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Mathematical gems of Debrecen old mathematical textbooks from the 16–18th centuries

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Abstract. In the Great Library of the Debrecen Reformed College (Hungary) we find a lot of old mathematical textbooks. We present: Arithmetic of Debrecen (1577), Maróthi's Arithmetic (1743), Hatvani's introductio (1757), Karacs's Figurae Geometricae (1788), Segner's Anfangsgründe (1764) and Mayer's Mathematischer Atlas (1745). These old mathematical textbooks let us know facts about real life of the 16–18th centuries, the contemporary level of sciences, learning and teaching methods. They are rich sources of motivation in the teaching of mathematics.

Key words and phrases: old Hungarian mathematical textbooks, interaction with other subjects, lower and middle secondary.

ZDM Subject Classification: A 303 History of mathematics and of mathematics teaching.

Introduction

Debrecen is the second largest city in Hungary with an important historical and cultural heritage. It was also called *the Calvinist Rome*, because the inhabitants converted to the new faith in the 16th century and the town became the centre of Calvinism. As in western Europe the Protestant Church applied three tools in Debrecen too: pulpit, school and the press.

The pride of Debrecen is the Great Calvinist Church. The Calvinist College (called the Debrecen Reformed College these days) was established in 1538 on the basis of the mediaeval town school. It was among the first Colleges to teach in Hungarian language.

In the 16–18th centuries Debrecen was one of the country's most advanced towns in industry and commerce. The teaching of mathematics was excellent in the Calvinist College. The students needed a lot of experience in solving everyday mathematical problems, because their aim was to become merchants, clerks, clergymen, burghers, so the professors taught mathematics in a more lifelike and practice-oriented way.

In the Great Library of the Debrecen Reformed College we find nowadays a lot of old books and maps, especially old mathematical textbooks. The authors of these textbooks on mathematics adapted to the needs of everyday life. They were mostly professors. The trend of applying mathematics was a very important viewpoint in the teaching.

The professors of the Calvinist College – G. Maróthi, S. Hatvani, F. Kerekes – were well-versed in the exact sciences. They had opportunity to learn at different universities of Europe (Swiss, Dutch, German), where they obtained their doctor's degree. They had the possibility to carry home new books on mathematics and physics which they used in their practice of learning or teaching. The level of teaching mathematics did not differ from the European standard. The Calvinist College trained well its students in sciences. Some students later became well-known, outstanding personalities.

We have to mention another important date too. In 1561 a press was founded by the family Hoffhalter in Debrecen. The above mentioned facts implied that the first arithmetic book written in Hungarian appeared in Debrecen, in 1577. (Nowadays we can find only a few examplars of its first and second editions in the National Széchenyi Library of Hungary, or in Transylvania).

Also other Hungarian mathematical books were printed in Debrecen, such as a Multiplication table of Julius Padua (1614), F. Menyői Tolvaj Arithmetica (1675), G. Maróthi: Arithmetica (1743, 1763, 1782).

The aims and benefits of studying old mathematical textbooks

We do mathematics under various aspects. I want to show that the instruction of mathematics by applications was always a very important viewpoint in the teaching process in order to understand more fully our surroundings and different sciences.

If we study these old mathematical textbooks we find that the *main aims of teaching mathematics* were:

- to give the pupils mastery of counting connected with real world problems,
- to make the pupils acquainted with different kinds of measures, moneys, which they have to utilize in their everyday life,
- to make the pupils able to gather information about the real world and other sciences,
- to teach them to draw, to construct, to measure,
- to develop their phantasy and space perception.

What are the *benefits* of studying these old mathematical textbooks?

These old *mathematical textbooks* let us know a lot of special old measures, measures of length, area, volume, weight, money with their conversions (i.e. metrological facts), facts about real life of the 16–18$^{\text{th}}$ centuries, e.g. the real prices, data of military character, social conditions, historical facts, the contemporary level of sciences, learning and teaching methods. They are rich sources of motivation.

How can we use this historical material in the classroom?

We collected a problem – book (worksheets) from the old textbooks. These problems are very useful and interesting. For the pupils (aged 10–16) they are a source of motivation. The *main topics* of these problems were:

Historical facts (King Matthias, King Attila and the Conquering Hungarians, Turkish occupacion of Hungary), problems of everyday life (vineyards, households, contributions, income, repay of loans, real costs), metrological facts, social and economical relations, amusing problems, Bible-stories, problems of arithmetical series, proportionality, problems of mixing.

Our methods were:

1. We posed a problem from an old mathematical textbook.
2. For a start we told a story about the life and the scientific work of famous mathematicians (Segner, Hatvani).
3. We discussed historical and national aspects.
4. We did some practical activity or reconstructed the steps of the old solutions (Mayer, Hatvani).
5. Our pupils constructed texts to the drawing book of Karacs (measuring the inaccessible).

6. On the basis of the work of Hatvani we could compare the mortality tables of different ages.
7. We got acquainted with old measuring instruments.
8. In the learning process of geometry we made use of T. Mayer's ideas.
9. We constructed a vocabulary (old Hungarian-Hungarian).
10. The pupils collected historical events, and other cultural facts about the centuries examined.

The forms of the instruction were:

Lecture by the teacher, lecture by pupils, team work (practical work or research work), individual work of pupils.

Presentation of some excellent mathematical textbooks from the 16–18th centuries

1. Arithmetic of Debrecen (Debrecen 1577, 1582)

This book is the first mathematical textbook written in Hungarian. It was printed and published in Debrecen first in 1577 and later in 1582 by Rudolf Hoffhalter, and it is known as Arithmetic of Debrecen. This name was created by Professor L. Dávid, first Professor of the Mathematical Seminar at the University of Debrecen in the 20th century.

The Arithmetic of Debrecen is a very nice and good mathematical textbook. The cover page is framed by Lilies of Florence and in its middle stands the Lamb of God (i.e. Agnus Dei) in black and red colours (Figure 1). The Agnus Dei symbol is part of the coats-of-arms of Debrecen. It is a very good summary for learning to count. Even nowadays it is easy to understand its language.

We don't know who the author of this book was. On the cover-page Hoffhalter says that it was translated from the work of Gemma Frisius, but this is not true. G. Frisius was a mathematician of the Netherlands (1508–1555). The Arithmetica written by him was published in Antwerp in 1540 and was reprinted more than fifty times. The Arithmetic of Debrecen is an original and very clear Hungarian arithmetic.

In the introduction Hoffhalter writes a romantic explanation: "It is not me who is the translator of this work, but it was brought to my office by a pious kin of mine by the grace of God, asking whether I would print it, and stating that he himself would have not been able to tell me the name of that pious person, to whom I could refer in my publication." (See Figure 2.)

For a time it was supposed that one of the professors of the Calvinist College of Debrecen, J. Laskay, had been the author. J. Laskay began his teacher's work at the time of the first edition. He was professor of the College between 1577–1596. One sentence in the preface of the second edition (1582), written by Rudolf Hoffhalter, leads us to the conclusion that he himself could be the author of the Arithmetic of Debrecen: "Although there are a lot of people writing about science, still I can consider the arithmetic of Frisius as the easiest and more general to use in the education of children, because it covers the whole science in a short and well ordered form, and now I am publishing it in Hungarian."

The reference to the work of G. Frisius seems to be a business trick. Both J. Laskay and R. Hoffhalter were too young at that time, they had no name. In my opinion it seems more probable that Rudolf Hoffhalter is both the author and the editor of the Arithmetic of Debrecen. In the introduction he emphasizes that arithmetics is useful for those people who want to deal with geometry, astronomy or philosophy.

The Arithmetic starts with an introduction and has two parts. The major part of the 144-page text explains the mathematical operations with Arabic numerals: enumeration, addition, subtraction, multiplication and division with integers and fractions (except the division by fractions), division to a given ratio, arithmetic progression, discussing Hungarian and German money and measures of weight. The second part of the book explains calculation with calculuses, that is the calculation using pebbles or stones. This book differs from the German textbooks, because after the numeration of integers it presents the addition of fractions, and similarly after the subtraction of integers it presents the subtraction of fractions, and the multiplication of integers is followed by the multiplication of fractions.

Examples from the Arithmetic of Debrecen

Multiplication

– I have got 30 soldiers. I have to pay to each of them 3–3 forints for a month. How many forints do I have to pay to the 30 soldiers for a month? (Figure 3)
– How many times does a clock strike a year? In a year there are 365 days, and in a day there are 24 hours. (Figure 4)
– There are 12 friends, each of them has 12 shops, in each shop there are 12 bags, in each bag there are 12 loaves of bread, in each loaf of bread there are

12 holes, in each hole there are 12 mice, and each mouse has 12 little sons. How many little sons are there in all? (Figure 5)

The Arithmetic of Debrecen presents the methods of inference, and standard problems related to profit sharing among the partners in a joint business venture under the heading social rule (Regula societatis). The author of the Arithmetic of Debrecen remarks that "in Hungary this rule has not much practical use as the Hungarians are hardnecks and reluctant to pay."

De Regula Societatis (Figure 6)

Examples. Three men formed a company. One of them gave 50 Ft, the other man gave 60 Ft, the third man gave 70 Ft into the business. They won 100 Ft. They wanted to share this sum in proportion of their given money. How many Forints are due to the different members of the company?

De Regula Falsi (Figure 7)

Examples. A copper merchant wants to buy 60 quintals of copper from another merchant. This merchant says that he has less than 60 quintals of copper. He explains that if he had the present weight of copper and had once it again, then the half and the quarter of the weight, and still 4 quintal then he would have the 60 quintal.

Question. How many quintals of copper has the merchant?

Remark. It is very easy to solve this problem by an equation:

$$x + x + x/2 + x/4 + 4 = 60, \quad \text{so} \quad x = 204/11.$$

The author solves this problem with the help of false numbers (regula falsi). The rule of false position, as the mediaeval Hungarians called it, is a root approximation method, regula falsi meant a tool for them to solve equations by trials.

One of the most conspicious features of the first Hungarian arithmetics is their emphasis on usefulness, or practicism. These early Arithmetics are illuminating and fascinating readings for other reasons as well. Their examples give a lot of information about the life and work of mediaeval Hungarian people.

2. G. Maróthi: Arithmetic (Debrecen 1743, 1763, 1782) (Figure 8)

G. Maróthi (1715–1744) was a famous professor of the Calvinist College of Debrecen. (Figure 9)

His father was a town counsellor, later the chief justice of the town of Debrecen. G. Maróthi was an infant prodigy. He finished his studies in the Calvinist

College when he was sixteen and went to study abroad (1731–1738). After returning to Debrecen he became professor of the Calvinist College. He taught there only six years, he had a short career. He worked too much and died as a young man.

He wrote a very good arithmetics book for the Calvinist schools. He had heard about the Arithmetics of Debrecen, (1577, 1582) and Kolozsvár (1591) (Figure 10), but in Maróthi's days they could not find these books anywhere. Both the professional and methodological aspects of Maróthi' s arithmetic compare well with his day's European textbooks of the highest standards. In the preface he modestly stated:

"I did not leave anything that I thought would be necessary for our country".

It was published three times, and it was still in use in schools as late as the beginning of the 19$^{\text{th}}$ century. Professor Maróthi was an excellent textbook writer, and one of the successful creators of the mathematical language. His language was very easy, his terms are the same as we use nowadays. His aim was to create the language of mathematics in Hungarian. He wanted that "even the womenfolk could understand them".

In his Arithmetic, Maróthi gave advice to teachers and also to students about teaching and learning methods.

"In calculation, one had better put everything to paper, if possible, nothing should be left to memory, for it will deceive one before long... If anyone of you should know a better rule, and can show a better way, I would be only too glad to learn about it."

Maróthi was a very good teacher, his methodological clues are true nowadays too, e.g.: "It would be better to hurry slowly."

In Maróthi's Arithmetic we find 4 operations: addition, substraction, multiplication, division. He mentions the abacus, the calculating with calculi (pebbles), he calls this method computing with *peasant numbers*. We find a lot of old liquid or cubic measures (bucket, bushel, can, quarter, vat, and special Hungarian measures: akó, icce, etc.), and currency denominations (gold coins, imperial thaler, short thaler, German forint, 30-, 20-penny coin, silver coin, different small coins, half penny).

Examples from Maróthi's Arithmetic (1763)

1. IV. Példa (pp. 14) (Figure 11)

I bought a vineyard for 400 Ft. In this year the vineyard was producing 150 akó wine. I would like to know how much costs my wine.

– I spent on the cultivation Ft 77.59 d.

– I payed Ft 37.56 d tithes.

– I payed for the grape gathering Ft 21.06 d.

– I payed to the carman Ft 19.50 d for carrying home the grape.

– I could have obtained 24.00 Ft as interest in a year for my 400 Ft.

This way the costs of 150 akó wine were Ft 179.71 d.

I would like to sell it for not less than this sum. If I couldn't realise this amount then it would have been better to put my 400 Ft out at interest.
(1 akó $\approx$ 50 litre)

2. V. Példa (pp. 14) (Figure 12)

I bought 6 porkers for Ft 58.34 d.

They had eaten as much of my barley as I could have sold for 17 Ft. I payed for the milling Ft 1.27 d. How much did cost me the 6 porkers? We write it down:

1. The price of the six porkers:	Ft	58.34 d.
2. The value of the barley:	Ft	17.00 d.
3. The expenses of the milling:	Ft	1.27 d.
Total:	Ft	79.61 d.

3. V. Példa (pp. 79) (Figure 13)

I bought 81 icce sack wine for 30 Forints. How much does 1 icce sack wine cost? (Icce is an old liquid measure. 1 icce = 0.88 litre or about one-fifth of a gallon.)

3. S. Hatvani: Introductio ad principia philosophiae solidioris (Debrecen 1757)

S. Hatvani (1718–1786) was one of the well-known professors of the Calvinist College of Debrecen. (Figure 14) His interesting and colourful character stirred the imagination of writers and poets (M. Jókai, J. Arany, etc.), who invented stories about Hatvani's scientific experiments in order to attribute him magic power. He got the name Hungarian Faust from one of the most popular writers of the 19th century, M. Jókai, who wrote a short story with this tittle about him. Hatvani was the first professor at the Calvinist College who made physical and chemical experiments at his lectures. He lectured on mathematics, geometry, philosophy, physics and astronomy. He was the first to give regular lectures on chemistry in Hungary.

He started his studies at the Calvinist College of Debrecen. After graduating from the Calvinist College he continued his studies in Basel. He got doctor's degrees in theology and medicine, but he learnt mathematics on the lectures of J. Bernoulli (1667–1748) and D. Bernoulli (1700–1782).

S. Hatvani wrote two mathematical works: *Oratio inauguralis de matheseos* . . . (1751) and *Introductio ad principia philosophiae solidioris* (1757). He wrote this book for students (aged 18–20) and for educated people. We find two essential applications of mathematics in it.

a) In the *Appendix "Observatio elevationis poli Debreceniensis"* Hatvani gives a construction for measuring the geographical latitude of Debrecen using the shadow of a high stick. His measuring was fairly good (47° 25'), nowadays the punctual value is 47° 33'. (Figure 15)

b) This book contains a part which deals with the elements of probability and statistics. The author shows the fundamental concepts of insurance mathematics by problem solving (probability of mortality, life expentacy, average age) and applies them to Hungarian public health and mortality, and finally he draws his practical conslusion about the bad state of national health. (Figure 16)

If we arrange these data in a table, we get:

Year	1750	1751	1752	1753
Number of birth in Debrecen	1022	890	832	936
Number of death in the first year of life	235	304	260	250
Percent of death in the first year of life	23%	34%	31.25%	26.70%

4. F. Karacs: Figurae Geometricae (Debrecen 1788)

F. Karacs (1770–1838) was a student of the College of Debrecen. He could draw beautifully, and he joined the engraver students of the town. Later he became an engraver and made a lot of maps. *Figurae Geometricae* is a little drawing book, which contains 10 drawings to illustrate geometry. These drawings are showing that applications (i.e. architecture, measuring of distances, measuring volume of a barrel) (Figure 17) form an important part of geometry.

5. J. A. Segner

In the Great Library of Debrecen Reformed College there are the following books of Segner:

a) *Anfangsgründe der Arithmetik. Geometrie und der geometrischen Berechnungen* (Halle, 1764)

b) *Deutliche und vollständige Vorlesungen über die Rechenkunst und Geometrie* (Lemgo, 1747, 1767).

J. A. Segner (1704–1777) (Figure 18) is one of the first Hungarian mathematicians known and recorded by the history of mathematics. He went to school in Pozsony (now Bratislava) and in Győr, and most probably studied a year at the College of Debrecen (1724). From 1725 he studied medicine, natural sciences and mathematics in Jena. He graduated as a physician. Between 1730–1732 he was doctor of the town of Debrecen. Then he worked at the University of Jena. From 1735 to 1755 he was professor at the University of Göttingen, where he taught mainly physics, mathematics and also chemistry. He took part in founding the new observatory of the University in Göttingen. In this observatory Tobias Mayer followed him, because after the death of Ch. Wolf he became professor at the University of Halle. He invented the prototype of water-turbine, the so called *Segner-wheel* (1750) (Figure 19), which was constructed on the theory of action and counteraction. In his textbook *Cursus Mathematici I–III* (Halle, I. 1757, II. 1758, III. 1767–1768), he began the chapter on solid geometry by presenting the forgotten *Cavalieri principle* (1626). (Figures 20–21) He explains and applies that principle for determining the volume of the sphere. For a long time it was believed that Segner was the discoverer of the principle of Cavalieri. The book *Anfangsgründe der Arithmetik* is the German version of Cursus Mathematici I.

Segner's mathematical books are very good. He has a subtle sense to discover long-forgotten values in the heritage of the past and an ability to elaborate the achievments of his age systematically so as to be understood by a wide readership. His books were very popular. His independent achievments include a proof of Descartes's rule of signs. Segner's examinations of inertia, acoustics and optics are also well-known.

6. T. Mayer: Matematischer Atlas (Augsburg 1745) (Figure 22)

There exist only a few exemplars of T. Mayer's Mathematischer Atlas. I know that in Germany there is one exemplar in Munich (Deutsches Museum) and the original, written and coloured by hand, is in the Landesbibliothek Stuttgart. In the Great Library of the Debrecen Reformed College we can find a coloured exemplar. (Figure 23)

How could the Calvinist College obtain this Mathematischer Atlas?

We know that one of the later mathematical Professors of the College, P. Sárvári, studied in Göttingen in 1792 with Professor Kästner. A contributor to the history of parallels, Professor Sámuel Hegedüs (Nagyenyed Reformed College, Transylvania), visited Göttingen in 1807, and he was corresponding with Gauss and Johann Tobias Mayer (1752–1830). This is a possible connection.

We have to mention that two persons had the same name: Johann Tobias Mayer, the father (1723–1762) and his son (1752–1830). Both were professors of mathematics and physics at the University of Göttingen. The son of Tobias Mayer wrote a lot of books on mathematics and physics. In the Great Library of Debrecen Reformed College we find one of his books: *Gründlicher and ausführlicher Unterricht zur praktischen Geometrie* (5 Band, Göttingen, 1777) (Figure 24). Its content is essentially the same as that of the father's *Mathematischer Atlas*, but the Mathematischer Atlas is more beautiful, descriptive, lifelike. It gives an intuitive method of mathematical instruction.

The other fact is that J. A. Segner was professor in Göttingen from 1735 to 1755, and Tobias Mayer (the father) became his successor. J. T. Mayer (the son) was professor in Göttingen between 1773–1779 and from 1800 too.

The Matematischer Atlas consists of 68 etchings, from which 60 plates deal with mathematics and applied mathematics (arithmetics, metrology, geometry, trigonometry and their applications, physics, mechanics, optics, astronomy, geography, cartography, chronology, gnomics, pyrotechnics, military and civil architecture) and 8 plates are from higher mathematics: arithmetical and geometrical exercises, conic sections (parabola, hyperbola, ellipse), geometrical places, curves (spiral, cissoid, conchoid of Nicomedes, conchois, logarithmical curve, cyclois), Thomas Baker's main-principal, method of excess and defect, infinitesimal calculus and applications (subtangents, subnormals of the parabola, hyperbola and ellipse), circle, length of arc of the circle and the area of parts of the circle.

Tobias Mayer's Mathematischer Atlas is a book for individual study. It is a very peculiar work. The origin of its form and contents follows from the life and personality of Tobias Mayer.

This Atlas was published by the firm of J. A. Pfeffel (Augsburg) for which Mayer worked from 1744 to 1746. During this time Mayer extended his scientific and technical knowledge, learned languages (French, Italian, English). Mayer left Augsburg to take up a post with the Homann Cartographic Bureau in Nuremberg. From 1751 he became professor of mathematics and economics at the University of Göttingen.

Some of Mayer's lectures (1752–1762) were printed in *Göttingische Anzeigen von gelehrter Sachen.* In these years he invented a new goniometer and explored application of the repeating principle of angle measurement, developed a new projective method for finding areas of irregularly shaped fields and transformed the common astrolabe into a precision instrument, he applied the repeating principle to an instrument of his own invention, the repeating circle, which proved to be very useful for sea navigation. The instrument used by Delambre and Méchain in their determination of the standard meter was a variant of Mayer's circle.

Presentation of the Mathematischer Atlas

Mayer had a special method: he gave graphic descriptions of mathematical definitions and their properties. He also made drawings – so this book is a mathematical picture – book. On the other hand on the 60 copper engravings he illustrated mathematics and its applications too. The whole work is very attractive. It is easy to read it in spite of being in Gothic letters. Each page consists of three big parts. On the left and on the right side in two columns there is theoretical information, in the middle there are drawings, figures and tables with data.

The contents of the 60 plates are:

Counting, addition, substraction multiplication, division, extracting of roots, prime numbers, different measures (length, area, volume, capacity), proportionality, concept and applications of logarithm, elements of plane and solid geometry, applications of geometry to geodesy, to the division of land, geometrical instruments, lenses and mirrors and the principles of forming pictures, measuring of weights, measuring of the capacity of a barrel, measuring length and height of different objects (towers, buildings, altitudes, etc.) with the help of trigonometry, spherical trigonometry and astronomy, 9 different kinds of sundials and the constructing of time at the sundials, problems of calendars (days, months, years, different forms of calendars), world concept of Copernicus, solution of the problem of Kepler, lunar occultations, solar eclipses, motion of comets, Moon, Sun, constructing of maps, stereographical projection, lines and curves at fortifications, principles of building starshaped entrenchments, some military problems, weapons and their geometrical constructions (canons, canon-balls, handgrenads, petards, bombs, etc.), ballistical problems of different kinds of shells, analysing the mathematical problems of their paths and the optimal arrangement of bullets and the optimal posing of guns at the artillery, building of living houses, especially different forms of columns and roofs, plan of the house, the characteristics of comfortable building, ground-plane of the building (first and second floor), principles of construction by the help of perspectivity, perspective pictures and their shadows, and a few physical problems.

Detailed analysis of some plates

Table IV (Figure 25) gives the basic concepts of plane and solid geometry in three parts a) Fig. 1–12, b) Fig. 13–16, c) Fig. 17–21.

a) The concepts of point and straight line are given in the same way as with Euclid. Mayer distinguishes between circle line and cirlice. He illustrates, the concepts of the centre of a circle, half-diameter, diameter, chord, arc curve, parabola, perpendicular lineals, parallel lines, straight and circular lines. The definition of parallel lines is that their distance is constant. We find the definition of an angle, rectangle, acute angle, obtenuse angle, sections with proportional length (arithmetical and geometrical proportions).

b) We find here the most important plane figures (triangles, square, rectangle, rhombus, rhomboid, trapese, trapezoid, delthoid, regular pentagon and hexagon, irregular hexagon, convex and concave polygons, circle, the different parts of the circle (segments, sectors). Three figures of the Fig. 16 are very interesting, namely the T, X and Z figures. I have never seen such kind of figure in schoolbooks but the idea is excellent. In Fig. 16 T. we see a plane figure which is limited not only by straight lines, it has a curved side too. The Fig. X and Fig. Y show the same idea. The author emphasizes that there are segments or sectors which are smaller or bigger than a halfcircle.

c) Fig. 17–21 illustrate the most important solids (cube, box, pyramid, prisms, oblique prisms, cone, truncated cone and pyramid, cylinder, oblique cylinder, sections of cylinders and cones, spheres, half-sphere, spherical segments, spherical sectors, regular polytopes).

I will draw the attention to 2 viewpoints:

1. At the Fig. 18 L, we see a concave pyramid. Why cant't we find this illustration in our schoolbooks?
2. The Fig. 20 T and V, resp. X and Y show the same principle of classification as we have seen at Fig. 16 U and X, resp. 16 Y and Z.

Table V (Figure 26) deals with different geometrical constructions: constructing perpendiculars, parallels, copying angles, bisector of an angle, bisector of a segment, dividing an angle into equal parts, dividing a line segment into equal parts, constructing numbers as a length, multiplication and division, constructing geometrical means of two segments (2 methods), golden section.

Table VI (Figure 27) deals with the construction of different shaped triangles from 3 given lengths of side, rectangle, rhombus, rhomboid and regular n-gons (square, regular 5-gon, 6-gon, 7-gon (approximately).

For constructing a regular 7-gon Tobias Mayer applies the method of Renaldini. He divides the diameter VF of a circle $(1,2,3,4,5,6,7)$ into 7 equal parts, with the distance $V7$ he makes an arc from V and from 7. The points of intersection are C and D. Then he connects C and D with 2, 4, 6. These lines determine the points B, A, Z and W, X, Y. The vertices of the regular 7-gons were V, W, X, Y, Z, A, B.

Table VII (Figure 28) deals with converting a surface into another surface with the same area. He converts a triangle into a rectangle, a rhombus, a rhomboid, a trapeze into a rectangle, into a square, a trapezoid, a square, a non-regular 4-gon, a circle, and a segment (approximately) into a triangle, a half circle into a circle.

The approximate construction applied by Tobias Mayer for the quadrature was the following:

He draws two perpendicular diameters of a circle (MO and PQ), divides into four equal parts the length of the chord PQ (P, 1, 2, 3, O), then connects Q with 1, projects orthogonally 1 to PQ, so he gets the point R. The length of the square's side will be equal to RQ. If we calculate the length of QR, assuming that the length of the diameter MO is $2r$, we get $d_{QR} = 1.75r$, $d_{QR}^2 = 3.0625r^2$, so $\pi \approx 3.0625$. (It is not a very good approximation.)

Summary

We found that the pupils' attitude to following the historical way was very positive. This kind of instruction made them curious, they became more open minded and more creative. We could show the continuity of mathematical concepts and processes over the past centuries. We motivated the learning process in the classroom, because our pupils deal with problems which centuries ago were objects of investigation. These problems allow the pupils to touch ancient past, they connect mathematics with various cultural and intellectual developments. We often can learn from the mathematical mistakes of the past. We brought biographies into the classroom. Some mathematicians have interesting lives. We can refer to Hungarian literature (stories, poems). Besides the mathematical discussion we can visit the Museum and Great Library of the Debrecen Reformed College. It would be a useful mathematical adventure to get acquainted with the mathematical gems of Debrecen.

Figures

Figure 1

Figure 2

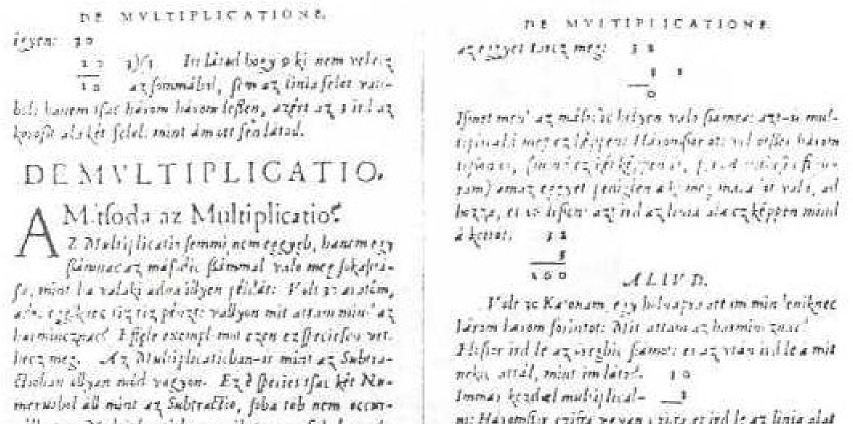


Figure 3


DE MVLTIPLICATIONE.

ALIVD. Vagyon 100 hordo borom akarnám ki árulni nyolcz nyolcz pénzen pintit: vallyon mit megyen ki mind az három száz hordo bor? Elsöben lásd meg mint egy hordóban hány czebar vagyon, no bár legyen csac 12 czebar. Elsöben multiplicáld meg az 100 hordo bort, es mind czebarra leszen, lásd meg azért igyen:

100
12
600
900
9600

Immár azt-is lásd meg hán' pint vagyon egy czebarban. No legyen 20. Multiplicáld meg azért az húszszal imez diéte summát igyen 9600

20

192000 Immár az min akarod ki árulni azt-is írd alaia, es multiplicáld meg, es valamint az multiplicatioból ki iu, anni leszen az ára.

192000
8
1536000

Somma szerint azért ennit teszen az ára vel, ennit megyen ki az bor mindenestül fogvan, tudni illic tizen öt ezer, három száz es hatvan forintra.

ALIVD. Vöttem négy Bál Barasztát: annak-is adom fingit 25 pénzen: vallyon mennit megyen ki? Elsöben lásd meg az eg' Balban hán' vég Baraszt vagyon No bár legyen ebben-is ötven vég: azért igy tselekedgyél vele 50

4
200

Vagyon azért az nég' Balban két száz vég. Lásd meg immár azt-is, eg' végben hán' fing vagyon. No legyen 25 fing, ezzel-is multiplicáld meg az két száz igyen: 200

25

Vagyon azért a két száz vég posztóban öt ezer fing: azért immár azt-is lásd meg a min ki akarod mérni fingit. No mérjünc el fingit 25 pénzen. Azért multiplicáld meg huszon öttel az alsó számot, es meg látod mire megyen ki az négy Bál poszto. 5000

25

Látod hog' ezer, két száz, es ötuen forintra megyen ki a nég' Bál poszto.

ALIVD.

Itt lássuc meg esztendeig hányat üt az óra. Lásd meg azért, eg' esztendöben hány nap vagyon. Vagyon penig 365 nap az esztendöben: Azért azt-is iol tudod hog' eg' napban es egy éyben vagyon 24 óra, azért az nap számot multiplicáld meg, igyen 365

24

Vagyon azért esztendö által enni óra, tudni illic nyolcz ezer hét száz es hatvan óra, sem töb sem kevesebb 8760

Olly szeres az Multiplicatio, mellyen sokat el vegezhetz

Figure 4


DE MVLTIPLICATIONE.

43600 Látod hog' itt ez exemplomban felyül 120 két czifra vagyon, alúl penig egy czifra vagyon, azért ezzel így tselekedgyél, hogy mind a három czifrát írd le az linia ala igy mint im látod itt, es az többit multiplicáld meg a mint látod itt, tudni illic az ieggyzendö számokat kezdel igyen 43600

120

ALIVD.

Vöttem egy Bál posztot Karasiat: vagyon benne ötuen vég, mindenic vég tart huszon nég' nég' finget: Singit adom hatuan hat hat pénzen, vallyon mire megyen ki. Elsöber azért lásd meg az eg' Bálban hán' vég vagyon: No legyen ötuen vég: Immár lásd meg mindenic végben hán' hán' fing vagyon. No legyen huszon nég' fing: Immáran ezképpen tselekedgyél vele 50

24

Látod hogy enni fing vagyon benne tudni illic ezer es két száz. Azért immár azzal-is meg multiplicáld az min el akarod adni illyen képpen mint im meg mutatom:

Látod hog' két száz es kilenczuen két forintot teszen, mert amaz két czifra penz számot tenne, ha ieggyzö Numerus volna lé-

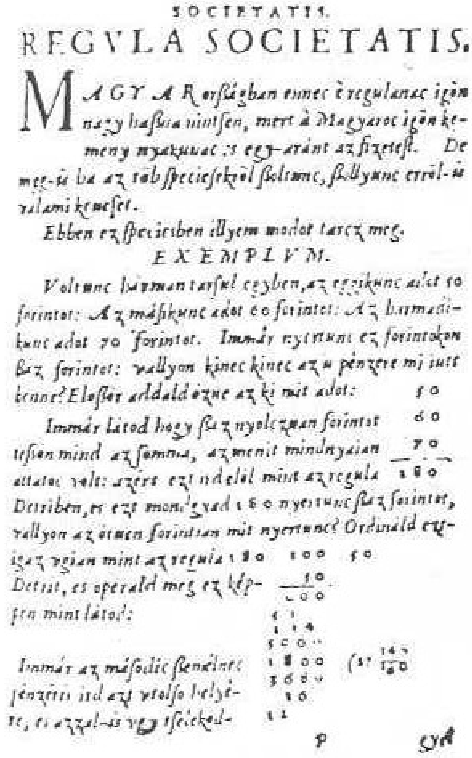
lyen, de hog' egy fincze, azért vagyon csac semmit teszen, mint ott sem-is meg tanittalac, hog' mikoron pénzel multiplicálod meg ackor a két utolsó figurának vony eg' két liniackát, es az linia elöt valameni számot teszen, de mind anni forintot teszen.

ALIVD. Vált 12 Barat: mindeniknek vagyon 12 bottia, mindenic botban vagyon 12 taska, mindenic taskaban vagyon mind tizen két két kenyér darab mindenic kenyér darabban vagyon tizen két egér, es mindenic egérnec vagyon tizen két két fia, vallyon mennit teszen somma szerint?

Barat. 12
Bot. 144
Taska 1728
Kenyer 20736
Lyuk. 248832
Egér 2985984
Eger fiac. 35831808

Somma szerint ennit teszen mint im látod az linia alat, harmincz ötször való ezerni ezer, nyolcz száz harmincz eg' ezer, nyolcz száz es nyolczat.

Figure 5


SOCIETATIS.

REGVLA SOCIETATIS.

MAGYAR orſzágban ennec è regulanac igön nagy haſzna vagyon, mert à Magyaroc igön kemeny nyakuuac és egy-aránt az fizeteſt. De meg-is ha az töb ſpeciesekröl ſzoltunc, ſzollyunc errül-is valami keueſet.

Ebben ez ſpeciesben illyen modot tarcz meg.

EXEMPLVM.

Voltunc haroman tarſul egyben, az egyikunc adot 50 forintot: Az máſikunc adot 60 forintot: Az harmadikunc adot 70 forintot. Immár nyertunc ez forintokon ſaz forintot: vallyon kinec kinec az ö pénzere mi iutt venne? Eloſzer addald öszue az ki mit adot: 50

Immár látod hogy ſaz nyolczuan forintot 60
teſzen mind az ſomma, az menit mindnyáian 70
attatoc volt: azért ezt ird elöl mint az regula 180

Detriben, es ezt mondgyad 180 nyertunc ſaz forintot, vallyon az ötuen forintban mit nyertunc? Ordinald ezt igaz vgian mint az regula 180 100 50

Detrit, es operald meg ez képpen mint látod: 50 / 500

Immár az máſodic ſzemélynec pénzétis ird az vtolſo helyére, es azzal-is vgy ſcécked- (27 140/180

Figure 6


DE REGVLA

DE REGVLA FALSI.

FALSI.

Exemplum.

REGVLA

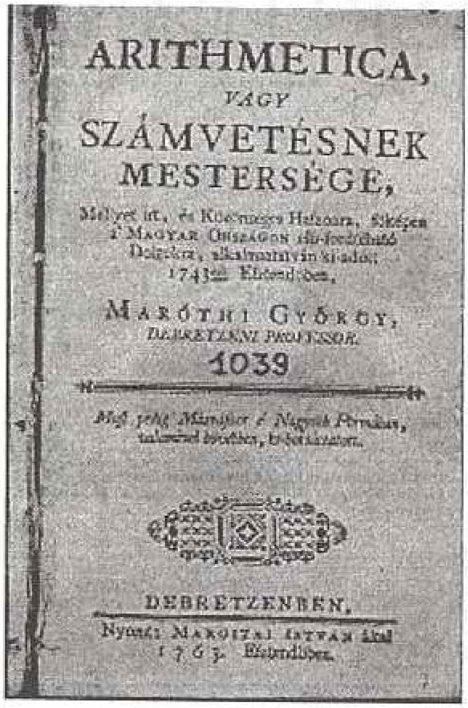
Es igy all.

ALIVD.

PROBA.

Enni	maſa	23	
Megint enni	maſa	23	
Fel enni	maſa	11½	
Egy fertal	maſa	5¾	
Es ahoz	maſa	6	

Figure 7



ARITHMETICA,
VAGY
SZÁMVETÉSNEK
MESTERSÉGE,
[illegible]
MARÓTHI GYÖRGY,
DEBRETZENI PROFESSOR.
1039
[illegible]
DEBRETZENBEN,
[illegible]

Figure 8

Figure 9. G. Maróthi

MAGYAR
ARITHME-
TICA, AZ AZ, SZÁMVE-
TESNEC TVDOMA-
NYA.

Most víyonnan az Frísiusnac Magyar Arithmeticayából soc wy és haß-nos példáckal ki adatot.

Joan. 1664. Zagoni

COLOSVARAT
Chrístus Wrunknac ßületése vtán, az
1 5 9 1.
Die 17 Martij.

Figure 10

IV. *Példa.* Van 400 forintos Szölöm. Termett benne 150 akó bor. Akarnám tudni, mennyiben van a' borom. Költöttem pedig.

1. Szölö-munkára mind öszve - *f.* 77, 59 *d.*
2. Dézmába fizettem - - 37, 56
3. Szüretelésre mindenestöl költöttem 24, 06
4. Szekeresnek a' haza hozásért - 19, 50
5. Mivel 400 forintra, törvény szerént, adtak vólna esztendeig 24 forint Interest, azt-is belé tudom. 24, 00

Summa - 179, 71

Ennyiben van azért a 150 akó bor. És ha ennyin el-nem adhatom, jobb vólt vólna ebben az Esztendöben azt a 400 forintot interesre adnom, ha az interest bé-fizették vólna.

Figure 11

V. *Példa.* Vettem hat hízó Diſznót 58 forinton 's egy máriás-áldomáson. Meg-ették annyi árpámat, a' mennyit el-adhattam vólna 17 forinton. Az árpa' daráltatásáért fizettem egyſzer-is máſzor-is, mind öſzve *f.* 4. *d.* 27. Mennyiben van a' hat Diſznó?

Le-írom így: 1. A' hat Diſznó' árra		*f.* 58, 34 *d.*
2. Árpa' árra	-	17, —
3. Daráltatásért fizett;		4, 27
Ennyiben van a hat Diſznó	*Summa*	*f.* 79, 61 *d.*

Figure 12

V. *Példa.* Vettem 30 forinton egy általag aſzſzú-ſzölö borát; mellyben vagyon 81 itſze ſzín bór. Mennyiben esett itſzéje? Itt a' 30 forintot nem lehet el-oſztanom a' 81 Itſzére; hanem pénzzé kell tennem. Már a' 3000 pénzből a' 81 Itſzének mindenikére eſik 37 pénz. Annakfelette az egéſz árrát megtóldottam 3 pénzzel. Akár így mondjam: A' 81 itſzének hárma esett 38 pénzen; a' többi mind 37-en

```
3000. R. 37.
 81 :
 243 :
 ----
 570
  81
 567
 ---
   3
```

Figure 13

Figure 14. S. Hatvani

APPENDIX.

OBSERVATIO ELEVATIONIS POLI DEBRECINENSIS,

INSTITVTA

a

STEPHANO HATVANIO,

M. D. Philoſ. & Matheſ. P. P.

Quoniam inter Mathematicos, nemo hoc vſque fuerat quod ſciam, qui latitudinem Ciuitatis noſtrae definiuiſſet: rem gratam me facturum credidi, cum popularibus meis, tum vero exteris, ſi in hanc rem inquirerem, obſeruationemque a me inſtitutam, cum iis communicarem. Saltem ſpero fore, vt nobilia illa ingenia, quae paſſim Hungaria noſtra alit, hoc exemplo ad tentamina huius generis ſuſcipienda incitentur. Cum enim quantum mihi experiri licuit, in horologiis ſolaribus conficiendis, Hungari noſtri, per totum fere qua late patet hoc Regnum, numero 48. graduum eleuationis poli vulgo vtantur: fieri nequit quin in hac re inſigniter fallantur, horologiaque tempori vero deſignando minime conuenientia

T 5 habeant.

(298)

habeant. Certum quidem eſt, eleuationem poli VIENNAE in Auſtria, eſſe aequalem 48. grad. & 13. minut. definiente Celebri Aſtronomo MARINONIO. (a.) Quis tamen erit tam lepido capite, vt eamdem latitudinem ſingulis regni huius vrbibus adſcribat: quarum tamen aliae ab aliis, a ſeptentrione in meridiem, pluſquam milliaribus 40. hungaricis remouentur?

Quamquamvero adparatu tali, qualem nobis Celeb. MARINONIVS, in *Aſtronomica ſua Specula Domeſtica* deſcripſit, (b.) pro filari meridiana conſtruenda, propter curtam, domi ſupellectilem vti non liceret: tamen in re tam nobili, ſed hocuſque neglecta, aliquid conari malui, quam nihil omnino agere. Tanto autem magis, quod Cl. ISACVM BROVKNERVM, in charta illa generali Globi Terreſtris, quam nuper edidit, (c.) ſed quam ſummus Mathematicus DANIEL BERNOVLLIVS adprobabat viderem, Hungariam noſtram parallelis includi, quae a gradu 45 latitudinis, vſque ad 50 excurrerent. Cupiebam proinde cognoſcere, quorſum latitudo Vrbis noſtrae referri deberet?

In hunc finem, vſus ſui gnomone, vltra ſeptem pedes Rhenanos alto. Pes Rhenanus, eſt mihi diuiſus in 10. pollices, & pollex quilibet in 200. partes aequales: quare totus pes rhinlandicus in 2000. partes aequales. Parieti igitur lapideo, firmiter infixi laminam horizontalem non valde craſſam, ſed tamen tres pollices cum dimidio latam: quae in medio pertuſa fuit foramine, quod ſuo diametro $\frac{1}{2000}$ vnius pedis aequabat.

Centrum

(299)

Centrum huius foraminis A, in figura hic adnexa, fungebatur vice verticis gnomonis: ex quo demiſſum filum, in planum horizontale ei ſubiectum, gnomonem ipſum exhibebat. Gnomon hic perpendiculariter horizonti imminens, indicatur linea obſcure ducta A B. Die igitur 21. Iunii, Anno 1757. in ipſa meridie, excepi lumen Solare lamina horizontali, quae dedit altitudinem gnomonis A B. ſeptem pedum 4 pollicum cum dimidio, & $\frac{21}{100}$ partes pollicis dimidii: ſeu 14921. lineas. Centrum vero orbiculi luminoſi, quod depingebatur ex S per A ad C, in plano horizontali, laminae ſubiecto, ſeu diſcus Solis; diſtabat a pede gnomonis B tribus pedibus, tribus pollicibus & $\frac{27}{100}$ partibus dimidii pollicis ſeu 6627. lineis; hinc igitur ſolis altitudo adparens per regulas Trigonometriae erui debet. In vſum tironum, en ſupputandi rationem adponam. Ex Principiis Trigonometriae notum eſt, pro inueniendo angulo C, ratio erit; vti C B ad B A: ita ſinus totus ad tangentem anguli C.

Quare

(300)

Quare iuxta Tabulas VLACCII erit:

CB = 6627. cuius Logar. eſt		= 3. 8213272
BA = 14921. cuius Logar. eſt (d.)		= 4. 1737980
Logar. autem ſinus totius eſt		= 10. 0000000

Ergo Logarithmus tangentis angulo C. oppoſiti erit (e.) = 10. 3524810

Huic autem numero, in Canone Tangentium VLACCII, reſpondent quam proxime 66 grad. & 3. minuta. Vel ſi pes vnus habeatur 100 pollex autem 10. linearum: poterit per numeros minores determinari.

CB = 332. cuius Logarithm.		= 2. 5211381
BA = 747. cuius Logarithm.		= 2. 8733206
Logarithmus ſinus totius vero		= 10. 0000000

Ergo Logarithm. tangentis anguli C = 10. 3521825 cui in Tabulis reſpondet rurſus quam proxime 66° 3' vti prius.

Quoniam vero declinatio Eclipticae, determinante Celebri Aſtronomo CASSINO, (f.) eſt = 23° 28': ſed etiam declinatio Solis borealis hoc eodem die Iunii eſt eadem; locus autem Solis erat, in ſigno Cancri grad. 0. min. 22: hinc ſi ab altitudine Solis antea reperta, ſubtrahatur declinatio Solis borealis; tum eleuatio aequatoris, erit aequalis 42 grad. & 35 min. Quare ſubtrahendo altitudinem aequatoris a 90° ſeu 89° 60' erit ALTITVDO POLI DEBRECINENSIS aequalis 47. grad. & 25. min.

Notum

Figure 15

286 CAPVT III.

infantibus, tres ex Epilepſia deceſſiſſe; quartum vero morbo alio. Anno 1751 Infantes intra annum mortui fuere 304. ex his in Epilepſia 210. obiere: Ergo $\frac{210}{304}=\frac{105}{152}=\frac{21}{30}=$ fere $\frac{4}{6}$ ſeu $\frac{2}{3}$. Anno 1752. Annotini mortui ſunt 260. quorum 214. deceſſere ex Epilepſia: Ergo $\frac{214}{260}$ fere $\frac{4}{5}$. Anno 1753, ex 312. mortui ſunt ex Epilepſia 236. Ergo ratio erit $\frac{236}{312}=\frac{118}{156}=\frac{59}{78}$ vel quod proximum eſt $\frac{60}{80}=\frac{6}{8}=\frac{3}{4}$ Anno 1754. ex 250. mortui ſunt Epileptici 210. Ergo $\frac{210}{250}=\frac{21}{25}$ fere $\frac{4}{5}$. Tenendum vero eſt Annis 1752. 1754. Variolas & Morbillos, graſſatas epidemice non fuiſſe: vti reliquis annis, in quibus etiam multi semeſtres his morbis ſublati decedebant. Hinc liquet, inter Annotinos apud nos decedentes, hunc vel illum morbo Epileptico obiiſſe, probabilitas eſt, cum variolae graſſantur $\frac{3}{4}$ morbo alio autem $\frac{1}{4}$ cum variolae non graſſantur autem $\frac{4}{5}$, morbo alio autem $\frac{1}{5}$. &c.

SCHOLION.

Cur tam ingens numerus infantum apud nos moriatur, non ſine graui cauſſa quaerere quis poſſet; cum primis autem, cur tanti Epilepſiâ, e medio tollantur? Iuxta Tabulas enim Halley ex numero viuen-

Figure 16

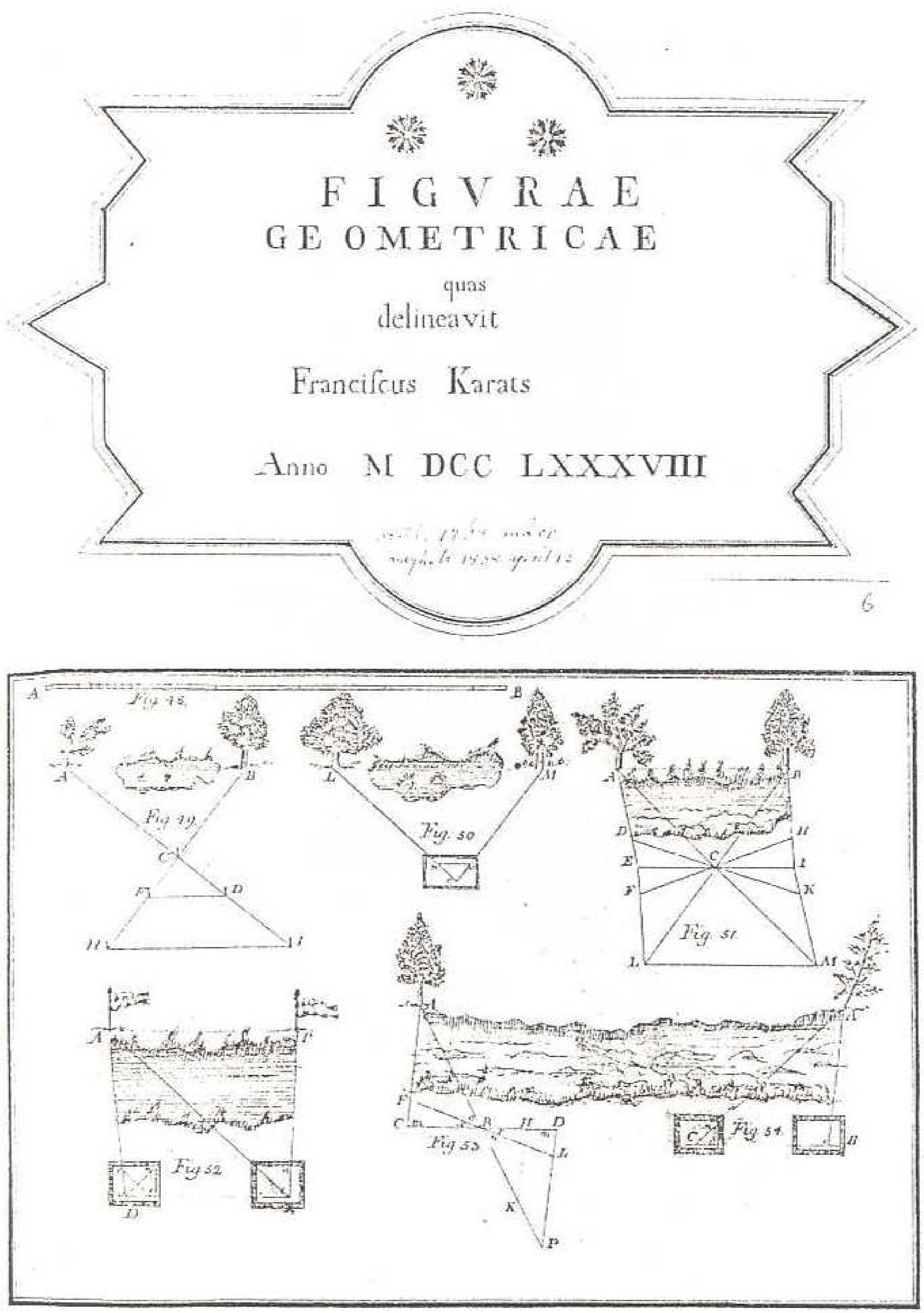

FIGVRAE
GEOMETRICAE
quas
delineavit
Franciſcus Karats
Anno M DCC LXXXVIII
Fig. 49.
Fig. 50
Fig. 51.
Fig. 52
Fig. 53
Fig. 54.

Figure 17a

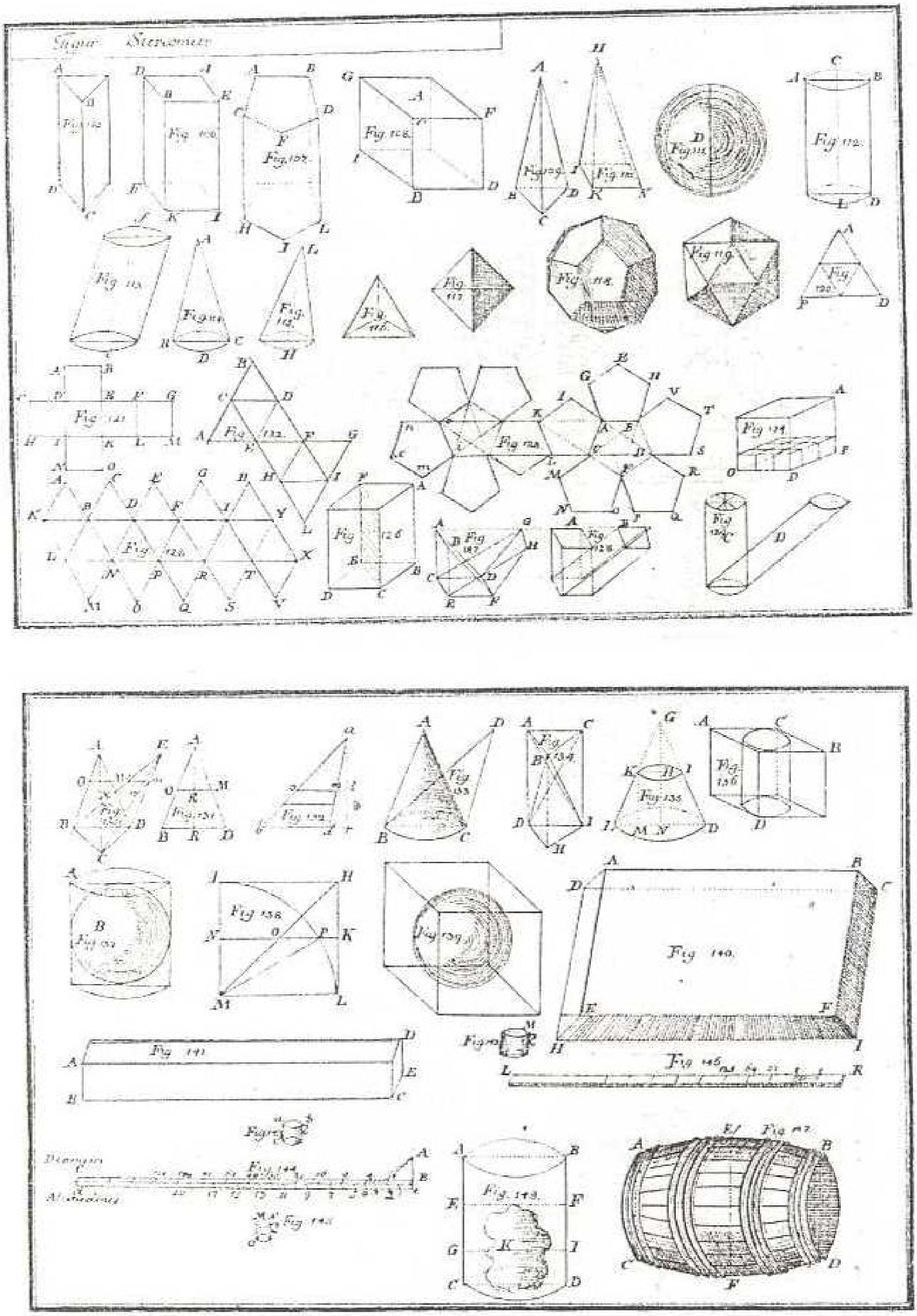


Figure 17b

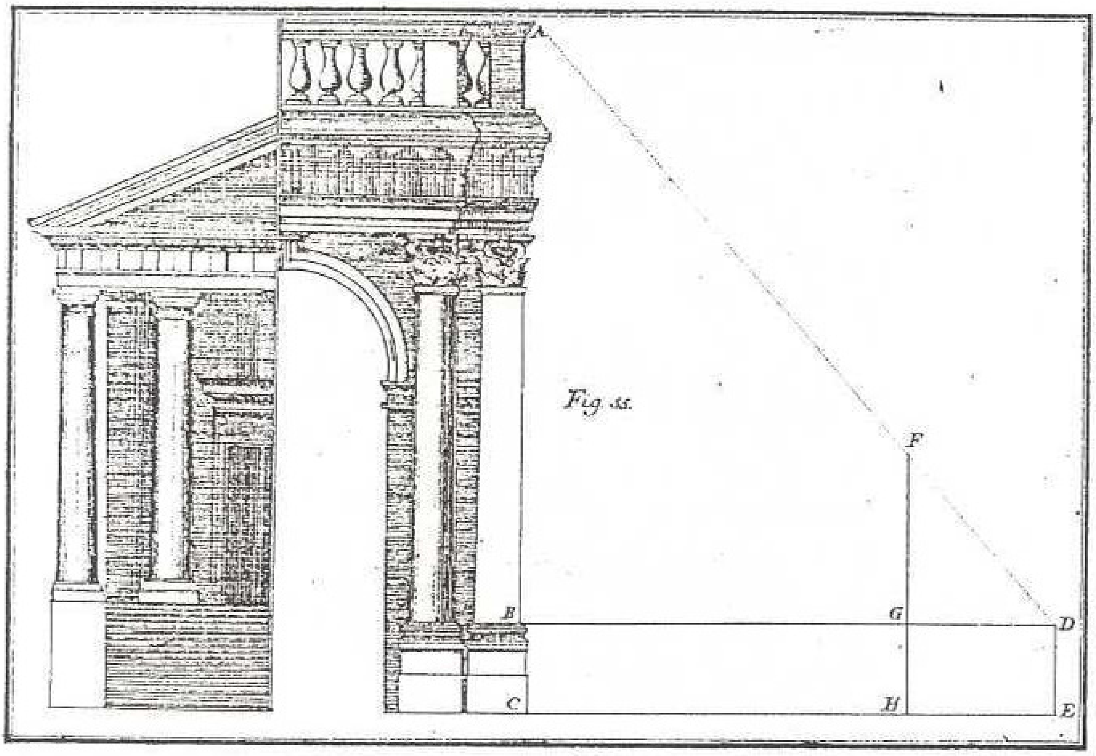


Fig. 55.
A
B
C
D
E
F
G
H

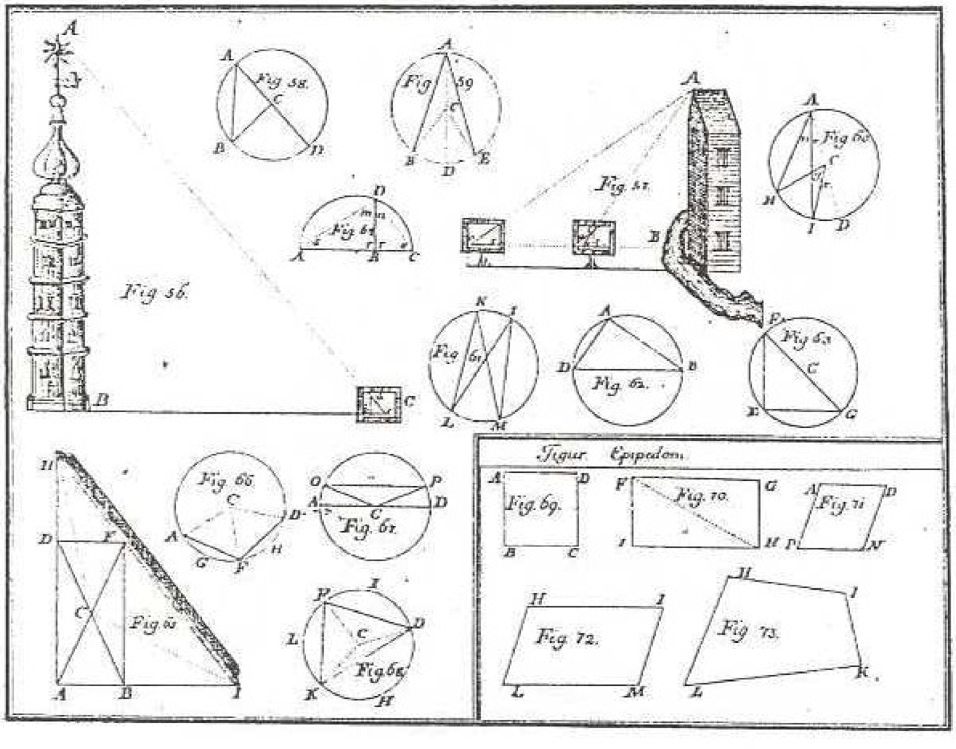


Fig. 56.
Fig. 58.
Fig. 59
Fig. 62.
Fig. 66.
Fig. 69.
Fig. 70.
Fig. 72.
Fig. 73.
Figur. Epipedon.

Figure 17c

Figure 18. J. A. Segner

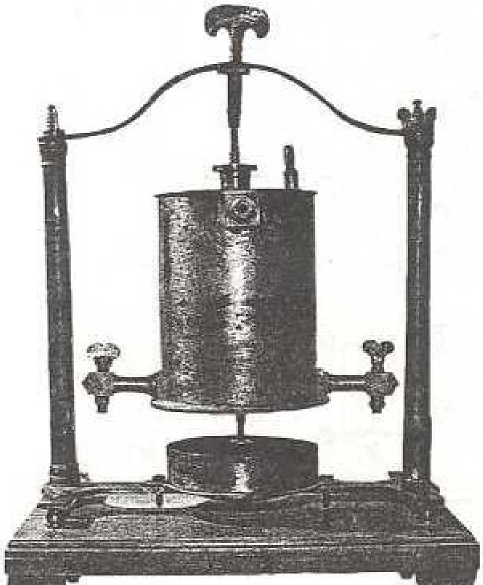


Figure 19

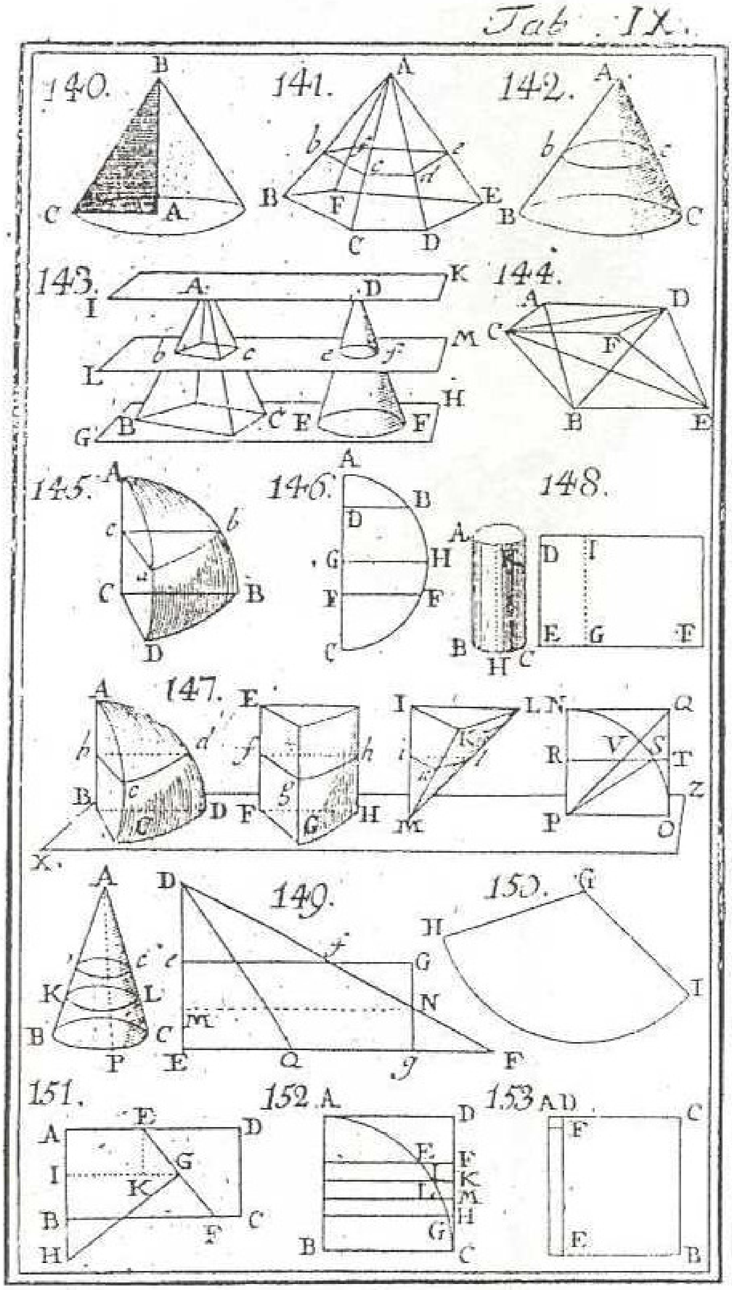


Figure 20

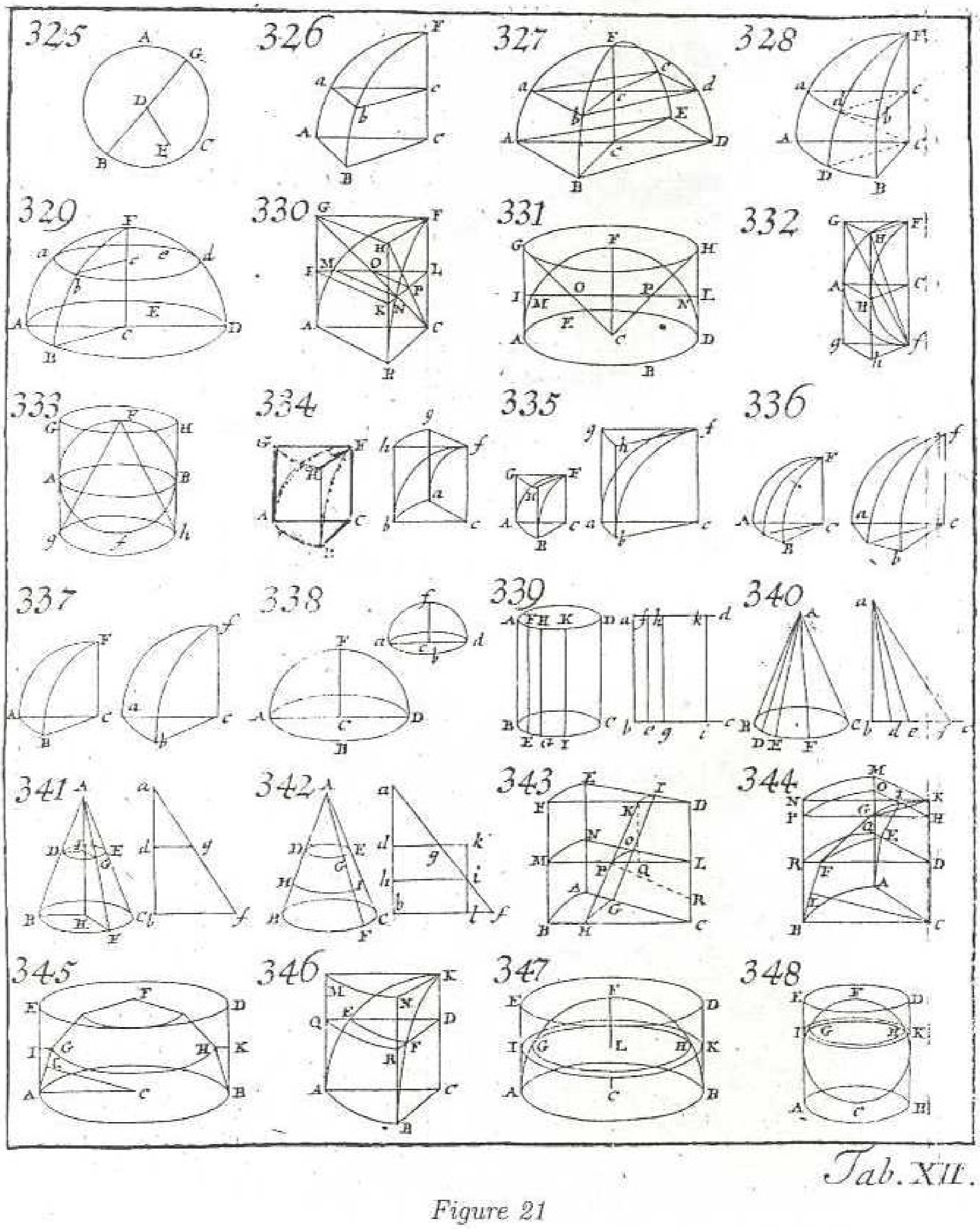


Figure 21

Figure 22

Figure 23

Gründlicher und ausführlicher

Unterricht

zur

praktischen Geometrie

von

Johann Tobias Mayer,

Königl. Grosbritt. Hofrath und Professor zu Göttingen.

Dritte verbesserte und vermehrte Auflage.

Erster Theil,

mit sieben Kupfertafeln.

Göttingen,

im Verlage bey Vandenhoek und Ruprecht.

1802.

Figure 24

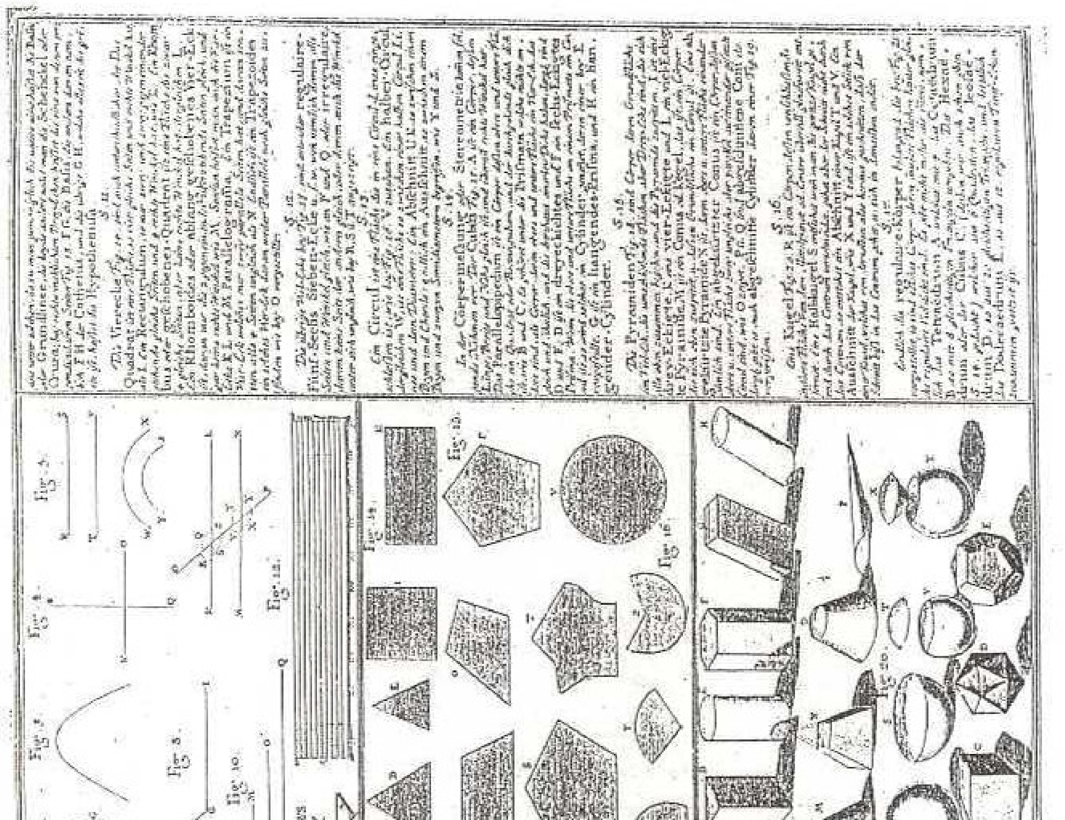

TAB. IV.
in welcher
Die in der Geometrie vorkommende Nahmen der Linien, Winckeln, Flächen und Cörper erkläret werden

Figure 25a

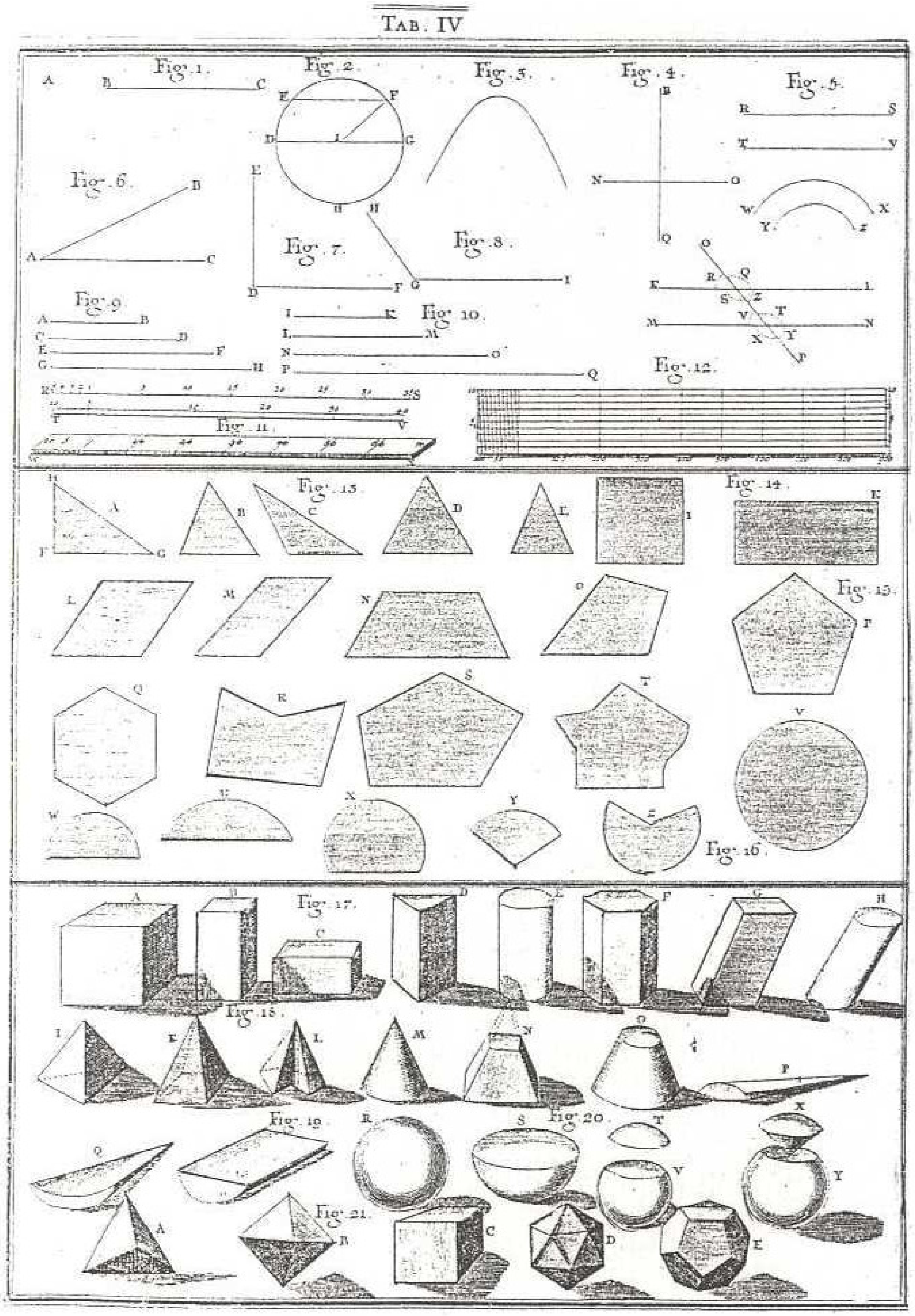

TAB. IV
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.

Figure 25b

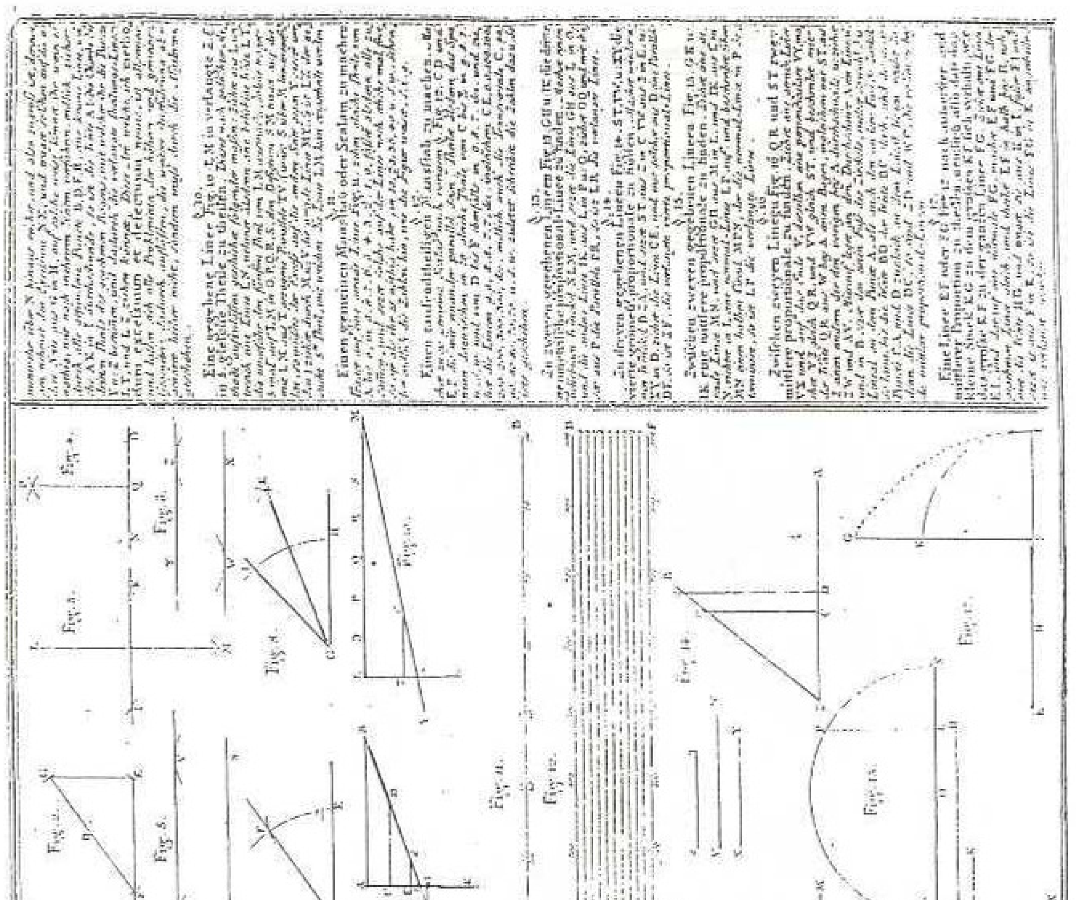


Figure 26

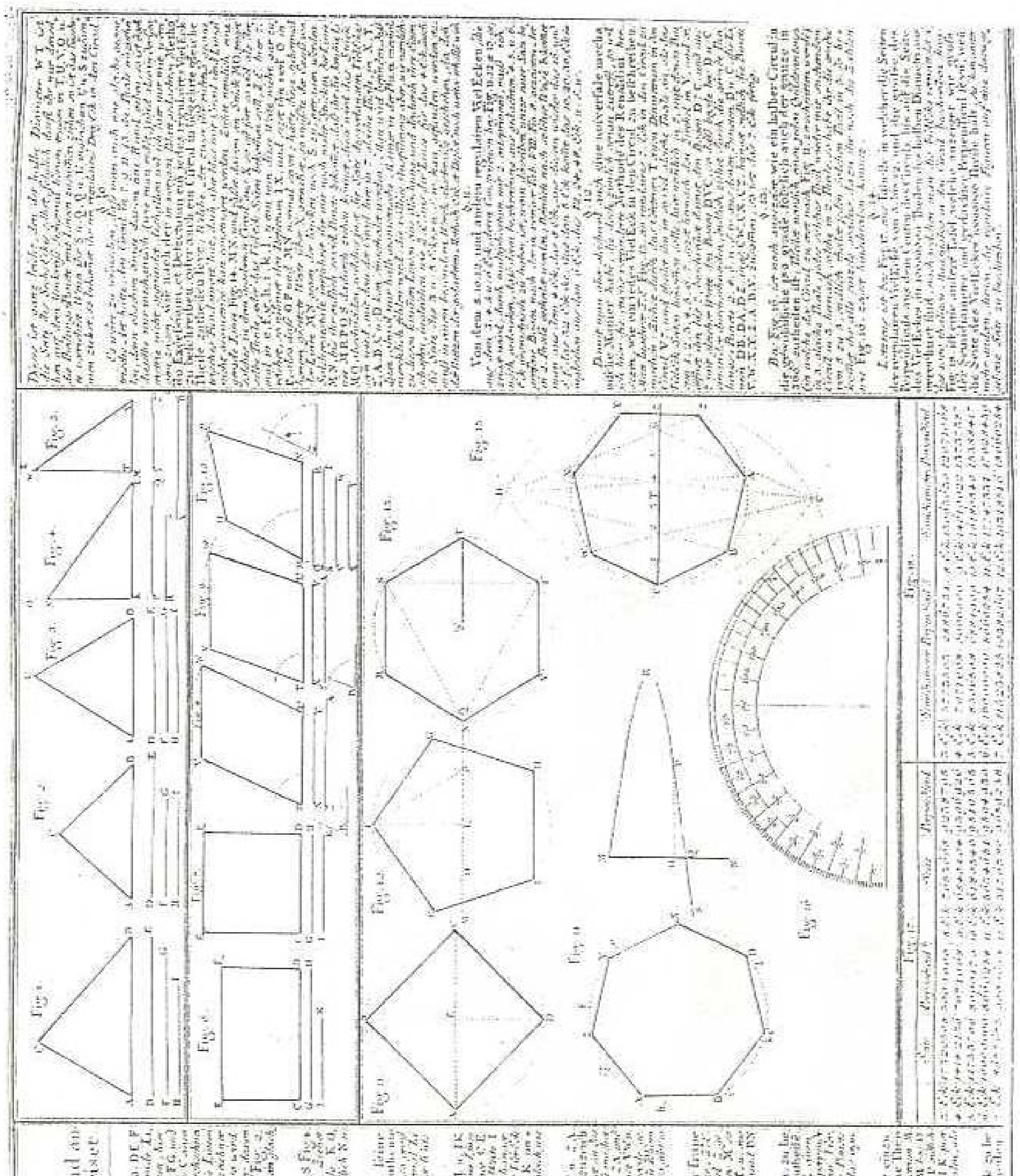


Figure 27

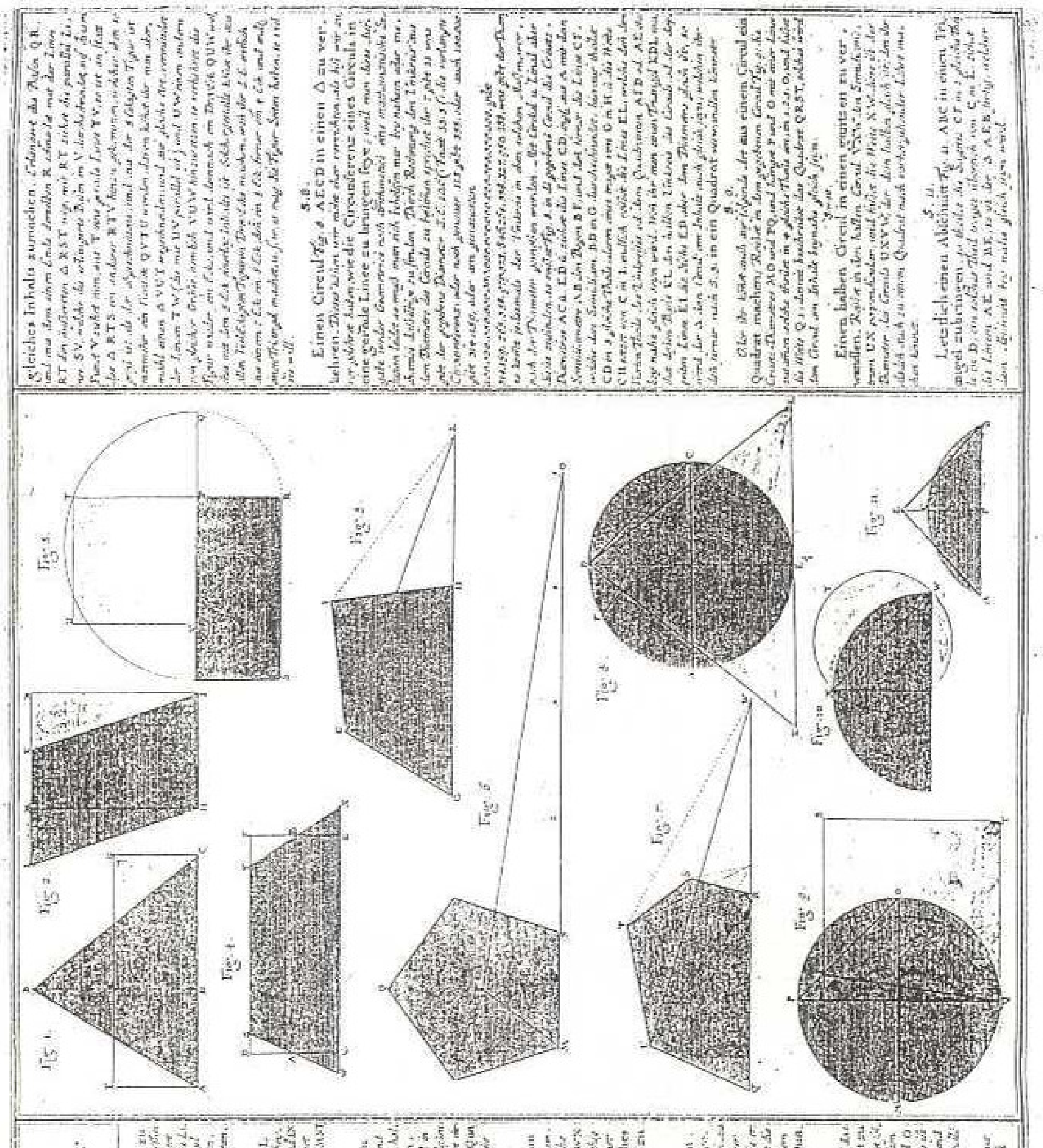


Figure 28a

Table VII.

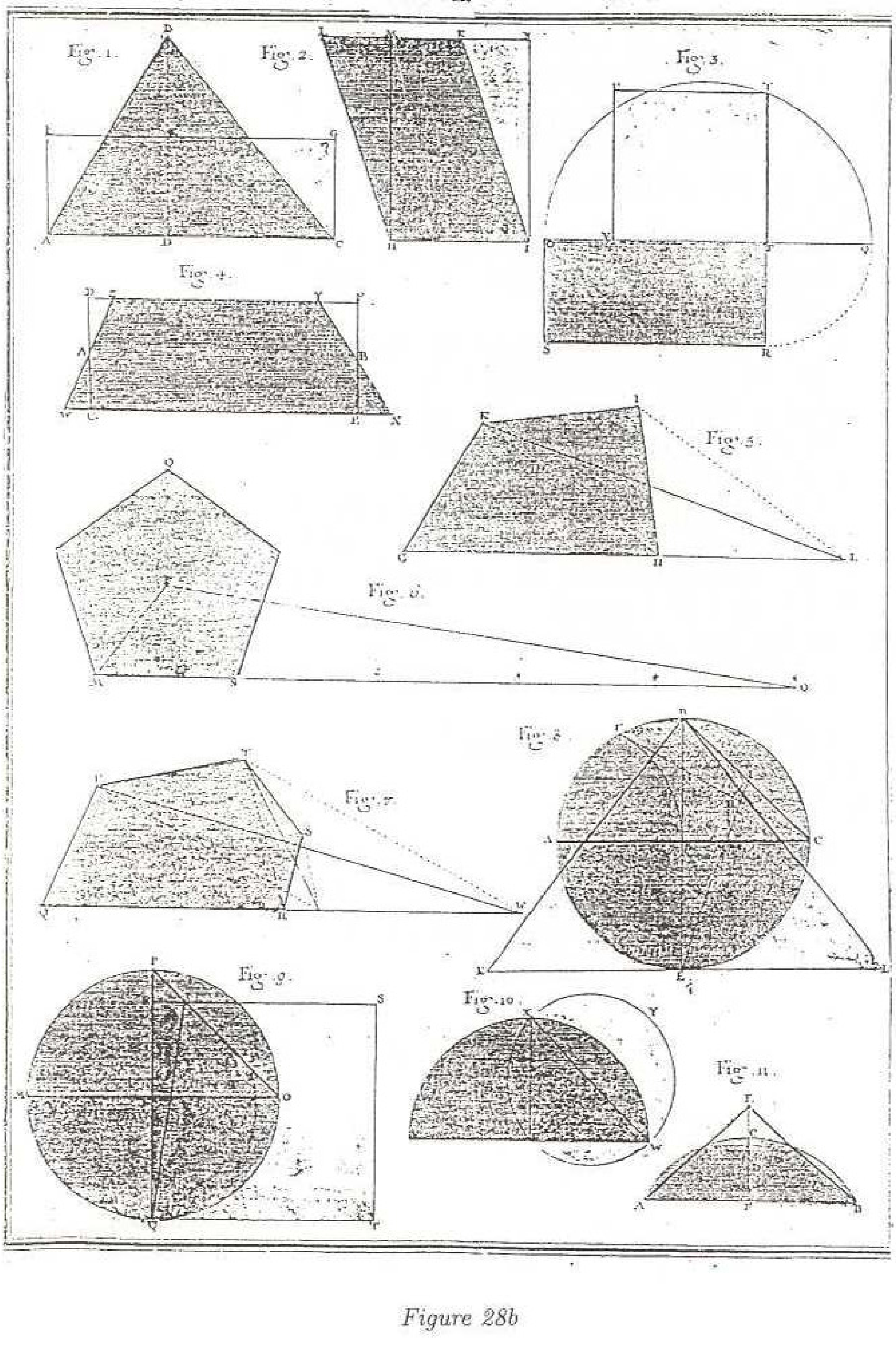


Figure 28b

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