

# Conversion between different symbolic representations of rational numbers among 9th-grade students

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*Abstract.* Our research involved nearly 800 ninth-grade secondary school students (aged 14-15) during the first weeks of the 2023/2024 school year. Less than 40% of students solved the text problems related to common fractions and percentages correctly. In terms of student solutions, pupils showed a higher success rate when the text of the problem contained common fractions, and the solution had to be given as a percentage. In this case, the success rate of switching between different symbolic representations of rational numbers (common fraction, percentage) was also higher. Observation of the methods used to solve also suggests that the majority of students are not flexible enough when it comes to switching between different representations.

*Key words and phrases:* fractions, percentage calculation, solution strategies, proportion problems, Hungarian mathematics education.

*MSC Subject Classification:* 97F80, 97D70.

## Introduction

In the first weeks of the 2023/2024 school year, 794 ninth-grade (14-15 years old) students participated in the survey created by us. The assessment prepared for the study contained seven open-ended, mostly word problems from the topics of proportionality, fractions, and percentage calculations. The exercises included word problems involving fractions, percentages, and proportionality. In the students' answers to the individual problems, we examined the success of solving

the exercises, and the solution strategies used, too. In the case of exercises related to rational numbers, we also studied the frequency and effectiveness of the conversion between different representations (common fraction, percentage).

Our goal was to map the existing knowledge of students having just entered secondary school regarding the mentioned topics. Evaluating the worksheets provided a fresh perspective on the strategies students employed, their proportional thinking development, and their current proficiency in the subject. We studied more than just the solutions to proportion problems among ninth-graders. Seventh-grade students solved similar sets of exercises in a classroom experiment implemented by us (Torma & Kosztolányi, 2025).

In this study, we will highlight only two exercises of the worksheets prepared for ninth-graders. When solving these, the students had to solve word problems related to common fractions and percentages, and in their answers convert from the given form to a different one.

## Theoretical framework

Most math teachers probably agree with the experience that most students have difficulty with topics related to fractions and calculations related to rational numbers. This is even more of a problem, because this knowledge is needed when discussing many other topics and solving most classroom exercises in mathematics. We must rely on a stable knowledge of operations with fractions, for example, when discussing proportionality and percentages, combinatorics and probability calculation, as well as the topic of equations. Their significance is especially remarking from the point of view that, according to research, the understanding of fractions is related to general mathematical performance, and the understanding of the size of fractions also plays a role in the development of the number concept (Torbeyns et al., 2015).

Another challenge for students in a problem-solving process is to find the right way of representation. Many educators, psychologists and researchers have defined and explained different aspects of representations in relation to the teaching and learning of mathematics. According to Bruner's theory, knowledge can be represented in three different ways (enactive-material plane, iconic plane, symbolic plane), which are developed in the individual in a sequential way. Lesh, Post and Behr have developed five categories: static pictures, written symbols,

manipulative models, real scripts, spoken language. They suggest that translation between representations and transfer within representations are important processes for effective learning of mathematics (Mainali, 2021).

The reason for the difficulties experienced can also be found in the fact that multiple interpretations of fractions appear in public education. According to Novita et al. (2022), five areas of representations of rational numbers can be distinguished:

- part-whole;
- quotient;
- ratio;
- measurement;
- operation.

Knowledge of representations is therefore essential, because teachers must be aware of how each area lays the foundation for and helps the understanding and learning of students in this topic.

Students encounter several different symbolic representational forms of rational numbers (common fractions, decimal fractions, percentages) during their studies. When introducing these, teachers may have several questions. One is the order in which they should be taught, and the other is with what kind of approach, and with the help of what kind of presentations and illustrations they should be taught.

To answer the first question, it is important to consider three aspects (Tian & Siegler, 2018):

- Which of the three representations is the easiest to learn?
- By using which one can we avoid possible misunderstandings the easiest way?
- Which is the easiest way to transfer the necessary knowledge?

Each representation has a rich illustration toolbox. Their application is also particularly important because, if instead of them, only the calculation approach is prioritised, the application of algorithms can hide the relationships between the distinct representations. Thus, instead of the possibilities becoming available using multiple representations, the teaching-learning process of rational numbers degrades into a series of monotonous and meaningless steps (Chick & Baratta, 2011).

It is even more important to put more emphasis on understanding and concept building because, otherwise, students will focus more on the form and symbols. In this way, the meaning of the fraction symbol narrows down, and it will be more

difficult to use in other contexts. This is particularly significant when discussing decimal fractions, since in that case not only the part-whole relationship discussed in connection with fractions appears, but also the concept of place value used in the case of whole numbers. However, without proper knowledge of the mentioned knowledge elements, students will not be able to establish a connection between the different forms of representations (Hiebert, 1984).

Regarding the various representations, an additional difficulty for students is to compare the magnitudes (Siegler et al., 2012). The use of reference points (such as  $1/2$  or 50%) can be an effective solution for bridging this problem (Clarke & Roche, 2009; Gay & Aichele, 1997; McIntosh et al., 1992). The correct interpretation of these pieces of information is successfully developed by the end of primary school age. According to the research results of Lembke and Reys (1994), 7th-, 9th- and 11th-graders did not have problems recognising the equivalence between half and 50%, one quarter and 25%, and three quarters and 75%. With the help of these values, students will be able to perform further approximations and checks.

However, these relations are not necessarily sufficient to achieve a flexible conversion between the various symbolic representations of rational numbers. According to the results of Lemonidis and Pilianidis (2020), the ability of conversion among the examined eighth-grade students is very low. The lack of success is particularly significant if the exercise includes percentages, too: barely a quarter of the students were able to convert percentages into fractions, and only 3.2% of the students succeeded in converting fractions into percentages.

## Methodology and data collections

794 ninth-grade students took part in the survey conducted in the first weeks of the 2023/2024 school year. Our goal was to use the assessment we created to map the current knowledge level of the students and the strategies used for solving the exercises. The seven open-ended, mostly word problems came from the topics of proportionality, common fractions, and percentage calculation. On the worksheets, the students mostly had to solve proportion problems, and also, at the end of them, word problems related to common fractions and percentages. In the latter exercises, we monitored the students' success, the scheme used for solving the exercises, and the realisation of the conversion between the various representations of rational numbers, too (or the lack thereof). The students had 45 minutes to solve the set of exercises.

We chose the beginning of the school year as the date of the survey so that we could examine the knowledge and problem-solving ability of the students accumulated in the first eight years of school. It is in middle school (grades 5–8) when students are introduced to fractions, and then they learn the operations related to them and their properties. According to the framework curriculum regarding the mathematical progress of middle school students, the development goal is that they become able to simplify and expand fractions, and to compare equal or different fractions. By the end of the eighth grade, students should know the concept of percentages and be able to solve simple percentage calculation problems (Education Office, n.d.). Due to the timing of the survey (it was not directly after the practice of the topic), the results may also provide insights into what knowledge elements can be mobilised after the summer break.

The first five exercises on the worksheet were as follows:

**Exercise 1.** *In a store, 6 bottles of the same mineral water cost a total of 720 HUF. How much will we have to pay in this store if we buy 9 bottles of this mineral water?*

**Exercise 2.** *Grandma made peach jam. 14 jars of 4 decilitres were filled. How many jars would she have needed if she had bought 7 decilitre jars?*

**Exercise 3.** *What kind of graph do you get if you connect the points corresponding to the related value pairs? Plot the points and the corresponding graph in the given coordinate system!*

$x$	0	1	2	3	8
$y$	0	1,5	3	4,5	12

Figure 1. Table for Exercise 3 of the worksheet

**Exercise 4.** *Fill in the missing cells in the table based on the graph. The domain of interpretation is the set of real numbers.*

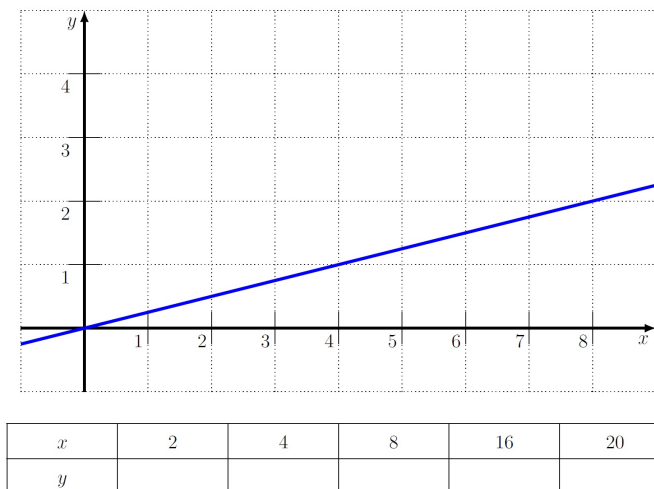


Figure 2. Coordinate system and table for Exercise 4 of the worksheet

**Exercise 5.** *Bronze is an alloy of copper. The most commonly used alloy is the so-called tin bronze, which is made using copper and tin. How much copper and tin are needed to make 20 kg of bronze if the ratio of tin to copper in bronze is 1:7?*

Exercises focusing on operations related to common fractions and percentages, and conversion between these two representations of rational numbers were placed at the end of the worksheet (Exercises 6 and 7). The texts of the exercises were as follows:

**Exercise 6.** *A beginner cycling team went around Lake Balaton in the summer. They covered 25% of the total distance (about 200 kilometres) on the first day, 35% on the second day, and 20% on the third day. What fraction of the total distance remained for the fourth day?*

**Exercise 7.** *4 children ate from a large cake. Peti ate  $\frac{1}{5}$  part, Judit  $\frac{1}{10}$  part, Anna  $\frac{1}{4}$  part, Zoli  $\frac{2}{5}$  part of it. What percentage of the cake is left?*

Eleven Hungarian secondary schools participated in the survey. A significant part of the 794 students writing the assignment (644 students) completed the

previous two grades in elementary schools, the rest studied in eight- and six-year grammar schools. Regarding their current school type, nearly three-quarters of the students attend four-, six- or eight-grade grammar schools, but students studying in vocational grammar schools, technical schools, and vocational training schools also took part in the survey (Figure 3).

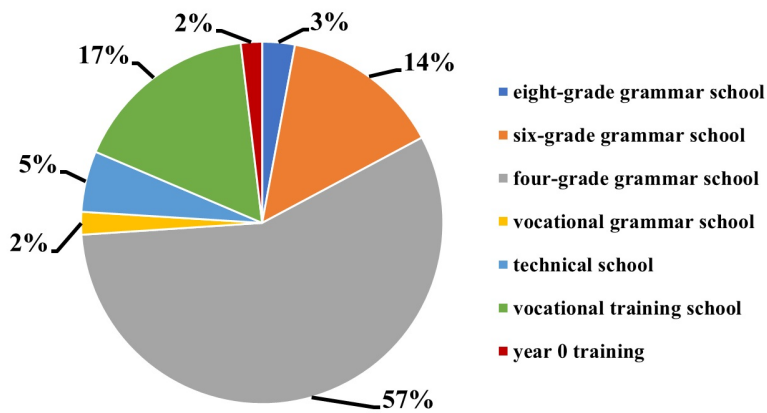


Figure 3. Distribution of school types

The location and type of schools participating in the survey and the number of students completing the worksheet are summarised in the Table 1 (the categories are based on the national ranking of secondary schools) (Balla, 2024).

## Research questions

Considering the theoretical background presented earlier, our research aimed to answer the following research questions:

- How flexible are students in switching between different symbolic representations of rational numbers?
- In the context of converting between common fractions and percentage forms, in which direction do ninth-grade students demonstrate greater success?
- Are there observable differences in the performance of students from different types of schools?

Location and type of school	Number of students
strong four-grade grammar school in a big city	59
middle-strong six-grade grammar school in a big city	64
middle-strong four-grade grammar school in a big city	156
four-grade grammar school in a big city	92
middle-strong vocational grammar school in a big city	17
middle-strong year 0 training in a big city	15
middle-strong six-grade grammar school in a small town	33
middle-strong four-grade grammar school in a small town	39
eight-grade grammar school in a small town	23
six-grade grammar school in a small town	19
four-grade grammar school in a small town	103
technical school in a small town	21
vocational training school in a small town	110
technical school in a large village	21
vocational training school in a large village	22

*Table 1.* Types of schools participating in the survey with the number of pupils

## Results and discussions

### Experiences related to Exercise 6

In Exercise 6, the students encountered the data of the word problem in the form of percentages, but in the answer, we expected the result as a common fraction. The solution was considered correct if not only the numerical value but also the representation of the rational number was appropriate. Only nearly a third of the students solved the exercise correctly, and 13% of the students left the exercise blank and did not even begin to solve it.



Comparing the success rate with the previous school type of the students, we find that the students having completed the 7th and 8th grade in eight- or six-class grammar schools solved the exercise correctly in higher proportions (42% and 36%) compared to their peers graduating from elementary schools (30%). In addition to all this, it was less typical for students attending grammar schools not to solve the exercise. These outstanding results can be explained, among other things, by the fact that most of these institutions are schools that nurture mathematical talent. The proportion of incorrect solutions was similar for all three school types: 54% of elementary school graduates, 58% of eight-grade grammar school graduates, and 59% of six-grade grammar school graduates made mistakes during the solution.

Comparing the success rate with the current school type of the students, a higher proportion of students in grammar schools or vocational grammar schools solved the exercise correctly. In the case of students in technical schools or vocational training schools, this proportion does not reach 15%. In line with this, these students have the highest proportion of incorrectly solved or blank exercises (Table 2).

	Eight-grade grammar school	Six-grade grammar school	Four-grade grammar school	Vocational grammar school	Technical school	Vocational training school	Year 0 training
Blank	0%	4%	8%	6%	7%	46%	0%
Incorrect	48%	60%	52%	63%	81%	51%	60%
Correct	52%	36%	40%	31%	12%	3%	40%
Average success rate	79%	65%	58%	53%	47%	30%	63%

*Table 2.* Distribution of success seen in Exercise 6 by school type (Average success rate: average of the percentage of correct solutions regarding Exercises 1–7)

The data show that the percentage of correct solutions for this task was lower than the average success rate for each exercise on the worksheet for all types of schools. One reason for this may be that students have difficulty in interpreting the text and constructing a mathematical model for the problem. The result also

reflects students' difficulty in understanding the relationships between percentages and common fractions.

The picture of the effective solution of the exercise can be further nuanced if we observe the solution paths and schemes of the students. Most of them basically chose two types of strategies for the solution. The first possible direction (on the left side of Figure 4): they have determined how much of the distance the group completed in the first three days by adding up the given percentages. The second possibility (on the right side of Figure 4) is that, knowing the total distance, they calculated the distances travelled on each day using percentages. By these basic ideas, we identified several possible paths of the solutions, which are shown in Figure 4. On the first-mentioned branch, 156 of the 392 students, while on the second branch, 72 of the 214 completed the exercise correctly. In addition to these discussed possibilities, there was also a strategy where the student calculated with common fractions from the beginning of the solution, so the conversion between representations in the case of several fractions appeared already at the beginning of the calculation. Eleven students chose this path, and nine of them solved the exercise correctly. Additional cases not shown in the figure, appeared in the case of several students: error in understanding the text, guessing without calculations, and illogical, aimless calculation. In most cases, these did not lead to the correct result.

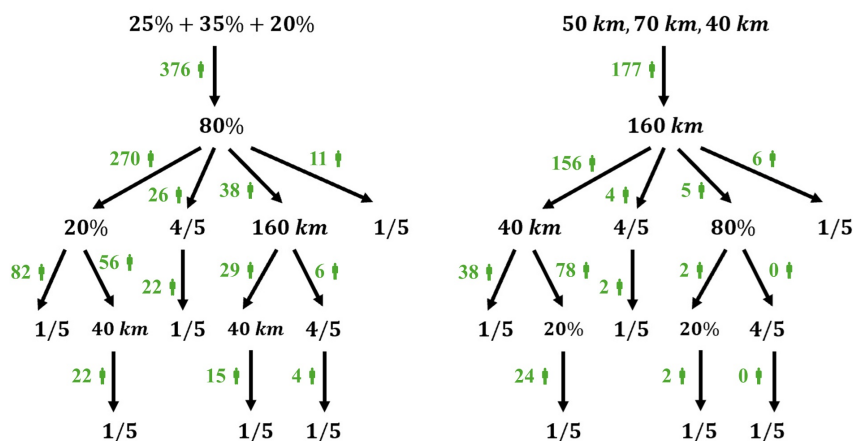


Figure 4. The most frequently used steps followed during the solution of Exercise 6

Evaluating the realisation of the expected conversion between representations (conversion from percentage form to fractional form) during the solution of the exercise, we established three categories: correct conversion, incorrect attempt, and no information about it. 36% of the students performed a correct conversion, 10% attempted it with incorrect results, and 54% did not try it at all. Like what was seen in the categories showing the success of the solutions, students having already studied in grammar schools in the 7th and 8th grades performed better in terms of the conversion between representations (34% of the elementary school students, 42% of the eight-grade grammar school students, 40% of the six-grade grammar school students did it correctly). Table 3 shows the distribution of the current school types. The rate of successful conversions only reaches 50% in the case of students at eight-grade grammar schools, and the lowest rate is experienced in the case of vocational training school students.

	Eight-grade grammar school	Six-grade grammar school	Four-grade grammar school	Vocational grammar school	Technical school	Vocational training school	Year 0 training
No infor- mation	35%	46%	44%	50%	77%	92%	47%
Incorrect attempt	13%	14%	11%	19%	7%	4%	13%
Correct conversion	52%	40%	45%	31%	16%	4%	40%

*Table 3.* Distribution of the success of conversion between representations by school type in the case of Exercise 6

Our success rates are on average better than the results of Lemonidis and Pilianidis (2020). However, it is obvious that learners do not move between the two symbolic representations with sufficient flexibility. When observing the students' solution schemes, it is noticeable that the specific numerical data provided (the length of the tour) also influenced the timing and success of the switching. These results suggest that it would be particularly important to allow sufficient time to learn about the relationships in the lower grades. In addition, it would be useful to plan the learning process by using a wide variety of exercises, visual models and drawings during the learning of procedural skills.

### Experiences related to Exercise 7

In Exercise 7, the students encountered the data of the word problem in fractional form, and they had to give the result in percentage form in the answer. Several students solved this exercise correctly (38%). Prominently, compared to Exercise 6, nearly twice as many students did not write anything for this one (this may also be because we are talking about the last exercise of the worksheet).

Comparing the success rate with the previous school type of the students, we find that the students having completed the 7th and 8th grade in eight- or six-grade grammar schools worked correctly in this very exercise, too, in higher proportions (65% and 61%) than their peers having graduated from elementary schools (32%). Missing the exercise was the least typical of students at six-grade grammar schools, while in the case of students in elementary schools, this proportion was the same as the ratio of those answering correctly. The proportion of incorrect solutions regarding the three school types was as follows: 36% of those graduating from elementary schools, 16% of those graduating from eight-grade grammar schools, and 32% of those graduating from six-grade grammar schools made a mistake during the solution.

Comparing the success rate with the current school type of students, grammar school students solved the exercise correctly in a significantly higher proportion. This proportion does not reach 25% in the case of students in vocational grammar schools, technical schools or vocational training schools (Table 4).

	Eight-grade grammar school	Six-grade grammar school	Four-grade grammar school	Vocational grammar school	Technical school	Vocational training school	Year 0 training
Blank	17%	9%	22%	25%	35%	67%	7%
Incorrect	13%	29%	36%	56%	42%	31%	60%
Correct	70%	62%	42%	19%	23%	2%	33%
Average success rate	79%	65%	58%	53%	47%	30%	63%

*Table 4.* Distribution of the success rate experienced in Exercise 7 by school type (Average success rate: average of the percentage of correct solutions regarding Exercises 1–7)

Examining the strategies used by the students, we can basically distinguish two ways. The first possible direction (on the left side of Figure 5) is that the students determined how much of the cake was consumed by adding up the given fractions, and then at the end of the solution, they calculated from this what percentage of the cake was left. So, in this case, the conversion between representations appeared only as the last step of the solution. The second option (on the right side of Figure 5) is that the students wrote down the fractions in the form of percentages at the beginning of the solution process and continued to calculate using these values. 232 of the 405 students on the first-mentioned branch, while 60 of the 66 students on the second branch completed the exercise correctly. Of the two schemes, the one outlined first was more sympathetic to the students, while the proportion of successful solutions was much higher in the case of the second scheme. This is mostly explained by the calculation errors seen in the operations performed with fractions, and the failure experienced when having a conversion between different representations. In addition to the options discussed above, the most frequent response was the one without any calculation. 53 students attempted to do this, of which only 4 gave the correct answer.

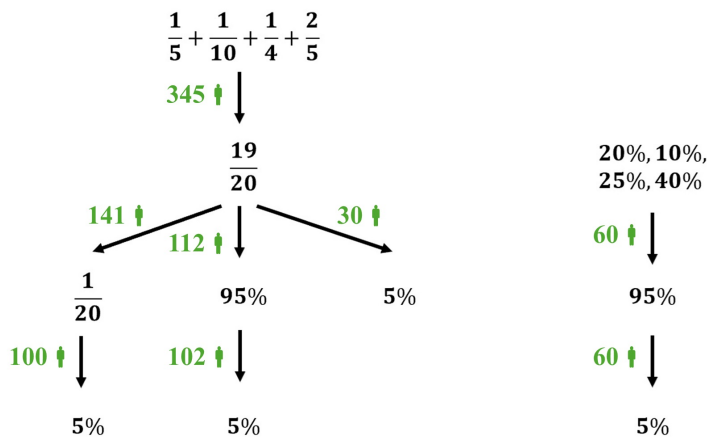


Figure 5. The most frequently used steps followed during the solution of Exercise 7

When evaluating the implementation of the conversion between representations expected during the solution of the exercise (conversion from common

fractions to percentages), we established the three categories presented in the previous exercise, as well. 43% of the students performed a correct conversion, 15% attempted it with incorrect results, and 41% did not deal with it at all. Like what we have seen so far, students having already attended grammar school in the 7th and 8th grades performed significantly better in terms of the conversion between representations, too (38% of elementary school students, 71% of eight-grade grammar school students, 68% of six-grade grammar school students did the conversion correctly). Regarding the current schools, the previously mentioned differences can be observed also in this exercise. In this case, as well, grammar school students performed better than their peers studying in vocational grammar schools, technical schools and vocational training schools (Table 5).

	Eight-grade grammar school	Six-grade grammar school	Four-grade grammar school	Vocational grammar school	Technical school	Vocational training school	Year 0 training
No infor- mation	18%	18%	35%	44%	58%	82%	27%
Incorrect attempt	4%	15%	15%	25%	16%	16%	27%
Correct conversion	78%	67%	50%	31%	26%	2%	46%

*Table 5.* Distribution of the success rate of the conversion of representations by school type in the case of Exercise 7

It was unexpected for us that (in contrast to the results of Lemonidis and Piliandis (2020)) students in this case switched between symbolic representations with a higher average success rate than in the opposite direction. The result may be strongly influenced by the denominator of the common fractions in the exercise. It would be useful in a future study to investigate how students' performance depends on the choice of common fractions in the text of the problem.

## Summary, implications

794 students starting in the ninth grade participated in the research we conducted. For the exercises focusing on operations related to common fractions and

percentages, we examined the success rate, the conversion between representational forms and the proportion of visual aids that may have been used.

31% and 38% of the participating students were able to solve the exercises correctly. Among the students, those having attended eight- or six-grade grammar schools already in grades 7–8 and currently attending eight-, six-, or four-grade grammar schools performed better. Since the survey was conducted in the first week of the school year, the effect of the current school can be considered insignificant, except when the given student already studied in grammar school in the previous two years. Based on these, the survey in this form confirms that the secondary school admission system further increases inequality and strengthens the selection experienced in Hungarian schools (Berényi, 2019).

Regarding the conversion between different representations of rational numbers (common fraction, percentage), we found that less than half of the students have been able to successfully convert from one form to another. The conversion could be considered more successful when it was about converting from a common fraction to a percentage. However, all in all, we state that the participating students cannot be considered flexible in terms of the conversion between the various representations of rational numbers. It is significant to point out that, when solving the exercises, in addition to the difficulties in understanding the texts, most of the students have problems performing operations with fractions correctly, even at secondary school age. Great emphasis must be placed on this even during secondary school education, too, so that gaps can be made up and mistakes can be corrected.

To understand the problem in more depth and to reduce these gaps, it would be worthwhile to develop a case study to provide information on whether the topic can be more easily understood and processed by using visual models, technological tools, financial or economic projects.

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