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Teaching
Mathematics and
Computer Science

On the relationship between Mathematics- and Computer Science Education

Karl Josef Fuchs

Abstract. In the first half of the paper, the profile of the two scientific disciplines of Mathematics Education and Computer Science Education is traced. In Mathematics Education, the description has been given in a short longitudinal section of its preying cornerstones since the beginning of the 1960s. In Computer Science Education, this is done through the description of an emancipatory science that has been taking place since the beginning of the 1990s. The second half of the contribution, with the discussion of the different perspectives of the two disciplines on the common topics of modeling and competence models, finally leads to the identification of the two disciplines as two autonomous and independent sciences.

Key words and phrases: modeling, competence model, educational principles, emancipation.

MSC Subject Classification: 97xxx, 68xxx.

On the genesis of Mathematics- and Computer Science Education

A short longitudinal section of traditional models of Mathematics Education

At the beginning, it should be noted that the following presentations concentrate exclusively on the German-speaking countries.

As early as 1969, the German mathematician Helge Lenné presented an analysis of Mathematics Education in conjunction with a working group for curriculum studies (Lenné, 1969). According to the topic, they conducted their analyses primarily in the context of the grammar school curricula. Special emphasis was placed on the knowledge that can be queried as a track record. The dominance of teaching focuses on detailed knowledge in the form of theorems, formulas and calculation techniques paired with teaching drills.

In his *Basic Questions of Mathematics Teaching* (1974) Erich Wittmann published a very first systematic model of Mathematics Education as science. In the first chapter of his book, Wittmann describes Mathematics Education as a professional science and borderline discipline. Kaufman and Steiner (1969, p. 317), cited in Wittmann, see its position in the field of established disciplines as follows:

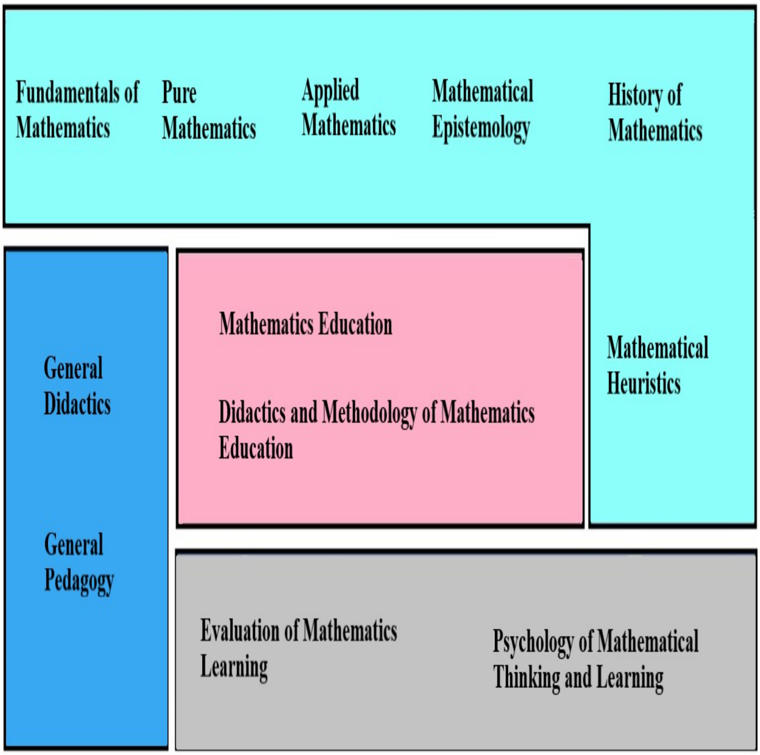

Figure 1. Mathematics Education in the fields of established disciplines

From the publications of the psychologist Jerome Seymour Bruner (1960), Wittmann derives the following didactic principles: *Spiral Principle* (learning takes place over different levels in a retrospective and forward preview);

EIS Principle (concept of multiple presentation; E (= Enactive), I (= Iconic), S (= Symbolic)).

Bruner's *Fundamental Ideas* (strategies that run through all mathematics and mathematics teaching at all school levels and forms) as well as the *Genetic Principle* (learning focuses on the dialogical development of the object of learning and considers the learners as subjects of learning) can be found in the book in chapter 'Methods for the construction of mathematical learning sequences'.

In the 90s of the last century, the German scientist Lutz Führer published his *Pedagogy of Mathematics Teaching* (1997).

The scientist sees Mathematics Education as being disciplined in two ways:

First of all, there is the technical/theoretical side (topics are, for example, subject areas: Geometry, Stochastics or The Use of New Media), secondly, the practical/socially relevant side (attitudes, skills, knowledge).

Führer discusses the Genetic Principle in the chapter 'Education as a process' and the Fundamental Ideas in the chapter 'Education as background knowledge' in his *Pedagogy of Teaching Mathematics.* The idea of *Application Orientation* in the chapter 'Education as foreground knowledge' joins the list of Fundamental Ideas.

What was missing so far was a compact, systematic textbook for education and training teachers. In 2001, the two German scientists Günter Krauthausen and Petra Scherer published a book entitled *Introduction to Mathematics Education*, which is suitable for closing this gap.

Individual aspects are discussed in the subchapter 'Discoveries learning and productive practice'. The discussion is concomitant with explanations in detail – separated into the sections 'Learning: Step-by-step predetermined paths vs. holistically on their own ways' and 'Practice: Reproduction and quantity vs. productivity and quality'.

In the section 'Learning and practising through play', first of all, the importance of playful learning is pointed out. Brain teasers and strategy games that are particularly suitable for promoting general learning objectives are outlined. Indicated Criteria which are helpful for the selection of suitable games are given. In addition to the pedagogical principles already discussed by Wittmann and Führer, Krauthausen and Scherer consider the *Principle of Progressive Schematization* (individual, informal solutions supported by visual aids lead to calculus, regardless of whether they are expressed in images, symbolic notations, language or manifesting actions (Glade 2011)) as well as the *Principle of Prior Knowledge Orientation* as worthy of mention.

In 2013, the German scientists Kristina Reiss and Christoph Hammer published their *Fundamentals of Mathematics Education*. On the one hand, the focus of their presentations is on mathematics teaching (objectives of the lessons, the framework and, for the first time, *Educational Standards* in the context of the potential of tasks (see Fuchs, 2021). On the other hand, the basic techniques of working in mathematics (modeling, proving, arguing), the educational principles (Genetic Principle, Spiral Principle and Principle of Action Orientation) as well as the diagnosis of and dealing with errors are discussed in the book.

The recent work on Mathematics Education is the *Handbook of Mathematics Education* published in 2015 by the authors Regina Bruder, Lisa Hefendehl-Hebeker, Barbara Schmidt-Thieme and Hans-Georg Weigand. The numerous individual contributions by a wide variety of authors, divided into five parts:

- Mathematics as an Object of Education (pp. 3–76)
- Mathematics as Teaching and Learning Content (pp. 77–254)
- Mathematics as Thought Processes (pp. 255–410)
- Mathematics in the Teaching Process (pp. 411–538)
- Mathematics Education as a Research Discipline (pp. 539–662),

provide a multifaceted picture of Mathematics Education.

The emancipation of Computer Science Education as a separate discipline by concepts of Computer Science Education

A first step towards the emancipation of Computer Science Education as an independent discipline was taken in 2007 and 1989 with the publications 'Projection, computer use – Two fundamental ideas and their significance for Geometric Drawing lessons' by Karl Josef Fuchs, as well as *Fundamental Ideas of Computer Science in Mathematics Education* by Petra Knöß (1989).

In his doctoral thesis, Karl Josef Fuchs considered the use of IT to be a Fundamental Idea. In her dissertation, Petra Knöß considered the development and application of algorithms as a Fundamental Idea.

At the beginning of the 1990s, basic information and communication technology education was introduced in Austria's schools at the lower secondary level. In the accompanying material *Computer Science Education – Information and Communication Technology Basic Education* (1990), Albert Rieder and Anton Reiter presented the structure of this educational initiative. It was a model consisting of the three pillars:

- Information Technology Literacy in the Handling of Computers (in all subjects),
- Professional and Application-Oriented Training (in the subjects Geometric Drawing, Mathematics, German and English) (Neuwirth, 1998; Fuchs, 1990),
- Computer Science as an Independent Subject (Fuchs & Stöckl, 1991).

In 1994, Helmut Caba and Karl Josef Fuchs published didactic prerequisites as a reaction to the highly heterogeneous structure of Computer Science teaching in schools in their “Attempt at a methodology and didactics of computer use in teaching” (1992).

The further emancipation of Computer Science Education was accomplished in 1993 by Andreas Schwill, and in 1994 by Karl Josef Fuchs, with their publications “Fundamental ideas of Computer Science” and “Computer Science Education: The logic of fundamental ideas”. Schwill’s contribution were primarily the *Criteria of Fundamental Ideas.*

The *Horizontal Criterion* is described by Schwill as thinking principle with “a widespread applicability in multiple areas, and they integrate and put the multitude of phenomena in order”.

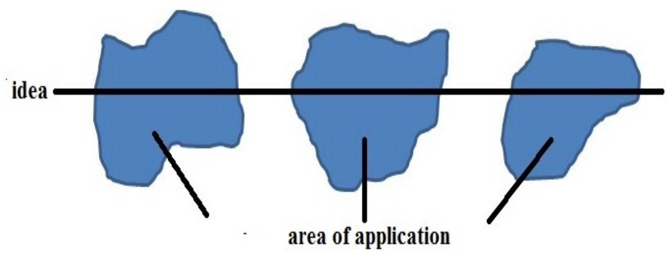

Figure 2. Schwill’s Horizontal Criterion

The *Vertical Criterion* is described by Schwill as thinking principle that “structures the contents within an application vertically”, which means “fundamental ideas can be communicated at nearly any arbitrary level (from primary level students to university level students) successfully”.

Following Schwill’s list of Fundamental Ideas as thinking principles they need not only have *wideness* expressed in the Horizontal Criterion and *richness* expressed in the Vertical Criterion but “must have [...] an anchorage in day-by-day thinking”, and own “realms of life relevance”. This attribute is labelled by *Criterion of Sense.*

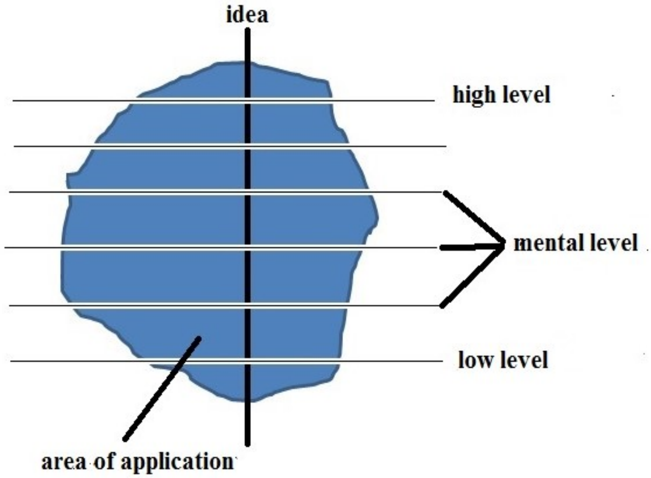

Figure 3. Schwill's Vertical Criterion

Finally the thinking principles must own a historical dimension. Schwill expresses this characteristic as *Criterion of Time*.

Fuchs's contribution was the discussion of Fundamental Ideas *Data and Relationship Structures*, *Modularization* and *Modeling*.

In his contribution to Anton Reiter's book in 2003, Rüdeger Baumann coined the term IT system, i.e. "systems that represent knowledge of different types and origins, these knowledge representations in the form of process data and programs and make them available to users in a suitable form", and he understands Computer Science as "the science of the draft and design of IT systems" (Baumann, 2003, p. 63).

Additionally, he formulates the following three guidelines for teaching Computer Science:

- Problem Solving with IT Systems,
- Principles of Action of IT Systems,
- Fundamentals and Limits of Informatical Knowledge Processing.

In 2003, in view of the heterogeneous implementation of teaching Computer Science in school, Fuchs posed the question of Computer Science competencies in his publication "School Informatics, quo vadis?" (see also chapter 'Competency models of Mathematics and Computer Science').

At the Conference on *Informatics in Secondary Schools – Evolution and Perspectives* (ISSEP) at the University of Klagenfurt in 2005, Karl Josef Fuchs presented the Boundary Structure Educational Model of different subjects (see Figure 4):

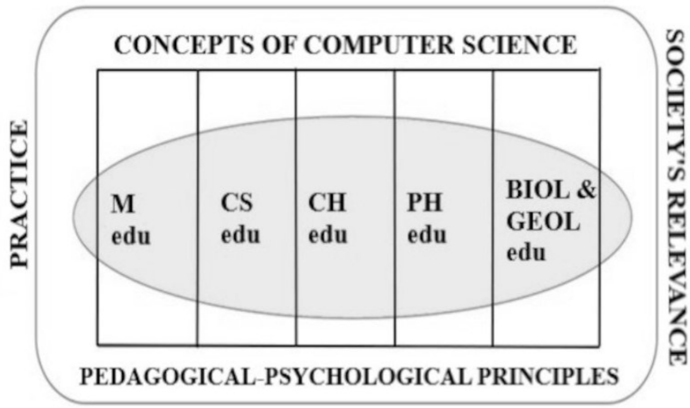

Figure 4. Fuchs's Boundary Structure Educational Model (Fuchs, 2005)

Fuchs answered the question of the boundaries of the didactics of the individual subjects in the following way: On the one hand, the boundaries must not be drawn too strictly due to the overlapping of numerous topics, on the other hand, the demarcation of boundaries is unavoidable due to the 'independence' of the didactics of the individual subjects through their own perspectives (see the next chapter 'Common topics – different points of view'). What remains is eminent potential for further educational discussions.

The previous descriptions are followed by the individual textbooks on Computer Science Education.

In 2006, the German scientist Ludger Humbert presented his *Computer Science Education With Tried-and-Tested Teaching Material.* In ten chapters, the book covers the topics

- Computer Science – Formation and Development of the Subject (contents, methods),
- as the subtitle of the book says, the Tasks,
- the Preparation, the Planning of Lessons as well as Performance Measurement.

In 2007, the German scientist Peter Hubwieser published his *Computer Science Education – Basics, Concepts, Examples.* The focus of his book is on the Model of Comprehensive Computer Science Education (also affectionately called *Hubwieser's triangle*). The vertices of the triangle are named with the *use of medium and learning aid*, *on-job-training of using* and *mastering basic concepts.*

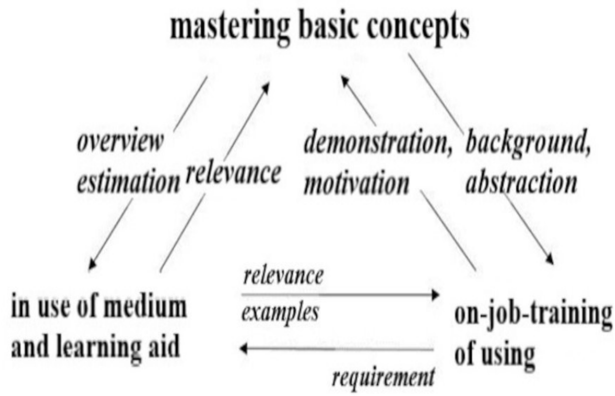

Figure 5. Model of Comprehensive Computer Science Education (Hubwieser's triangle) (Hubwieser, 2007)

In 2011, Sigrid Schubert and Schwill Andreas presented their *Computer Science Education.* In addition to the Criteria of Fundamental Ideas already described, the two authors among others discussed in this book in a total of eleven chapters

- What is Computer Science? (Contents, Objectives, Teaching Methods)
- The Legitimation of Computer Science as a School Subject. The authors concretize the legitimation in three questions:
  - Plans (How can we solve problems through planned development, design and application of IT Systems?),
  - Languages (What are the possibilities and what are the limits of formal language knowledge processing?),
  - Systems (How are IT Systems structured, what are the principles of the interaction of their components and how do they fit into larger system contexts?).
- Problem Solving and the Paradigm of Object Orientation.

The book also contains an embedding model of Computer Science Education in other disciplines, similar to Fuchs's model, which he presented at ISSEP in 2005.

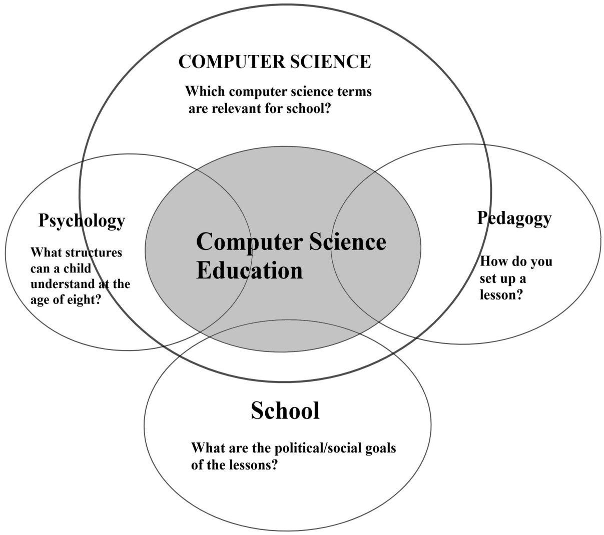

Figure 6. Schubert and Schwill's boundary structure model (Schubert & Schwill, 2011)

In 2016, Eckart Modrow and Kerstin Strecker presented their *Computer Science Education*. In the *Educational Framework*, the authors deal with

- Fundamental Ideas (whereby the ideas of Algorithmizability, Contextualizability, Digitizability, Networking and Realizability are added to the well-known *Idea of Modeling*, which is discussed in detail in the chapters 'Fundamentals of Computer Science' and 'Programming'),
- Aspects of Learning Theory,
- Tasks in the Context of Competencies.

Database Systems and Data Models as well as Networks together with teaching examples and their *methodology* are dealt with in the chapter 'Computer Science and society'.

Common topics – different points of view

Each discussion of the following topics will be introduced by a question.

Modeling in Mathematics and Computer Science

What are the different points of views of Mathematics and Computer Science on modeling?

The authors Karl Josef Fuchs, Claudio Landerer and Simon Plangg describe the differences in the point of view of modeling in the books *Mathematics and Computer Science Education and Methodology* (Fuchs & Landerer, 2021) as well as in *Teaching and Learning Medium Computer* (Fuchs & Plangg, 2022) as follows: The point of view on the Idea of Modeling in Mathematics can be simplified schematically in a three-step way:

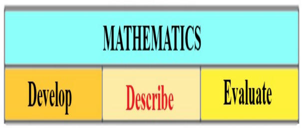

Figure 7. Mathematical Modeling

- Step 1: *Develop* in Mathematical Modeling. This explicitly means Constructing, Understanding, Simplifying, Structuring and Abstracting.
- Step 2: *Describe* (*translate* into the language of mathematics) in the Mathematical Modeling Process. This is explicitly called Mathematization and Mathematical Work.
- Step 3: *Evaluate* in the Mathematical Modeling Process. This explicitly means interpreting, Arguing, Documenting, Expounding, Explaining, and Criticizing.

Andreas Schwill and Marco Thomas formulate the Mathematical Modeling Process as follows: The "Models of Mathematics" are "in the end, mostly symbolic" (Schwill, 1995, p. 24). They form as "symbols, descriptive, static structures" (Thomas, 2000, p. 41).

The point of view on the Idea of Modeling in Computer Science can be simplified schematically in a three-step way, too:

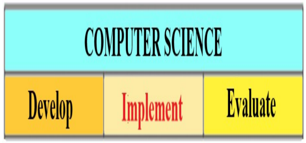

Figure 8. Computational Modeling

- Step 1: *Develop* in the Computational Modeling Process. This step is equal to Step 1 in the Mathematical Modeling Process.
- Step 2: *Implement* (programming the computer) in the Computational Modeling Process. Coding (in a programming language or subject-specific application), explicitly simulate and play are meant.
- Step 3: *Evaluate* in the Computational Modeling Process. This step, too, is equal to Step 3 in the Mathematical Modeling Process.

Marco Thomas describes the goal of Computational Modeling as the creation of enactive models for the "mastery and use of complex processes and structures" (Thomas 2000, p. 41).

What modeling paradigms does computer science provide for the implementation of models?

In this article, we would like to mention *Procedural* and *Functional Modeling* as paradigms (Fuchs & Plangg, 2020, Chapter 2.3 'Programming with a Computer Algebra system', pp. 21ff).

By Procedural Modeling, we mean a program as a sequence of instructions. Control structures (Sequence, Branching, Repetition) describe the execution of each statement.

Individual language elements for implementation according to the Functional Paradigm are functions in a strict mathematical sense. The execution of programs is essentially a concatenation of the individual functions.

Each modeling paradigm is now illustrated by a coding example.

Example: *The Interval Halving Method*

Restriction: The real function f, the zero x of which we determine on an interval $[a, b]$ which is a subset of the x-*axis*, is strictly monotonically growing over the entire interval.

In a *first step*, we choose our interval in such a way that the values of the function at the endpoints of the interval are: $f(a)$ is negative and $f(b)$ is positive.

In the *next step*, we determine the halving point $H1$ in $[a, b] : H1 = ((a + b)/2, 0)$.

Now, if $f((a + b)/2)$ is negative, the new interval we are looking at is $[(a + b)/2, b]$. But if $f((a + b)/2)$ is positive, so the new interval is $[a, (a + b)/2]$.

We also halve this interval again and determine the halving point $H2$. Again, we make sure that for the newly formed interval, the function value for the lower bound of the interval is less than zero and for the upper bound of the interval is greater than zero.

Abort condition: We repeat the procedure as long as the absolute value of f of the abscissa of Hi with $i = 3, 4, \dots$ is bigger than s. As s being a positive real number, we denote a given error bound therewith. An approximation of $(a+b)/2$ must be output when the absolute value of f of the abscissa of Hi with $i = 3, 4, \dots$ equals the given error bound or goes below it.

```
Define bisection()=
Prgm
  Request "Lower Bound a: ", a
  Request "Upper Limit b: ", b
  Request "Error barrier s: ", s
  While |f((a+b)/2)| > s
      If f((a+b)/2) < 0 Then
        a := (a+b)/2
        b := b
                    Else
        a := a
        b := (a+b)/2
      EndIf
  EndWhile

... Disp "(Approximate) Zero: ",approx((a+b)/2)
EndPrgm
```

Figure 9. Interval Halving Method in Procedural Modeling

```
Define bisec(a,b,s)
Func
If |f((a+b)/2)| > s Then
  If f((a+b)/2) < 0 Then
    bisec((a+b)/2,b,s)

                  Else
    bisec(a,(a+b)/2,s)
  EndIf
                Else
  approx((a+b)/2)
EndIf
EndFunc
```

Figure 10. Interval Halving Method in Functional Modeling (Programmed with the TI-Nspire)

```
=IF(1+INT(SQRT(81-ROW()^2))=COLUMN();"*";"")
```

Figure 11. Coding

	A	B	C	D	E	F	G	H	I
1									*
2									*
3									*
4									*
5								*	
6							*		
7						*			
8					*				
9	*								

Figure 12. Discretized Quarter Circle

The example coding in a *subject-specific application* applying the *Idea of Discretization* is taken out of *Computer Science Education* (2007) of Peter Hubwieser, who proposes the use of a spreadsheet for 'exaggerated raster representation' of geometric objects.

Competency models of Mathematics and Computer Science

What are the differences in the competency models of Mathematics and that of Computer Science?

The Mathematics Competence Model of Austria is two-dimensional for Vocational Secondary Schools (Fuchs, 2013, p. 228, Figure 1), and three-dimensional for General Secondary Schools (Fuchs, 2023, p. 9, Figure 8):

- (Dimension 1) *Operative Dimension.* General Secondary Schools – Secondary Level I and II: The items of the dimension cover the topics Representation and Modeling, Calculating and Operating, Interpreting, Arguing and Justifying. // Vocational Secondary Schools: The items of the dimension cover the topics Modeling and Transferring, Operating and Use of Technology, Interpreting and Documenting, Arguing and Communicating.
- (Dimension 2) *Content Dimension.* General Secondary Schools – Secondary Level I and II: The items of the dimension cover the contents Numbers and Measures, Variable and Functional Dependence, Geometric Figures and Bodies, Statistical Representations and Parameters for Secondary Level I; and Algebra and Geometry, Functional Dependences, Analysis, Probability and Statistics for Secondary Level II. // Vocational Secondary Schools: The items of the dimension cover the topics Numbers and Measures, Algebra and Geometry, Functional Relationships, Analysis and Stochastics.
- (Dimension 3) *Complexity Dimension.* Only General Secondary Schools: The three items of the dimension cover the topics Applying Basic Knowledge and Abilities (low level), Making Connections (mean level) and Reflection (using reflection knowledge = high level) (see, Fuchs, 2021).

Very similar to the Austrian model is the model published by the German KMK (*Standing Conference of Ministers of Education and Cultural Affairs*) in 2023. This model is also three-dimensional and consists of the dimensions Process-Related, Content-Related Competencies and Requirement Areas (KMK, 2023, p. 16):

- (Dimension 1) *Process-Related Competencies.* The items of the dimension cover the topics Mathematically Arguing, Communicating, Modelling and

Depicting, Solving Problems Mathematically, Dealing with Mathematical Objects and Working Mathematically with Media.

- (Dimension 2) *Content-Related Competencies.* The items of the dimension cover the Guiding Principles of Data and DataChance, Space and Form, Structures and Functional Context, Quantities and Measurements, as well as Number and Operation.
- (Dimension 3) *Requirement Areas.* The individual levels of this dimension are called Reproducing (low level), Making Connections (mean level), Generalizing and Reflecting (high level).

A competency model of Computer Science was already published by Karl Josef Fuchs and Claudio Landerer in 2005. Against the background of a different approach to mathematics apart from Modeling (for a different point of view, see Modeling in Mathematics and Computer Science) and Communicating, they formulated the following two-dimensional Competence Model:

- (Dimension 1) *Content Components*:
  - *System Competence* (Structuring, Functioning, Limitations and Effects of IT Systems)
  - *Application Competence* (User Systems: Documenting, Publicating, Calculating and Presentating)
  - *Communication Competence* (Communicating, Knowledge Organization with IT Systems)
  - *Problem Solving/Modeling* Competence (applying subject-specific IT Systems).
- (Dimension 2) *Action Components* (based on the Bloom's taxonomy, see (Fuchs, 2023, p. 8 Figure 6)):
  - *Literacy* (Knowledge, Reproduction)
  - *Skills* (Applying, Understanding)
  - *Creativity and Cognition* (Designing and Explaining)
  - *Evaluation* (Evaluate).

Another competency model was published in Austria in the course of the Introduction of Digital Basic Education in the Lower Secondary Schools and Middle Schools. The digi.comP Version 2016 Competency Model is described as a circle divided into eight sectors.

These sectors include the following areas:

- A: Digital Literacy and Computer Science Education
- B: Living, Teaching and Learning under the banner of Digitality; Elevation of Technology Ethics; Media Education and Biography; Accessibility
- C: Designing and Modifying Materials for Teaching; Use of Works and Copyright
- D: Planning, Implementation and Evaluation of Teaching and Learning Processes with Digital Media and Learning Environments; Formative and Summative Assessment
- E: Subject-specific Use of Digital Media, Software and Digital Content
- F: Promoting the Digital Competences of Learners
- G: Efficient and Responsible Class and School Administration; Communication and Collaboration in the School Community
- H: Lifelong Learning; Further Education and Training with or on Digital Media.

In 2008, the German Informatics (GI) Society presented its Competency Model (see Figure 13) in the course of the discussion on educational standards in all subjects.

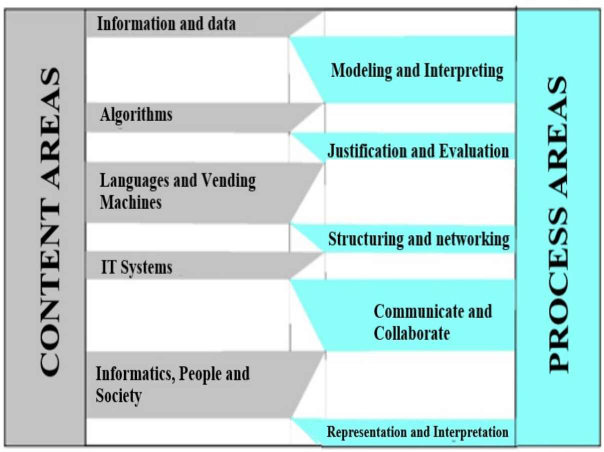

Figure 13. GI Competency Model

State of the art

With the 21st century came the emancipation of computer science education from mathematics education.

In 2009, Karl Josef Fuchs and Hans-Stefan Siller described this emancipation as a process between theory and practice as follows:

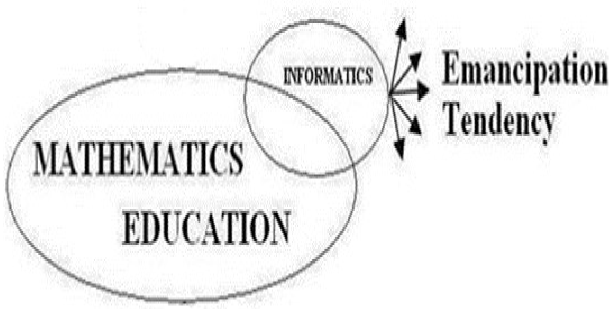

Figure 14. Process of emancipation (Fuchs & Siller, 2009)

For this reason, the two disciplines must be regarded as well-established, independent subjects.

This independence was accompanied by the emergence of its own research communities. People from these communities have given presentations at national and international conferences (M: Konferenzen der Gesellschaft für Didaktik der Mathematik [*Conferences of the Society of Mathematics Education*, *International Commission on Mathematical Instruction* (ICMI)] / Inf: *International Conference on Informatics in Schools: Situation, Evolution and Perspectives* (ISSEP), *International Federation for Information Processing Conferences* and *EUROLOGO*). Additionally, they have published in journals (M: *Journal of Mathematics Education*, *Zentralblatt für Didaktik der Mathematik* [*The International Journal on Mathematics Education* (ZDM)] and *Beiträge zum Mathematikunterricht* [*Contributions to Mathematics Education*] / Inf: *LOG IN Informatische Bildung und Computer in der Schule* [*LOG IN Computer Science Education and Computers at School*]. The publications were essentially strongly practice-oriented, exemplary individual solutions for the design of computer science lessons.

In addition, teaching and research in the two disciplines were established at German-speaking universities through the establishment of special departments, for example at the Paris-Lodron University Salzburg or the Friedrich Schiller University Jena. Since that time, these departments are also responsible for the educational components of teacher training.

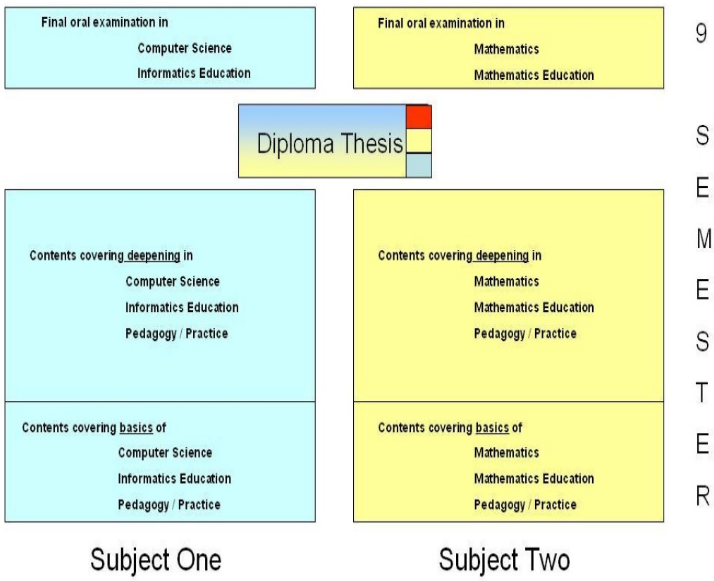

Figure 15. Course Scheme of the Combination Informatics and Mathematics (Fuchs, 2008, p. 53)

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