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Sage and scribe – asymmetrical pair work that can easily fit into any mathematics lesson, yet still have cooperative benefits

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Abstract. This article uses a case study experiment to learn the characteristics of a pair work, called the sage and scribe method (Kagan, 2008). We also wished to explore the positive and negative effects of the systematic application of this single cooperative element without any other structural changes during the lessons. In the case study experiment, we asked two teachers, accustomed to traditional frontal teaching methods, to substitute individual work tasks in their standard lesson plans with the sage and scribe method. Our experiments indicate that this method wastes insignificant time, requires little extra effort on the part of the teacher, yet has many of the positive effects of cooperative methods: in our experiments, students received immediate feedback, corrected each other's mistakes, learned from each other in meaningful discussions and engaged in collaborative reasoning to address emerging problems.

Key words and phrases: sage and scribe, cooperative learning, pair work, case study, cooperative benefits.

MSC Subject Classification: 97D40.

Introduction

This article presents some of the results of a PhD project. It aims to identify and investigate a cooperative method that could be seamlessly integrated into the 45-minute daily mathematics instruction of overburdened teachers with minimal additional effort. Our objective was to demonstrate to teachers that the systematic application of this method yields benefits characteristic of cooperative learning approaches. The research concentrated on students' involvement, communication, and thinking processes through a case study of the cooperative instructional method known as the sage and scribe method. Owing to the experiment's limitations, including the small sample size and the short duration of the intervention, it was not possible to investigate students' achievement.

Theoretical framework

Cooperative teaching and learning

"Cooperative learning is the instructional use of small groups through which students work together to maximize their own and each other's learning" (Johnson & Johnson, 1999). Several studies report on the positive effects of cooperative methods in mathematics education (Alrø & Skovsmose, 2003; Capar & Tarim, 2015; Johnson & Johnson, 2009; Kyndt et al., 2013; Pásztor-Kovács, 2019). Slavin and his colleagues state that programs affecting daily teaching and students' interactions are more likely to achieve the method's positive benefits (Slavin et al., 2009). Group work (applied in any field) positively affects achievement although effect sizes change from 0,09 to 0,91 without any detected reason (Hodgen et al., 2018). It is also known, that "effect sizes of studies in sciences and mathematics are significantly higher than the effect sizes of studies in social sciences and languages" (Kyndt et al., 2013, p. 143). Capar and Tarim calculated that the effect size of cooperative teaching in mathematics is 0,59. They say that the effect sizes tend to increase with students' age, and the method is the most effective in the field of algebra and geometry. Furthermore, a new cooperative program should be used for at least five weeks to make its impact. (Capar & Tarim, 2015)

However, to use cooperative teaching methods effectively, some principles must be fulfilled: The groups must be small (2-6 members), students have to experience positive interdependence and individual accountability, and they need to encounter face-to-face promotive interaction. The presence of social and interpersonal skills is needed as well as group processing (Kyndt et al., 2013). Kagan identifies four key pillars of cooperative learning: positive interdependence, individual accountability, equal participation, and simultaneous interaction. Unlike the previous model, Kagan does not stress the necessity for promotive, face-to-face interactions; rather, he focuses on the importance of increasing the frequency of communication. Furthermore, Kagan posits that social and interpersonal skills, as well as group processing, emerge as natural outcomes of utilizing cooperative learning structures, rather than prerequisites. Notably, Kagan places significant emphasis on equal participation, a component which is less central in the framework proposed by Kyndt and her colleagues. (Kagan & Kagan, 2009)

There are also many difficulties that the teacher should manage. Students can avoid working within a group (Kyndt et al., 2013), they can find it hard to work together due to social tensions (K. Nagy, 2015), behavioral problems could emerge and the class easily becomes noisy. "Complex cooperative lessons require much time and effort, and they could be time-consuming" (Kagan, 1999). Ability differences could be hard to manage (Kagan & Kagan, 2009; Slavin, 1995), however, several solutions are available, as discussed in the references of this paragraph as well as on Kagan's website (Kagan, n.d.) and several other printed or web-based platforms.

In Hungary, there is no data about the everyday usage of cooperative teaching and learning in the main educational papers during the last 20 years (Magyar Pedagógia, Új Pedagógiai Szemle, Iskolakultúra, Érintő Elektronikus Matematikai Lapok). Although mathematics teachers probably know some of the basics of the theory, they probably do not use it often in high school. (Kovács-Kószó, 2024)

The sage and scribe method

The sage and scribe method is a cooperative pair work in which the sage solves the problem without a pen by thinking aloud and dictates to his/her partner, the scribe. The latter's task is to record what and only what his/her partner says and, furthermore, to help when the sage needs it, taking care not to reverse roles. Moreover, it is the scribe's responsibility to ensure that the correct solution is written on the paper, so if (s)he notices a mistake, (s)he should report it. The equal participation is fulfilled by frequent role reversals (Kovács-Kószó & Kosztolányi, 2020). It is worth underlining that this method carries within its structure three pillars of cooperativity: positive interdependence, individual accountability, and supportive interactions (Kyndt et al., 2013). In this article, pair work will be used synonymously with the sage and scribe method.

Consistent with Kagan's theory (Kagan & Kagan, 2009), Sari and colleagues' experiment – among 16-year-old students learning geometry – suggests that thinking aloud in pairs can support students to make meaning of learning activities and help them to understand the material (Sari et al., 2018). However, it is also well-known and widely supported by literature that it is difficult to generate meaningful student-student dialogues (Sfard, 2015). Furthermore, students need to learn from their peers' thought processes, thus enhancing metacognitive activities, which are useful for understanding the learning material and contributing to the overall learning process. When only one of the pair understands the topic, students can teach each other. It is known, that learning by teaching is one of the most effective learning strategies. (Nemirovsky, Rosebery, Solomon, & Warren, 2005)

No published empirical studies have been found that focus exclusively on the sage and scribe method. This conclusion is based on a systematic search using four closely related terms ("sage and scribe", "sage'n'scribe", "sage & scribe", "sage&scribe") across four different databases (namely, Scopus (https://www.scopus.com/), ERIC (https://www.eric.ed.gov), JSTORE (https://www.jstor.org/), and Google Scolar (https://scholar.google.com/)). Whenever the sage and scribe method appears in the literature – based on the above search – it is always mixed with other types of cooperating strategies. When Kagan discusses the method, he only gives examples, but no proofs to support his statements (Kagan & Kagan, 2009; Kagan, 2008, 2014).

The sage and scribe method is probably effective in procedural learning. Working with this method, students get immediate feedback and formative assessment, which have high effect sizes as well as feedback is likely to count more from a fellow student than a teacher. "With Sage-N-Scribe students feel themselves to be on the same side; the structure creates a community of learners eager to help each other. Another advantage, of course, is that students are verbalizing their thinking. As they verbalize, they listen to themselves. They become more aware of their thinking, more focused, and more likely to self-correct. The structure fosters meta-cognition – thinking about one's thinking. At the same time, the students become more aware of the thinking of others. They listen to their peers. Lower achieving students have the advantage of listening to higher achieving students who model correct ways to approach problems" (Kagan, 2008). One elderly teacher claimed that since she is implementing the sage and scribe method, the process of checking homework has become significantly more efficient, because the students could complete them in much better quality in the first place. Kagan does not describe any particular difficulties concerning the sage and scribe method (Kagan, 2008).

The advantage of the sage and scribe method is that it can be incorporated into a traditional mathematics lesson with a small but significant change: individual work tasks could be replaced by completing the tasks with the sage and scribe method. The teacher-student relationship can also undergo a major change, as student thoughts become hearable to the teacher.

Pilot study

The sage and scribe method was rigorously tested in a pilot study during the 2019-2020 school year. This contained nine brief classroom experiments (Table 1), each comprising 2 to 7 lessons, and it was applied to a variety of age groups in different school types, in three distinct cities. In our experiments, we asked the teachers to replace the individual work tasks in their usual lesson plans with the sage and scribe method, and do not change any other aspects of their lesson plans.

We formulated our research questions and developed some parts of the methodology during this pilot study. Firstly, how and what to communicate with the teacher at the beginning of the experiment and what the key points that should be re-emphasized later are. Secondly, how should we introduce the method to the students, and in what areas do they need reminders? Thirdly, we developed the questionnaires. These experiments along with the theoretical framework were the basis of our research questions and hypothesis, and we built on these during the planning of the main two experiments of this case study.

Research questions

The next section lists our research questions and related hypotheses. There are several aspects to be investigated, although we could concentrate only on some of them.

Research Question 1

When Kagan presents a set of 31 frequently asked questions from teachers (Kagan & Kagan, 2024), the second and fourth points specifically address concerns related to the time demands and the level of effort required for implementing cooperative methods. This observation aligns with the authors' own experiences during discussions with Hungarian educators. Consequently, the first research question was formulated as follows.

Is there a significant time loss or extra teacher energy involved in systematically using the sage and scribe method instead of individual work once the students have become accustomed to the method?

The pilot study led to two hypotheses on this question.

The application of the method as described in the research question does not result in a significant loss of time (Hypothesis 1a).

Teachers do not feel that considerable extra effort is required (Hypothesis 1b).

Grade	Type of the school[1]	Class size	Topic	Length of the experiment (number of lessons)
Grade 9	A strong vocational school in a big city	12	Revising at the end of the year	6
Grade 7	A strong grammar school in the capital	17	Functions	2
Grade 9	Vocational school in a small city	8	Equations	1
Grade 9	Vocational school in a small city	12	Equations	1
Grade 11	A middle-strong grammar school in a big city	17	Algebra, exponential equations	7
Grade 9	A middle-strong grammar school in a big city	32	Basics of algebra	7
Grade 9	A middle-strong grammar school in a big city	16	Equations	6
Grade 10	Vocational school in a big city	13	Functions	6
Grade 7	Primary school close to the capital	14	Letters in algebra	6
Grade 8	Primary school close to the capital	18	Algebra	6
Grade 7	Primary school close to the capital	24	Algebra	6
Grade 5	Primary school close to the capital	25	Decimal fractions	6
[1] In Hungary, students start primary school at the age of six and continue for eight years. Secondary school lasts four years.				

Table 1. Details of a series of experiments with sage and scribe method. Information on the type of schools is based on a comparing website (Sulinavigátor, 2023)

Research Question 2

It is uncommon to employ a single cooperative learning method alone, as these strategies are typically integrated in lessons. Accordingly, the following research question was posed:

Which benefits of cooperative methods present themselves when the sage and scribe method is used instead of individual work during lessons?

It is important to emphasize that we did not intend to make any other changes to the structure of the mathematics lesson, but only to replace the individual work already included in the original lesson plan. As discussed in the theoretical framework, cooperation could yield several positive effects. Kagan also highlights specific potential benefits of the sage and scribe method (Kagan, 2008). We intend to investigate some of them in this article. Although the general conclusions about achievement could be based on a much bigger sample size and longer experimental period, we intended to get some information through the lens of Dekker and Elshout-Mohr's framework, in Hypothesis 2e (Dekker & Elshout-Mohr, 1998).

Several hypotheses were formulated to specify the research question:

- Students work in pairs; they do not leave the topic of the lesson (Hypothesis 2a).
- Students verbalize their thoughts (Hypothesis 2b).
- Students correct each other's mistakes (Hypothesis 2c).
- Students receive immediate feedback (Hypothesis 2d).
- Students' work displays regulating and key activities that can contribute to mathematical understanding and learning (Hypothesis 2e).

Research Question 3

It is important to conduct a detailed analysis of the key characteristics that describes student cooperation during sage and scribe pair work. How the assigned roles influence students' interactions, a key component of cooperative learning, is a critical area of investigation. During the pilot study, students frequently required reminders that the scribe plays an active role, and (s)he is allowed to do more than just writing. It was useful to highlight often that (s)he is not only permitted but also obligated to assist the sage when necessary. The following question also addresses one of Kagan's four pillars of cooperative learning: Does student participation remain equitable throughout the task, or does the rotation of roles simply result in equal participation? In other words: Do the roles of sage and scribe equally require active participation from students in task completion, or does the role of the scribe offer a more passive option for a less engaged student? While it was not possible to address all of these questions, the following research question and hypothesis were formulated:

How is the asymmetry of the structure of the sage and scribe method realized in practice?

Symmetrical dialogue and joint thinking can develop despite asymmetrical roles (Hypothesis 3).

Methodology and data collection

In September 2020, we were given the opportunity to try out the sage and scribe method for 11 and 15 lessons in two 10th classes. One of the classes (hereafter Class A) had humanities as a special interest. It had 16 students from one of the best high schools in a large city (Sulinavigátor, 2023). The teacher was a strict and demanding lady with about 20 years of experience, who was keen to renew her professionalism because she felt that the frontal method was becoming less and less effective. However, before the experiment, she had made little use of alternative methods. Although the students did not particularly like mathematics, they were hard-working, and they aimed to get a good mark and a good result at the end of the course. The other class (hereafter Class B) had 38 members from a good high school in a large city (Sulinavigátor, 2023). Their mathematics teacher, who was also head of the class, was less harsh with the students, trying to understand them, in many cases being rather friendly towards the class, but preparing them for the final exam with a strong sense of purpose. Also, with nearly 20 years of experience, she was good at disciplining the class but saw a strong need for renewal.

In both cases, the pupils had three mathematics lessons per week. Both experiments started in the second and third lesson of September 2020, with one method-learning lesson in each class, and lasted for 11 and 15 lessons. The pairs were assigned by the teachers based on Kagan's principles (Kagan, 2001), ensuring that pairs did not consist of close friends or adversaries, and that the abilities differences between partners were not big.

Twice the pairs were rematched. After each lesson, a 1-2-minute-long, five-question mini-interview was conducted with both teachers, and three longer interviews took place before, during, and after the experiment. Students completed two (in Class A) and three (in Class B) questionnaires during the experiment, and on one occasion each student was interviewed for 5 to 10 minutes after the 11th (in Class A) and after the 12th lesson (in Class B). Due to the differences, the two classes are not comparable, but both experiments provided valuable information. Furthermore, in Class A, all pairs' work was audio-recorded, but due to lack of parents' permission, there were no such data in Class B. We did not collect specific data about achievement, as the length of the experiment and the number of participants were too small. All lessons were observed by one of the authors of this article.

46 recordings of pair work were transcribable and analyzable. In the diagrams below, we will plot information about these recordings. In each lesson, there were different amounts of analyzed dialogues (6, 2, 4, 6, 2, 4, 4, 3, 5, 6, 4), and there was a lot of variation among pairing, therefore we did not intend to draw conclusions about individuals or pairs.

During the experiments, our intention was not to modify the overall structure of the mathematics lessons, but rather to replace only the independent work components already involved in the original lesson plans. These original lesson plans were usually structured in the following way: checking the homework, presenting the new material for the lesson, and modeling some exercises on the blackboard in an interactive way. Then, students were instructed to solve similar tasks independently or, during the experiment, in pairs, followed by a classroom discussion as necessary and the assignment of homework. There were also practicing lessons, where more individual work was usual. The topic was algebra in both classes, with easily dictatable tasks.

Detailed description of quantitative analysis methods

In Class A, the pairs' dialogues were audio-recorded, and 46 dialogues, totaling 779 minutes, were processed and analyzed.

Sequencing, coding units

The transcripts of the dialogues were broken down into coding units. A new unit was started when the speaker's identity or the focus of the speech changed (Wood & Kalinec, 2012).

Examples of changes in the focus of speech include:

- when the student evaluated the correctness of the solution while solving the task: "This is not going to be good.";

- when the student first talked about what he or she was going to do: "Let's look for an example like this in the notebook.", then started dictating the problem: "$x^2 - 26 \cdot x \ldots$";
- when solving the equation, the students reached a new line;
- when in the exercise "Simplify!" they reached an equal sign;
- when a new idea appeared while solving the problem.

If the pair worked independently without teacher assistance, each coding unit was assigned two codes.

1st Coding system: Based on regulating and key activities

Dekker and Elshout-Mohr (1998) distinguish three types of activities in their process model for cooperative task solving: key activities, regulating activities, and mental or cognitive activities.

Regulating activities	Key activities
Students **ask** each other to **show** their work.	Students **show** each other their work.
Students **ask** each other to **explain** their work.	Students **explain** to each other their work.
Students **criticize** each other's work.	Students **justify** their work.
	Students **reconstruct** their work.

Table 2. Regulating and key activities

The cells of Table 2 were first formatted into a code system with two additional codes: 'Students are engaged in a mental or cognitive activity' and 'Neither'.

The resulting nine-item code system was discussed by four independent coders based on four transcripts and applied on one transcript (Dr. Zoltán Kovács, Dr. Eszter Kónya, PhD students Emőke Báró and Eszter Kovács-Kószó). The following final system was developed based on joint discussions, which were applied for the whole 779 minutes:

- **Ask to explain** (regulating activity): students ask each other to explain their work. This code should also apply when a student asks a question aloud although (s)he is not necessarily expecting relevant answers from his/her peer.
- **Criticize** (regulating activity): students criticize each other.
- **Ask to confirm** (regulating activity): students ask for confirmation.

- **Explain** (key activity): students explain their work to each other. This code also includes instances of reconstruction.
- **Confirm** (key activity): students give each other confirmation of their work. This includes praises for the work.
- **Task-solving** (mental or cognitive activity): solving tasks out loud (sage's utterances), thinking about solving a task, or expressions that show thinking about it (e.g., "um").
- **Neither**: any manifestation in which students give voice to their thoughts about the lesson but cannot be included in the above categories. For example: negotiating the roles of sage and scribe or looking for a calculator.
- **Private conversation**: a conversation unrelated to the classroom, to mathematics, or the roles of sage and scribe.
- **Non-coding**: transcriptor's comments, teacher's whole-class calls.

This coding system was used to examine Hypotheses 2a and 2e.

2nd Coding system: Analyzing the nature of the sage and scribe pairing

Adapting several code systems from the literature for analyzing cooperative methods was unsuccessful. We tried to apply and adapt the matrix of collaborative problem-solving for PISA (Graesser et al., 2018) and the Reconsidered Inquiry Co-operation Model (Alrø & Skovsmose, 2003), but most of the codes were almost absent from the discussions, firstly because of the tasks' aims were mainly practicing an already discussed material, and secondly due to the asymmetrical nature of the sage and scribe method. The second direction was two other theories: four types of collaborative roles (Tatsis & Koleza, 2006) and the face strategies in Tatsis and Dekker's work (2010). Although the collaborative nature of the sage and scribe method did not accommodate to these roles well, face-saving strategies were also not predominant in the conversations. The theory of the MOST moments (Leatham et al., 2015) is highly useful for identifying mathematically significant moments in student-teacher conversations, although it was not adoptable for student-student discussions. To get more information about achievement, we tried Miles and Huberman's coding system (1994), which meant to identify periods where the students' mathematical understanding changed positively or negatively, but the coders found it highly subjective. For similar purposes, we intended to use theories related to relational and instrumental understanding, but the coders found it hard to maintain objectivity. Another potential coding system is described by Wood and Kalinec (2012), but more than half of the codes were almost absent from the dialogues, as there were few off-task utterances. The other half of the codes were not specific enough for our present research. Therefore, ones of our own were built, with the help of Emőke Báró (PhD student).

The codes were developed considering the hypotheses based on the four dialogues mentioned above. Every problematic episode was discussed, and an 11-element system was finalized after three modifications (Figure 1). This system was applied to the whole 779 minutes.

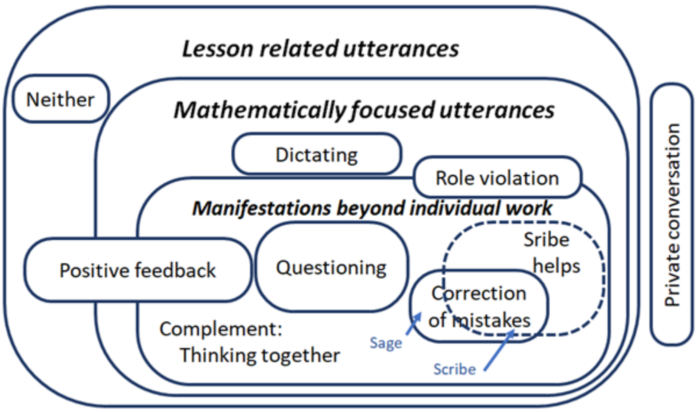

Figure 1. Coding system for analyzing the nature of the sage and scribe pairing

- **Dictating**: dictating or guiding the task in some way, in the role of the sage. This code also applies when the sage thinks aloud, but only to himself rather than to his partner.
- **Correction of mistakes**: a brief call or request from the scribe (sometimes from the sage) with the intention that the sage (sometimes the scribe) should notice his/her mistake and correct it. The scribe assumes that the sage has made an error only through inattention. No more than two such codes may be given in succession, and in the third case, the **Thinking together** code should be used.
- **Questioning**: a question or request for help from the sage or scribe that is formulated because (s)he does not understand the mathematical idea of his or her partner. (The results pertaining to this code are not discussed in this article.)

- **Scribe helps**: manifestations of the scribe acting as a competent helper. In this case, the scribe believes that (s)he knows or understands something that the sage does not.

- **Role violation**: manifestations that violate the roles of sage and scribe. This code should be chosen even if another code would be valid for the given manifestation. (The results pertaining to this code are not discussed in this article.)

- **Positive feedback**: correction, praise, or acknowledgment of a previous manifestation. Any non-negative laughter or positive communication.

- **Thinking together**: mathematically focused manifestations that would not arise from individual work and do not fall within the above codes. For example, a dialogue about a problem that arises, in which the manifestations are not considered to be role misconduct, since dialogue has a place in the problem-solving process. Invocations to think together, questions like "Am I right?" to check one's work, acknowledgments like "Yes, I understand", and comments like "Wait" also fall under this code. This code applies where the sage accepts or rejects a correction from a partner. Fill-in-the-blank words expressing thought may also be included here (well, then, ish, so). This may apply if the scribe is assisting the sage by using a calculator. Comments on the difficulty of the task, or the scribe's expressions while writing, can also be added to this code.

- **Neither**: statements relating to the lesson which cannot be included in the above categories. For example: negotiating the roles of sage and scribe or looking for a calculator.

- **Private conversation**: a conversation unrelated to the classroom, mathematics, or the roles of sage and scribe.

- **Non-coding**: transcriptor's comments, teacher's whole-class calls.

Note: The **Scribe helps** code is part of the **Thinking together** code and is not well identified according to the coding tests. Therefore, this code is only suitable for quantifying the subjective opinion of the coder. The last three codes are the same in the two coding systems.

This coding system was used during the examination of Hypotheses 2a, 2b, 2c, 2d and 3. Examples of the codes above are presented in Table 3.

1st coding system	2nd coding system	Role of student	Utterance
Excerpt from the 3rd lesson (see also Figure 2)			
Task-solving	Scribe helps	Scribe	In my opinion this', this will be an addition also
Confirm	Thinking together	Sage	Aha (her voice is a little insecure)
Criticize	Thinking together	Scribe	No this is', this is not'. Oh! Each will be a subtraction!
Explain	Thinking together	Scribe	Because you have to look at the denominator only.
Non-coding	Non-coding		Silence for about 10s
Task-solving	Dictating	Sage	Ouch 7-3, so, 4.
Criticize	Thinking together	Scribe	What? I don't think you can deduce that from each other.
Task-solving	Thinking together	Sage	What? Wait! This one (a-b)·(a-b)=1·a2, right? [She is writing it down somewhere, not to the worksheet.]
Ask to confirm	Thinking together	Sage	(a-b)·$(a-b)^2$, doesn't it?
Criticize	Thinking together	Sage	Noooo.
Explain	Thinking together	Sage	Because they are both the same. It's like, it's like a plus. So if (a+b)·(a+b), it is a^2+2ab+b^2. This is the same, but with a minus sign.
Here stands seven utterances, when the pair were jointly thinking about the task.			
Non-coding	Non-coding		About 20s silence. The Sage gives meaningless noises like hm, khm, üühüüm, mmm.
Task-solving	Thinking together	Sage	Well, let's try anyway, I have no other idea.
Ask to explain	Thinking together	Scribe	How?
Non-coding	Non-coding		4s silence
Explain	Thinking together	Sage	Like a^2-2ab
Neither	Neither	Sage	Pause, the Sage sighs loudly.
Task-solving	Dictating	Sage	7^2-2√7√3, 8s silence [She did not dictate the multiplication sign.]
Excerpt from the 2nd lesson			
Neither	Neither	Sage	You write that also
Task-solving	Dictating	Sage	√50 ... well then it' [slowly, with pauses]
Excerpt from the method learning lesson			
Ask to confirm	Correction of mistakes	Sage	Yeah. I don't know how that should be done. Should we multiply the two of them with each other?
Confirm	Thinking together	Scribe	Yes, I think so.
Excerpt from the 2nd lesson (see also Figure 3)			
Task-solving	Dictating	Sage	[the Sage counts out loud] Then it's 75, so that if fif' fifty, that's nice, divide it with 10, then, it is 5*10, and 25, divided by 25, just a moment, 25, it is 5, it is 15 [Her thoughts are not easy to follow.]

Confirm	Positive feedback	Scribe	Yes [as soon as his partner says 15]
Neither	Positive feedback	Sage	That's right.
Neither	Positive feedback	Scribe	[they both laugh a little]
Four more similar rows			
Neither	Positive feedback	Sage	A creative solution!
Excerpt from the 1st lesson			
Neither	Neither	Sage	[he is counting under his breath]
Private conversation	Private conversation	Scribe	This time last year they had the diamonds, and now they haven't even put them out.
Private conversation	Private conversation	Sage	What has not been put up?

Table 3. Examples of the codes

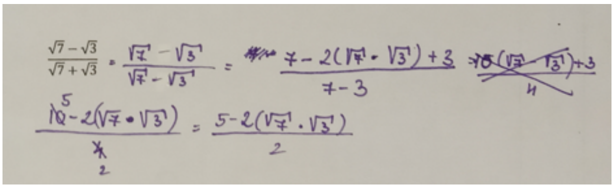

Figure 2. A picture of a worksheet that corresponds to the above discussion

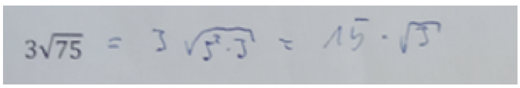

Figure 3. A piece of a worksheet that corresponds to the above discussion

Results and discussion

In the following chapter, we will answer our research questions concerning the sage and scribe method. In each case, we will first test our hypotheses in the light of qualitative and then of quantitative data.

It must be emphasized again that the experiment aimed to replace the independent work already present in the original lesson plan with the sage and scribe method. The experiment was not intended to change anything else in the structure of the mathematics lesson.

Hypothesis 1a: Time loss

After each lesson, we asked both teachers the question: "How much did the pair work slow down the lesson? Approximately how much faster would you have progressed if students had worked individually instead of in pairs?"

The answers showed that the method never slowed down the lesson in Class A, and only once, a little, in Class B.

The teacher of Class A pointed out that when there was only time for one pair work task, the advantages of using the method were more pronounced, while the disadvantages were less noticeable. In addition, in Class A, there were only eight pairs (out of 16 students), so the teacher was able to listen, help, and give feedback to each pair, which was not possible for 16 individuals. Joint checking could thus be omitted, which saved time. The previous two effects did not apply in Class B due to its large size.

At the end of the experiment, in Class A, most students felt that they would have had time for more tasks if they had worked individually, but this was not in line with the teacher's opinion. Class B students gave neutral answers concerning this topic.

In summary, the use of the sage and scribe method rarely slowed down the lessons and the occasional time loss was not significant, supporting Hypothesis 1a.

Hypothesis 1b: Extra energy invested by teachers

The teacher in Class A was enthusiastic, quickly seeing the benefits of the sage and scribe method that made the necessary sacrifices worthwhile. For example, asymmetry meant that not all the solutions were recorded in all notebooks. She compensated the class for this by uploading extra exercises with solutions to practice on the group's online platform. In Class B, students' resistance was a difficulty, but this required mental effort rather than extra preparation. Of course, matching pairs was an additional task, but none of the teachers felt it was a burden. Apart from these, no other extra effort was highlighted by any of the teachers. Thus, the data from the case study supported Hypothesis 1b.

Hypothesis 2a: Students work in pairs; they do not leave the topic of the lesson

A question was asked at the end of each lesson: "Were the students more active as a result of the pair work in this lesson?" In the case of Class A, it received a very strong "yes" response after each session. According to the observations of the lessons, the pupils' movement, writing, and talking showed that they were active, furthermore, the audio recordings also proved that they were working continuously, based on the transcriber's subjective opinion, which is consistent with the objective data presented below. In the case of Class B, there was also a "yes" response after half of the lessons, in the other cases there was either no data or the teacher cited a disciplinary problem.

In the initial, method-learning lessons, both teachers pointed out that the pair work helped the students to work better, with a stronger compulsion to think without teacher assistance.

In all but one lesson in Class A, there was unanimous agreement that the use of pair work was worthwhile. Reasons given included that the topic was better understood by the students because they had to "put the task together themselves". According to the teacher, the most dominant benefit was that it forced students to work and think, rather than just copying the result from the blackboard, which often happens in a usual session. This compulsion to follow instructions provided sufficient motivation to try to work in this strange arrangement. The work paid off: both the teacher's opinion and the recordings showed that everyone was able to behave naturally in the pair work.

After the initial enthusiasm, towards the end of the experiment, the teacher of Class B repeatedly showed hesitation at the difficulties, but the need for change pushed her through them. It occurred to her that the method could be more effective if the sage could write. In Lesson 13, she asked whether the group had had enough and whether it had become boring. She gave more often short or evasive answers, which could be due to her uncertainty.

Student questionnaires from both classes show that students were active in the lessons. An example of this is that students scored themselves highly on a five-point Likert scale (mean: 4.3; standard deviation 0.2) in terms of how much thought had been put into the tasks when they were in the role of scribe.

The percentage of 'Private conversation' code to all coded units is shown in Figure 4. Different numbers of audio recordings were transcribed for each lesson. The dots represent zero values.

Out of 46 voice recordings, 30 times (dots on Figure 4) the couple made only class-related utterances. In the opposite direction, the third pair of Lesson 9 (dark green bar) stands out with a rate of 14.2%. This could have been because the girls had good skills, so they finished the task quickly, but the teacher did not give them any extra work, so they talked in their spare time. Overall, the data show

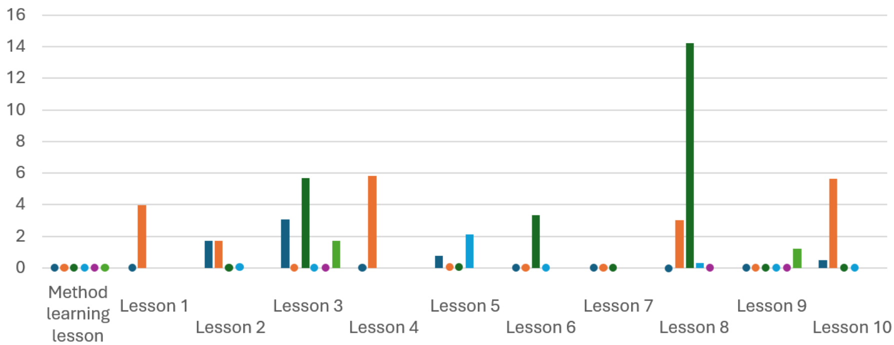

Figure 4. Percentage of the 'Private conversation' code about total coded units in each dialogue

high discipline in Class A. The low percentages in Figure 4 are strong evidence that the students have been mentally active.

Overall, both the qualitative and quantitative data support Hypothesis 2a: students in the experiments worked during pair work, and they did not get side-tracked. Pair work probably helps to motivate students to work actively.

Hypothesis 2b: Students verbalize their thoughts

One positive effect of verbalizing thoughts is qualitatively supported by the interviews. Many of the students in Class A expressed with their own words one of the commonly known advantages of cooperative work: when they could talk out loud while solving the task, they c ould remember what they were thinking in the long run. This idea did not appear in Class B.

In the second coding system, the codes for 'Thinking together', 'Questioning', 'Scribe helps', and 'Correction of mistakes' capture utterances where students articulate their mathematical reasoning in ways that differ significantly from independent work. In contrast, the 'Dictating' code aligns more closely with individual task-solving behaviors. Figure 5 shows the sum of these codes. The benchmark for the percentage calculation is the total number of units coded. Each column represents the dialogue of one pair in a lesson; the same colour cannot be associated with the same person.

The high percentages in the graph in Figure 5 indicate that students are indeed verbalizing their thoughts, far beyond the obligatory role of the sage.

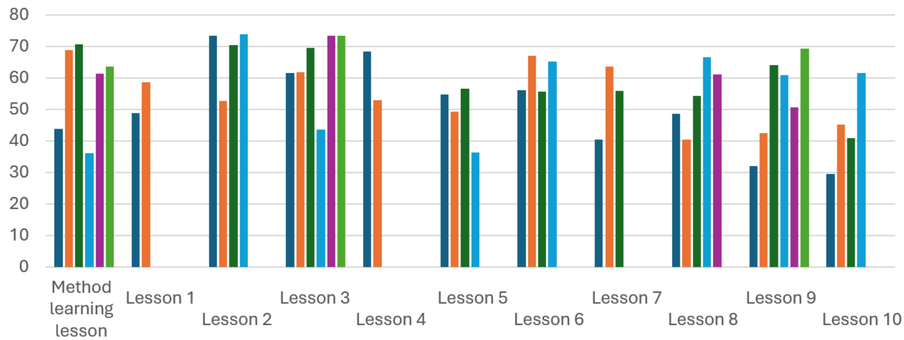

Figure 5. Percentage of the ‘Thinking together’ code and its subsets in each dialogue

The positive impact of this is supported by several research studies (Taljaard, 2016) in addition to the students’ experiences, therefore Hypothesis 2b is strongly supported.

Hypothesis 2c: Students correct each other’s mistakes

Hypothesis 2c assumes that students correct each other’s mistakes. To prove this, a variety of quantitative data were collected. In the second coding system, the ‘Correction of mistakes’ is represented in Figure 6.

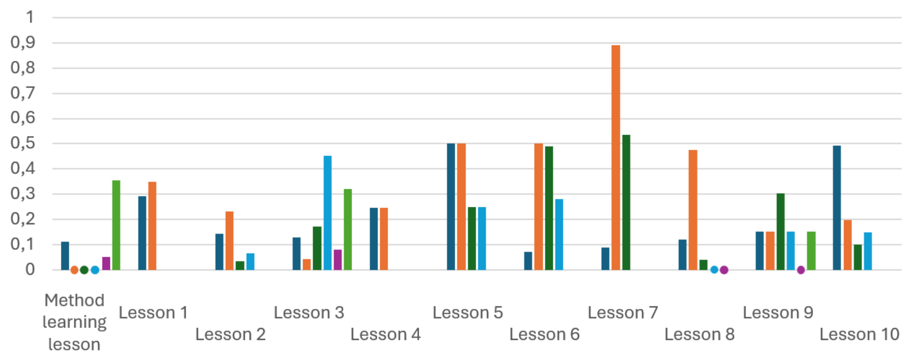

Figure 6. Number of occurrence of the ‘Correction of mistakes’ code per minute in each dialogue

In six dialogues, it occurred that students did not correct each other's errors (dots in Figure 6), but no common cause was found based on the following criterias: mathematical ability, temperament, gender, or friendship issues. The data for this claim are taken from the teacher's partly subjective opinion, the transcriber's subjective opinion and students' interviews.

In many cases, the number of 'Corrections of mistakes' is not frequent (about 1 mistake per 10 minutes), but it is not uncommon to correct errors once every 2-3 minutes (on average), which in turn implies regular feedback. The outlier (0.9 error corrections per minute, orange bar in Figure 6) belongs to a pair where the better-ability boy deliberately embarrassed his slightly less-able partner, who therefore made several errors. This dialogue was close to the phenomenon of bullying.

According to the students' admission in questionnaires, on average, there were about four corrections of mistakes per lesson within a pair. This is in line with Figure 6, calculated from the audio recordings: 3.4. In Class B, we could only rely on the questionnaires, according to which an average of seven error corrections were made in a lesson.

Overall, it can be said that the correction of each other's mistakes is a significant consequence of the pair work, therefore Hypothesis 2c is strongly supported.

Hypothesis 2d: Students receive immediate feedback

Students rated themselves as helpful on a five-point Likert scale (mean: 4.2; standard deviation: 0.05), which is necessary to give immediate feedback to their peers.

The number of 'Scribe helps' code from the second coding system, normalized for one minute is plotted for each dialogue (Figure 7).

In nine out of 46 dialogues, the scribe does not provide help at all (dots in Figure 7). Looking closely at the individual dialogues, there were several reasons for this: the sage did not need help, or the scribe did not have the necessary skills to do so. There were also many instances where, although help was needed and the scribe would have been keen to provide it, the scribe was also unsure of the situation and needed time to think. These cases typically developed into sessions with a high number of 'Thinking together' code and solutions were found together. Unfortunately, there were also cases where the scribe, lacking sympathy, did not help when (s)he could have.

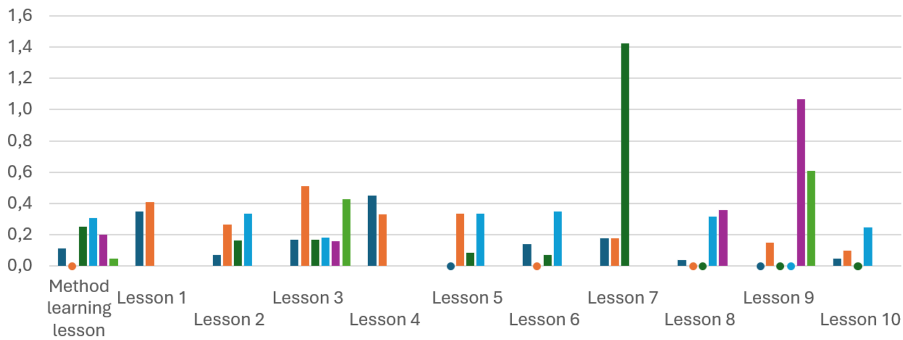

Figure 7. Percentage of 'Scribe helps' code in each dialogue, normalized per minute

The number of 'Positive feedback' was also significant, as shown in Table 4. Data are normalized to time (minute) and pairs.

Method learning lesson	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6	Lesson 7	Lesson 8	Lesson 9	Lesson 10
0,44	1,22	0,88	0,45	0,49	0,58	0,63	0,8	0,36	0,89	0,22

Table 4. The average number of 'Positive feedback' normalized to time (minute) and pairs

The dominant function of interactions coded as 'Scribe helps', 'Positive feedbac'k, and 'Correction of mistakes' (discussed in the previous section) is feedback. Thus, both the subjective (rating on the five-point-Likert scale) and the objective (Figures 6 and 7, and Table 4) data suggest that students often get feedback on their thoughts from their peers during sage and scribe pair work, which supports Hypothesis 2d.

Hypothesis 2e: Regulating and key activities

Both teachers repeatedly referred to the fact that the students worked together. They regularly perceive from the outside that students do work in pairs, not just side by side. However, they could not judge the quality of pair work.

The overall benefits of cooperative methods were partially experienced by the students. Several feedback indicated in both classes that pair work contributed to a deeper understanding of the topics and tasks. We present here some thoughts from the students from the interviews and the questionnaires' free comments: "[the pair work] guides me to the right solution, it checks me", "I get warnings if I commit a mistake", "it was more interesting", "We got along really well, our joint work improved over time. My partner helped me a lot, and I am trying to return it", "I will miss these pair work sessions", "sometimes my partner could explain the problem better than the teacher", "it is more interesting, I got on-the-ground feedback, you don't bother your partner when you ask for help, unlike during individual work", "it makes me free to ask my questions, helps me to focus", "I feel some pressure in front of my peer to give my best performance", "it makes the lessons less monotonous", "I improved a lot, we helped each other a lot".

We present the results of the coding system based on regulating and key activities in the following graphs, which provide quantitative feedback on the quality of pair work, as regulating and key activities can contribute to mathematical understanding and learning.

The total number of regulating and key activities per minute is plotted in Figure 8. Each dialogue has two consecutive lines: a right-hand side (regulating activities) and a left-hand side line (key activities) with the same colour. The dialogues are grouped by lesson. Lines of the same colour within a graph do not represent the same person.

On average, there are 1.3 regulatory activities per minute, compared to 1.7 key activities. The line chart illustrates that both regulating and key activities constitute essential components of the students' pair work, indicating substantial and meaningful collaborative engagement. In the vast majority of cases, the number of key activities is significantly higher, with only four cases where the ratio is reversed (light blue line in Lesson 9, dark blue lines in Lesson 7, 8 and 10). This finding may indicate that the framework of the sage and scribe method inherently facilitates the emergence of key activities, such that key activities can occur independently of regulating activities. It is also possible that a single regulating activity may cause several key activities.

By analyzing the regulatory and key activities individually, our findings can be summarized as discussed below. (The diagrams could not be presented here due to page limit.)

- **Ask to explain**: Students rarely asked their peers for explanations. In one dialogue, students asked for explanations on average 0.25 times per minute (standard deviation: 0.22). One exceptionally high value (1.22) was a dialogue in Lesson 10, which represents two students with poor mathematical skills

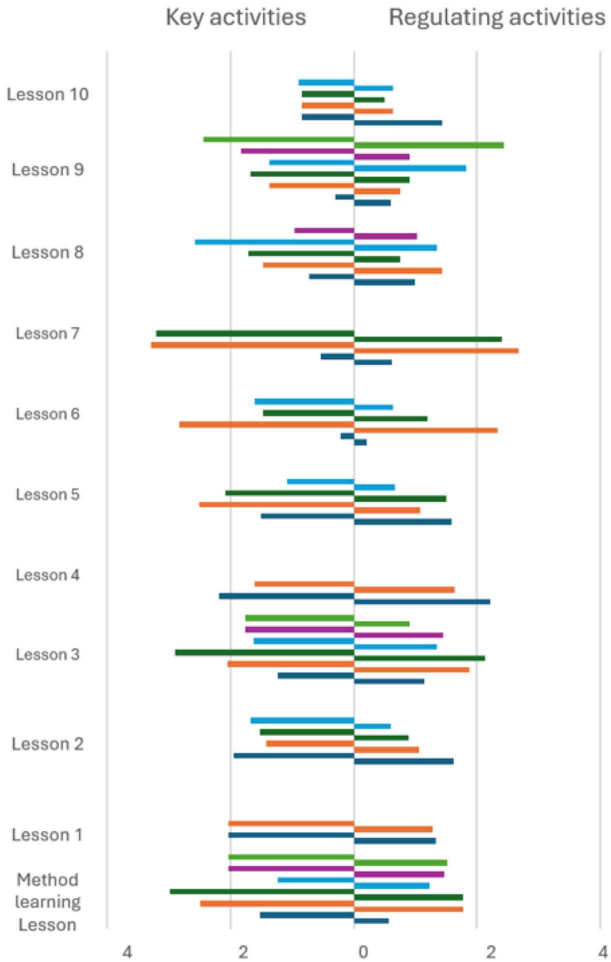

Figure 8. Total number of regulating (right-hand side) and key activities (left-hand side) per minute

compared to the class. They were not friends, and they worked on a simple task. In this lesson, in three other dialogues, the code 'Ask to explain' does not appear at all, while in the whole experiment, there are a total of six instances where the discussed code does not appear.

- **Criticize**: Criticism was more frequent and occurred in all lessons. On average, students criticized 0.65 times per minute in a dialogue (standard deviation: 0.34).

- **Explain**: Students explained to their peers moderately often. On average, students explained 0.38 times per minute in a dialogue (standard deviation: 0.23), which means that on average there was an explanation every 2.5 minutes in the work of each pair. Explanations were absent from only 4 out of 46 dialogues.

In Figure 9, two regulating activities are plotted ('Ask to explain' and 'Criticize'), and the closely related key activity ('Explain').

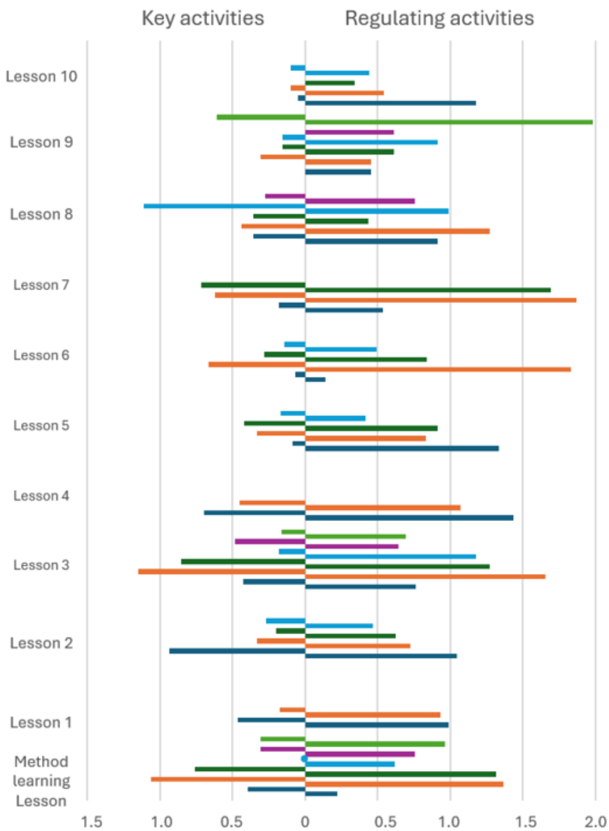

Figure 9. Total number of the codes 'Ask to explain' + 'Criticize' (right-hand side) and 'Explain' (left-hand side) codes per minute in each dialogue. Two consecutive, same-colour lines (right and left) belong to the same dialogue.

In the graph (Figure 9), the right-hand lines tend to have a higher absolute value, suggesting that the regulating activities – 'Ask to explain' and 'Criticize' – do not always, or only after several instances, entail the occurrence of the key activity 'Explain', which can of course occur without any regulating activity.

Figure 10 shows a similar relation between' Ask to confirm' (right-hand side, regulating activity) and 'Confirm' (left-hand side, key activity). 'Confirm' occurs significantly more frequently, often independently from the corresponding regulating activity ('Ask to confirm').

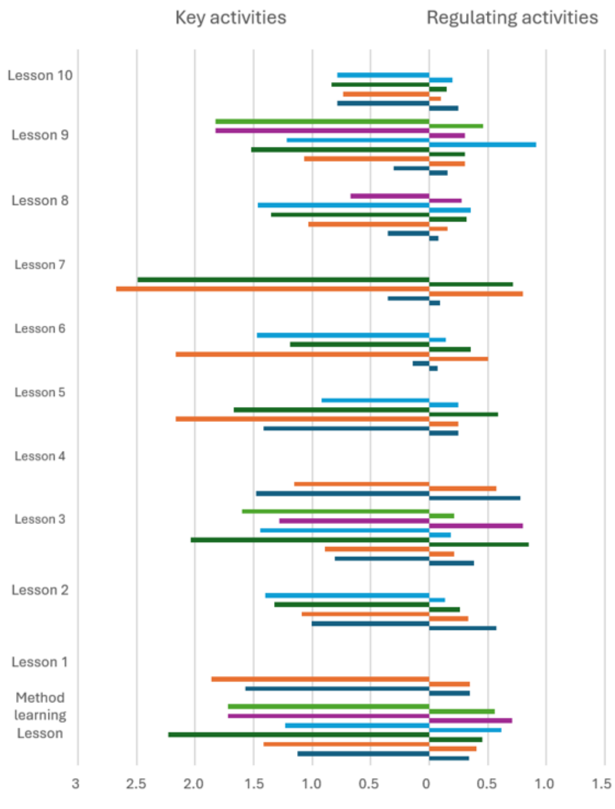

Figure 10. Total number of the codes 'Ask to confirm' (right-hand side) and 'Confirm' (left-hand side) codes in each dialogue, per minute. Two consecutive, same-colour lines (right and left) belong to the same dialogue.

Table 5 shows quantitative data on regulating and key activities.

Regulating/ key activity	Ask to explain	Criticize	Ask to confirm	Explain	Confirm
Total occurrence	191	510	288	325	990

Table 5. Total occurrence of regulating and key activities in 46 dialogues over 779 minutes

From Table 5, it is clear that 'Confirm' is the most common key activity that is believed to have a positive impact on students' self-confidence. Interestingly, this phenomenon was not expressed by the students either in the questionnaires or in the interviews. However, the low but stable presence of 'Explain' is also encouraging.

Regulating and key activities are prominent in the learners' dialogues, with 'Confirm' and 'Criticize' predominating most often. Based on research findings, these promote meaningful collaboration. Thus, data from this case study supports Hypothesis 2e: students' work displays regulating and key activities that can contribute to mathematical understanding and learning.

Plotting the regulating and key activities for individual students for each dialogue showed that there were significantly more of each regulating and key activity in the first, method-learning lesson than any other lesson, but no other information is highlighted in the graphs, therefore these are not presented here.

Research Question 3: Symmetrical thinking, dialogue

Our next research question aims to investigate the nature of the collaboration between students during sage and scribe pair work. We hypothesize that symmetrical dialogue and joint thinking can develop despite asymmetrical roles (Hypothesis 3.)

Firstly, we wish to emphasize that role asymmetry and regular role-swapping help balance equal work-share: the otherwise more shy or less capable party is given more opportunities to express his or her ideas. The framework set up by the roles also weakens the possibility of only one member working in a pair.

Class A students confirmed during the interviews that they corrected each other's minor mistakes and sometimes engaged in mathematical discussions or joint searches for the correct solution when disagreement arose. Meaningful mathematics discussion could also lead to conflicting beliefs or one or both students becoming unsure. In several cases, but not always, students tried to understand the solution, not necessarily accepting their partner's opinion without reflection.

According to interviews conducted in Class B, 12% of students reported that their peer's explanation or support was beneficial, 50% perceived the method as neither positive nor negative, and 38% believed that the pair work had negatively impacted their performance. Yet 60% of students reported that there had been real joint work on arising errors. When they could not solve a task, most tried to ask the teacher for help, but a third tried some other ways to overcome difficulties (brainstorming, searching in their notebooks, or asking another pair for help). Almost half of the students said they had learned from their peers through collaborative work.

The information from Figure 5 is repeated here: in 32 out of 46 dialogues in Class A, more than 50% of the manifestations were coded as 'Thinking together', 'Questioning', 'Scribe helps', and 'Correction of mistakes', furthermore, the lowest value was 28%. This provides clear quantitative evidence that asymmetric roles do not prevent symmetric dialogue.

The data support that symmetrical dialogue and shared thinking can (also) develop in students' work in the presence of asymmetrical roles.

Summary

This article uses a case study experiment to learn the characteristics of a pair work, the sage and scribe method (Kagan, 2008). In the case study experiment, we asked two teachers, accustomed to traditional frontal teaching methods, to substitute individual work tasks in their standard lesson plans with the sage and scribe method without any other structural changes during the lessons.

Both qualitative and quantitative data from the case study experiments support that this method wastes insignificant time, and requires little extra effort on the part of the teacher. In our experiments, students were active participants during the pair work periods. They verbalized their thoughts, received immediate feedback, and corrected each other's mistakes. They learned from each other in meaningful discussions and engaged in collaborative reasoning to address emerging problems. Regulating and key activities, most often 'Confirm' and 'Criticize', were prominent in the students' dialogue, which could help to make collaborative work more meaningful, based on previous research results (Johnson & Johnson, 1994). Our data support that symmetrical dialogue and shared thinking can (also) develop in students' work in the presence of asymmetrical roles.

There are some important aspects of this case study which due to page limit are not discussed in detail, but we wish to mention them shortly at the end of the article:

- Most students often do what the roles of sage and scribe require of them, even if many students are uncomfortable with the roles. The photos of the students' notes showed that the roles of the sage and scribe did not cause extra problems in writing down the tasks, nor did students complain about it in the interviews.
- Another important aspect is that only one solution is recorded per pair. There are several possible solutions to solve or neutralize this:
  - It can be incorporated into the lesson with minimal time loss by having the sage regularly copy the solution into his/her own notebook either after the problem has been solved or during the classroom discussion/checking of the task.
  - A digital photograph of the solutions can be taken, although it makes learning more difficult at home.
  - When several similar tasks appear during practice, it may render unnecessary for each student to record every exercise in their notebook.
  - Missing tasks can be resolved at home for practice.
  - Elaborated exercises can be published online by the teacher, but these also make learning more difficult at home.
- Increased noise level can also be a problem, but the teacher will hopefully find this a useful change, because this noise includes a lot of interesting student thoughts as well. Excessive noise should be disciplined. (K. Nagy, 2015)
- Bullying may arise during pair work and may remain hidden from the teacher.

The method is promising, and it should be tested in a large-scale experiment with control groups, with different age groups and school types, and with different teacher personalities. It would also be important to continue the research in middle-ability and low-ability classes with average teachers. Mathematical performance should also be investigated in the future.

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