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Teaching
Mathematics and
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Proof without words: beyond the parallelogram law

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Abstract. We present a visual proof of the parallelogram law and using it we can describe a visual proof of a classical theorem on convex quadrilaterals relating sides and diagonals.

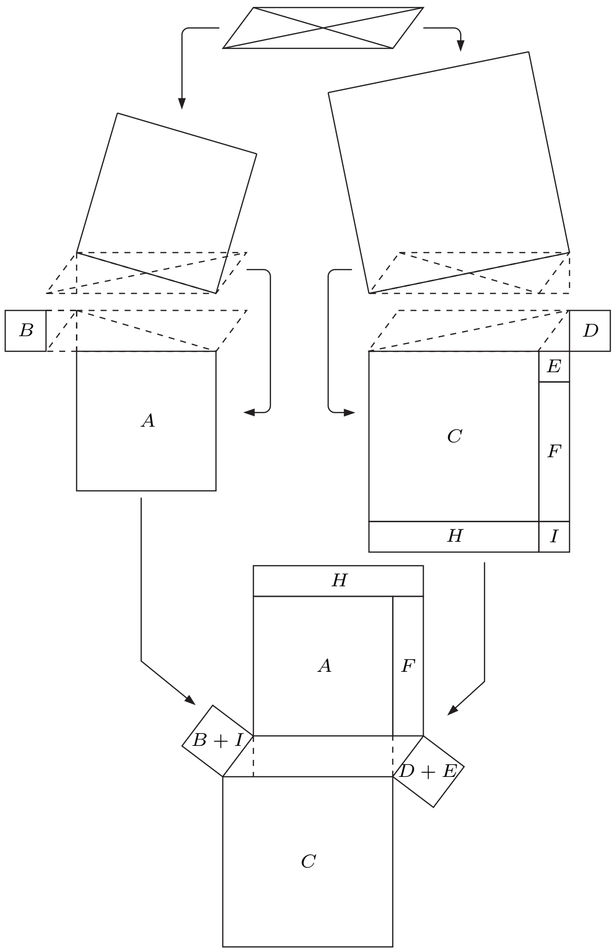
Key words and phrases: proof without words, parallelogram, quadrilaterals.

ZDM Subject Classification: E50, U60, G10, G40.

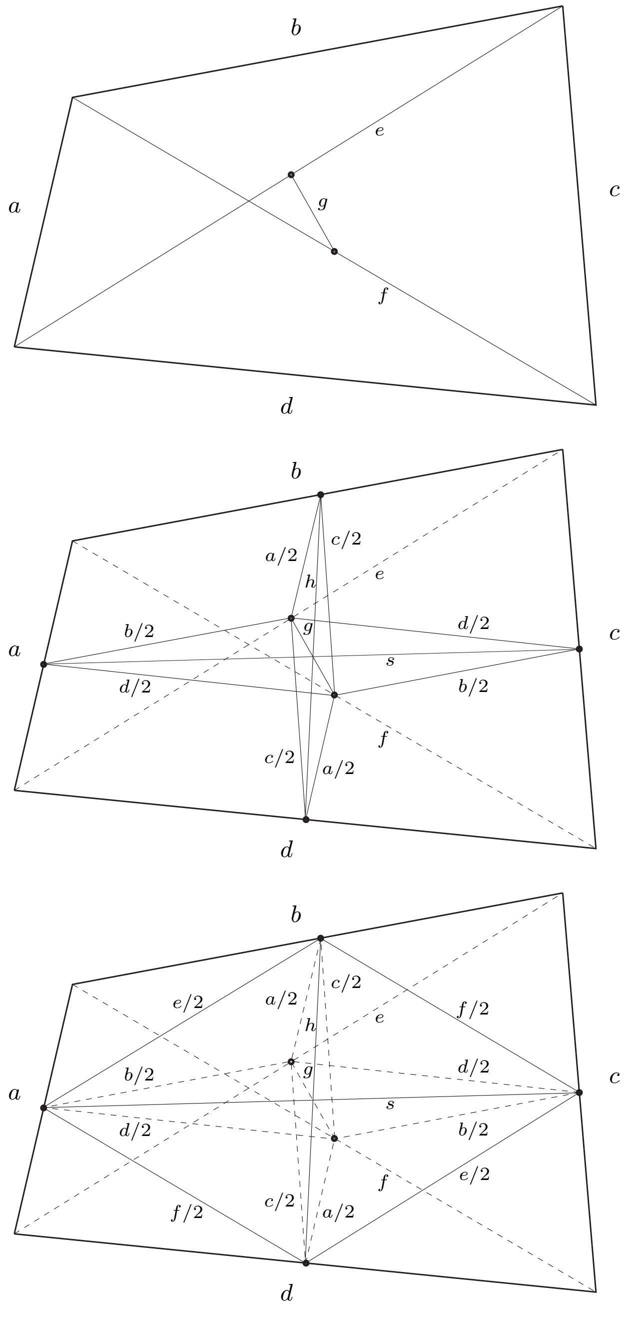
Lemma (Parallelogram law). *In any parallelogram the sum of the squares of the diagonals is equal to the sum of the squares of the sides.*

Theorem. *In any convex quadrilateral the sum of the squares of the sides is equal to the sum of the squares of the diagonals plus four times the square of the distance between the middle points of the diagonals.*

Proof of the lemma.



Proof of the theorem.



$$g^2 + s^2 = 2\left(\frac{b}{2}\right)^2 + 2\left(\frac{d}{2}\right)^2$$

$$s^2 + h^2 = 2\left(\frac{e}{2}\right)^2 + 2\left(\frac{f}{2}\right)^2$$

$$g^2 + h^2 = 2\left(\frac{a}{2}\right)^2 + 2\left(\frac{c}{2}\right)^2$$

$$2\left(\frac{a}{2}\right)^2 + 2\left(\frac{b}{2}\right)^2 2\left(\frac{c}{2}\right)^2 + 2\left(\frac{d}{2}\right)^2 = 2g^2 + 2\left(\frac{e}{2}\right)^2 + 2\left(\frac{f}{2}\right)^2$$

$$a^2 + b^2 + c^2 + d^2 = e^2 + f^2 + 4g^2.$$

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