing some hyperlinks does not become automatically electronic. There are also difficulties in writing only electronic attachment to a classical book.

In the talk we deal with the most important features (actually important from our point of view) of interactive mathematics textbooks, the arising mathematical, didactical and technical problems. The “principles” are illustrated with examples taken from the book-CD “Models of Impulsive Phenomena”.

Illustrated analysis of Rule of Four

György Maróti

Maroti@epigamma.hu

Rule of Four as a basic didactic principle was formulated among the NCTM 2000 standards and since then it is quoted by numerous books and publications. We can say it is generally accepted by didactic experts. The usage of the Rule of Four, however, has been realized mainly in the field of calculus, in fact certain authors restrict the wording of the principle to the calculus.

Calculus is a pleasant field, indeed. A sequence of a function provides us with example for numeric representation, while the formula describing the function and the graph of the function illustrate a symbolic and a graphical representation, respectively. In the end by wording the basic features of the function on a natural language we gain textual representation.

This idyllic scene becomes more complex when we leave the frames of calculus. In this paper we investigate the consequences of rule of four if we use it outside calculus. We discuss the different types of representations and show several examples which make the multiple features of representations evident. Next we introduce the concept of *base representation* and *realistic representation*, which is intended to be considered as the mathematical notion of “didactic usable” or “didactic rational” representations. In the end we generalize the notion

of numeric representation, which leads us to a more widely usable didactic principle. We name this principle as the *rule of multiple representations*, which can be considered as a generalization of Rule of Four.

### From iteration to discrete dynamical systems using CAS

Mihály Klincsik and Csaba Sárvári  
klincsik@witch.pmmf.hu, sarvari@witch.pmmf.hu  
University of Pécs  
Pollack Mihály Engineering Faculty  
Department of Mathematics

The chaos theory, developed in the last three decade, has caused a crucial change in the approach of the scientific researchers and users. Recognition of concept-system of the dynamical systems has become basic part of the higher mathematical culture.

In our paper we would like to present the basic didactical frame and approaches of such a course made up with the help of Computer Algebra Systems (CAS) for students familiar with fundamentals of analysis. First we review some didactical principles for teaching mathematics in general, write about the advantage of modularization for CAS and refer to the constructivistic view of learning. Then we deal with our own development, a CAS-based collection of programs for teaching Newton's method for calculation of roots of a real function. Included is the discussion of domains of attraction and chaotic behaviour of the iteration.

CSABA SÁRVÁRI  
DEPARTMENT OF MATHEMATICS  
POLLACK MIHÁLY ENGINEERING FACULTY  
UNIVERSITY OF PÉCS  
PÉCS  
HUNGARY  
*E-mail:* sarvari@witch.pmmf.hu

*(Received July, 2003)*