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Fostering engineering freshmen's shifts of attention by using Matlab LiveScript for solving mathematical tasks

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Abstract. We designed an experimental path including a summative assessment phase, where engineering freshmen are involved in solving mathematical tasks by using Matlab LiveScripts. We analyzed the students' answers to a questionnaire about their perceived impact of the use of Matlab on their way to solve mathematical tasks. The main result is that students show shifts of attention from computations to other aspects of problem solving, moving from an operational to a structural view of mathematics.

Key words and phrases: mathematics education, shifts of attention, University Transition, Matlab.

MSC Subject Classification: 97U70, 97H60.

Introduction

In this paper, we report the outcomes of an experimental path aimed at fostering a shift of attention (Mason, 1998) of engineering students from an operational approach and view of mathematics to a structural one. This shift, intended as a change of focus when a subject takes actions, is linked with the development of specific forms of awareness.

According to Sfard (1991), learning mathematics is a progressive construction of conceptions. Each conception includes two complementary views: a structural one, according to which the conception is an abstract object, and an operational one, according to which it consists of processes, algorithms, and actions. The author also stresses that to understand mathematics deeply, it is crucial to develop the ability to handle both the structural and operational views and flexibly go back and forth between them, independently of the definition of understanding we assume. In particular, the construction of structural conceptions goes through the acquisition of operational conceptions. It can be achieved in a hierarchical three-step path: (1) *interiorization*, where the student becomes able to perform procedures, first on more and more low-level objects and then just mentally performing; (2) *condensation*, where the learner can pack long sequences of operations into a block, no longer needed to fully detail; and finally, (3) *reification*, where a new 'conception' arises.

Recent literature (e.g., Jaworski et al., 2012) and our experience as teachers of mathematical courses for engineering freshmen suggest that typically students, mainly those attending applied studies, tend to have an operational view of mathematics and to perceive the correctness of computations and procedures as the core of the discipline.

It is worthwhile to notice that the teacher has a crucial role in making the students orient their attention like an expert in the discipline. This can be pursued by creating conditions and learning environments in which students could experience suitable shifts of attention, allowing them to focus on some structural aspects rather than (only) on operational ones and to become aware of specific mathematical facts (Mason, 1998).

Research hypothesis

This study relies on the hypothesis that engineering students are focused mainly on calculations and procedures when involved in mathematical activities. This focus is also linked with their sense of self-efficacy: typically students perceive to be good at math if they are able to do exact computations. According to this hypothesis, which is in tune with recent literature about the students' beliefs and views of mathematics (Jaworski et al., 2012), we designed an educational path, involving also the aspect of summative evaluation, aimed at fostering a shift of attention from the procedures (linked to the operational level) to the strategies (linked to the structural level), when our students face mathematical tasks.

To this aim, we introduced the use of Matlab LiveScript, which is an interactive environment that allows users to integrate code and its outputs with texts and images. This Matlab LiveScript feature allows the student to combine the verbal description of the solving process of the task with black-box instructions (Buchenberg, 1990) to carry out computations. This shift is in tune with Sfard's condensation step.

Our research question is:

Q1) *How do the students perceive the impact of the use of Matlab LiveScript on the possible shift from the operational approach to mathematics towards advanced mathematical thinking linked to a structural approach, too?*

Methods

Study setting

The study, which is part of a wider research about the use of Matlab Livescript to support summative assessment (Albano et al., 2023), was carried out with Computer Science Engineering freshmen of the University of Salerno attending the course of Geometry, Linear Algebra, and Logic, taught in the academic year 2020-21. Attending the course lessons is mandatory. The experimental path involved only the part of linear algebra.

Students have been introduced to the use of Matlab LiveScript after the introduction of matrices and linear systems. The use of LiveScript is based on the idea of considering matrices and systems as blocks of basic operations that are useful to achieve different goals. In other words, matrices and systems are considered tools to solve problems where, according to the context, they assume different meanings. In this respect, students have been expected to be able to handle linear algebra problems both at a low level, which means performing by hand all the needed computations, and at a higher level, which means making use of Matlab functions in Matlab LiveScript. Thus, the assessment concerned both capabilities.

In this study, we refer to the 174 students who took the examination in the first session.

The exam constituted of two open-ended tasks requiring the same operations on matrices and linear systems (calculation of the dimension of a vector subspace, extraction of basis, and representation of vector subspaces). The first task was to be performed in paper-and-pen style, but was to be delivered through a Moodle text box; whilst, the second task was to be performed through the use of Matlab LiveScript. According to our research goal, both tasks required students to describe the problem solving process they carried out. We notice that the Moodle text box restricts the possibility of using advanced mathematical symbols and formalism (e.g., the integral sign), but this is not a strong constraint for linear algebra.

The proposed tasks are shown in Table 1.

Task to be delivered by Moodle text-box	**Task to be performed by using Matlab**
Solve in a correct, complete, clear and efficient way the following task, progressively explaining the solving process and justifying it with suitable theoretical results. Let f be homomorphism with the canonical representation given by the following matrix $A = \begin{pmatrix} 1 & 3-h & 2 \\ h & -2 & 4h \\ 0 & 1 & -h \\ -1 & -1 & 1 \end{pmatrix}.$ 1. What values of h make f injective? 2. For the values of h making f not injective, compute the dimension and find a basis of $\mathrm{Im}(f)^\perp$. 3. Given three vectors $u1$, $u2$ and $u3$ and their images $g(u1)$, $g(u2)$ and $g(u3)$, find such conditions that the linear extension of g: i. exists and it is unique; ii. exists but it is not unique; iii. does not exist.	Solve in a correct, complete, clear and efficient way the following task, progressively explaining the solving process and justifying it with suitable theoretical results. Given the matrix $A = \begin{pmatrix} 0 & -15 & 120 & -60 \\ -90 & 15 & -360 & 180 \\ -18 & -6 & -12 & 6 \\ -36 & -12 & 36 & -18 \end{pmatrix}.$ 1. Establish if A is diagonalizable in R or in C. 2. For A diagonalizable in R, compute at least two diagonalization matrices and the corresponding diagonal matrices. 3. Let f be the homomorphism having A as the canonical matrix of representation; establish if f is injective and, if it is not the case, compute the dimension and a basis for $\mathrm{Ker}(f)$.

Table 1. The Moodle task and the Matlab task

Both tasks address the same aspects:

(a) fundamental vector subspaces associated with a homomorphism f, i.e., $\mathrm{Im}(f)$ and $\mathrm{Ker}(f)$;

(b) injectivity of a homomorphism;

(c) Cartesian representation of a vector subspace;

(d) basis of a vector subspace.

In particular, as regards point (c), the Cartesian representation of $\mathrm{Im}(f)^{\perp}$ is explicitly required in the Moodle task, and the Cartesian representation of $\mathrm{Ker}(f)$ is implicitly required for the computation of its basis in the Matlab task; as regards point (d), a basis is implicitly required to find the Cartesian representation of $\mathrm{Im}(f)^{\perp}$ in the Moodle task, and it is explicitly required for $\mathrm{Ker}(f)$ in the Matlab task.

The main difference between the two tasks concerns the task data. We chose the data of the task to be performed by Matlab as hard to handle without computational tools (high order of matrices, with big numbers). This aspect was designed on the one hand to foster the development of students' awareness of the available Matlab tool potential and, on the other hand, to induce their shift of attention from the calculations to the solving process.

Post-reflective survey

After the students submitted their examination test, they were asked to complete an anonymous questionnaire delivered through the Moodle Feedback tool. It consisted of the following four open questions.

(1) What kind of the two tasks do you prefer? Why?

(2) What differences did you grasp between the two kinds of tasks?

(3) In your opinion what are the pros and cons of the use of Matlab LiveScript to solve the task?

(4) About what kind of task do you feel more confident of your answer?

We underline that the questions were intentionally open and general to focus on the ways the students perceived the experience of the use of Matlab LiveScript (Connelly & Clandinin, 1990) and to highlight their shift of attention. This choice aimed to avoid influencing the students' answers and fostering their freedom to talk about the elements they think are the most significant ones (Di Martino & Zan, 2010).

Data analysis and research methodology

In tune with the research question, we analyzed the answers of the students highlighting their shifts of attention, revealed by the use of specific verbs or expressions such as "focusing on" or "concentrating on".

In tune with our previous study (Albano et al., 2023), we expected that the students' release of calculations would allow them to focus on other aspects;

in particular, we expected the emergence of the following categories of shifts of attention, that can be linked to features of structural mathematical thinking:

- *shift toward the theory*: the student describes his/her use of Matlab highlighting the opportunity to focus on theoretical aspects;
- *shift toward the solving strategy*: the student describes his/her use of Matlab highlighting the opportunity to focus on thinking about the overall solving strategy of the problem;
- *shift toward the reasoning*: the student describes his/her use of Matlab highlighting the opportunity to concentrate on the reasoning needed to identify how to proceed in the solving process from the procedural point of view;
- *shift toward the justification*: the student describes his/her use of Matlab highlighting the opportunity to focus on the explanation needed to justify the procedural steps of the solving process.

The researchers independently read the students' answers and coded them according to these categories of shifts until an agreement was reached (Cohen et al., 2018).

Results

Exactly 162 students answered the questionnaire, even if 17 students did not answer Question 2, and 9 did not answer Question 3. We carried out quantitative and qualitative analyses of the students' answers to face the research question. The outcomes show that the students largely prefer the Matlab task, even if the preference is not so large for the level of confidence in the answer (see Table 2).

	Matlab	Moodle	the same or N/A
What task did you prefer?	123	18	21
About what kind of task do you feel more confident of your answer?	106	19	39

Table 2. Quantitative analysis of Questions 1 and 4

The analysis of the whole questionnaire was carried out according to the expected kinds of shifts of attention, described below. From a quantitative point of view, 43,55% of the students reveal a shift toward the justification, 35,48% a shift toward the reasoning, 14,52% a shift toward the theory, and 6,45% a shift toward the solving strategy. In the next section, we will look at in-depth at all the students' answers by a qualitative analysis.

Shift toward the theory

The students described their use of Matlab highlighting the opportunity to focus on theoretical aspects, as in the following prototypical excerpt: *I quickly solved the problem since I did not make calculations, but only applied theoretical concepts and formulas and let the software make the rest.*

In some cases, it is underlined that the software executes the computations and this reduction of the cognitive load allows to shift the focus on theoretical aspects, concepts or algorithms to be applied to the specific task.

The students' comments suggest that the opportunity of using Matlab LiveScript enables the students to see the whole solving process from a global perspective, since the code encapsulates the procedural aspects in a black-box.

Shift toward the solving strategy

The students described their use of Matlab highlighting the opportunity to focus on the overall solving strategy of the problem, as in the following prototypical excerpt: *This way to solve the problem allowed me to focus on the relevant aspects, i.e., the solving strategy, without any worry about calculation mistakes.*

Generally, the students highlighted that thanks to the calculations done by the software, they could save time and avoid worries about mistakes. They added that *the problems can be solved in an effective way; moreover, this method induces to better understand the solving process and to learn the syntax and semantic of Matlab.*

The students' comments suggest that the use of Matlab induces the perception of a more effective problem solving process, focusing on its understanding rather than the operational details.

Shift toward the reasoning

Some students declared that the use of Matlab allowed them to focus on the reasoning behind the calculation, as in the following prototypical excerpt: *I solved* *more quickly the problem using Matlab, focusing mainly on the reasoning inducing the calculations rather than on the calculations.*

The excerpt suggests that calculations are perceived as the most boring and least interesting part of the solving process (*Matlab greatly facilitates the calculations, which in my opinion are the most boring and least interesting part of the problem*). This seems to concern the high-level shift in the view of the discipline, since typically engineering students perceive the calculation aspect as the most important in the mathematical activity (see the Introduction). Moreover, the calculations are sometimes perceived as a source of worry and the use of Matlab avoids it ([Matlab] *avoids any worrying about computational mistakes*).

Shift toward the justification

Some students described their use of Matlab highlighting the opportunity to focus on the explanation needed to justify the procedural steps of the solving process. The following prototypical excerpt refers to the opportunity to make explicit the solving process, either for what concerns the procedural steps or the comments to explain them: *The solving process of the problem by using Matlab was quicker, allowing to better explain the intermediate steps.*

In some students' comments, further awareness emerges about the potential of the tool with respect to the kind of the proposed problem either for fostering a high-level view of the whole solving strategy and for the communication: *The LiveScript was perfect for the kind of problem to be solved. It allowed solving the problem by adding the needed comments to clarify the strategy/solving process.*

Discussion

In this section, we discuss some effects of the different kinds of shifts of attention emerging from the students' answers to the survey.

From the qualitative analysis we carried out, some specific elements arose, concerning communicative, metacognitive and affective aspects.

Communicative aspects

According to Sfard (2008), mathematical activity can be conceived as a historically established discourse, and learning mathematics means taking part in this discourse, which can be with ourselves (when we are thinking) or with others.

From the communicative perspective, the analysis outcomes allow us to highlight two different aspects.

On the one hand, the students commented about the writing opportunities offered by the Moodle platform and Matlab LiveScript to make clear the steps of the solving process, also taking advantage, within the Matlab environment, of the use of mathematical symbols: *The advantages* [offered by Matlab LiveScript] *are the quick writing and the opportunity of using a more mathematical language, including formulas and symbols. Instead, Moodle does not allow to write symbols, so it is difficult to explain the algebraic solving process of a task.*

On the other hand, some students focused on a meta-level aspect. They put the attention on the more clear communication, in terms of writing and reading, i.e., for the solver and the reader, of the solving process as a sequence of high-level steps. Students used terms such as visualization, clarity, fluency, and intuitiveness to indicate how the solving process is perceived by themselves or others. This is in tune with Sfard's discursive approach recalled before.

Some students underlined the Matlab feature of visualizing the computations in a more convenient manner, i.e., step by step, or more condensed: *more quickness and effectiveness in calculations, with more organization*; *the Matlab computations are very quick and visibly clearer.*

Some students went more in-depth, grasping the opportunity offered by Matlab to regard the solving process as *more fluent, quick and intuitive.*

This seems to suggest that the students tended to achieve the typical vision of an expert mathematician about the solving process of the task.

In the next excerpt, the student noted the usefulness of Matlab LiveScript for communicating the solving process, which sometimes takes the features of discourse in text and code alternation. Indeed, the student of the next excerpt highlighted the discourse at two levels: with herself, when she focuses on the monitoring activity; with the others, when she refers to the writing of the solving process: *Matlab allowed me to monitor the calculations and to be sure of their correctness. Moreover, the alternation between text and code allowed me to clearly write the solving process.*

In this perspective, the solving process by using Matlab seems to foster a condensation process (see the Introduction), which can be linked with a better organization of the solution and the clear visualization of the output of each run routine: *The use of some functions allowed to completely solve the task in a few lines.*

Some students showed their strong awareness that the use of Matlab LiveScript made them shift to high-level thinking, inducing them to focus on a strategic level (goal-oriented actions) rather than on low-level thinking (detailed computations), as in the following excerpt: *The emergent idea is a high-level solution, focused more on what to do and to get rather than on the details of how to perform each computation.*

Metacognitive aspects

Some students focused on their feelings about the solving process due to their opportunity to monitor the calculations and the confidence in the correctness of computations: *I have the opportunity of monitoring step by step what I am doing.*

In some cases, students showed to understand that different methods can be used to check the correctness of a solving process. The following prototypical excerpt highlights that the computations being correct makes students change their control strategies, bringing their knowledge into play (What other algorithms do I know? What kind of verification can I do?), as opposed to what students usually do as a first form of control (that is, checking again the computations): *I feel more confident about the Matlab task because I could be sure of the arithmetic correctness applying various equivalent algorithms which allowed me to confirm the results.*

From some excerpts, a different way to handle the time at disposal during the exam seems to emerge. Indeed, the awareness of the potential of Matlab LiveScript, which is quick and allows the implementation of various methods, induces different scheduling for the solving process and planning of the activities.

Moreover, some students underlined that a good knowledge of theoretical notions is needed to effectively use Matlab. At a deeper level, it is grasped that the intertwinement between theoretical knowledge and the Matlab potential makes the student free and aware during the solving process, as the following excerpt testifies: *By using Matlab I could quickly control the obtained results by applying various alternative methods. If you have a good theoretical basis, it is possible to freely and consciously use various tools and algorithms.*

Affective aspects

According to Di Martino and Zan (2010), the attitude toward mathematics depends on three different elements: the emotional disposition, the self-efficacy sense and the view of mathematics. All these aspects are considered in the students' answers.

Concerning emotional disposition and self-efficacy sense, most students put their attention on less anxiety and more confidence in using a tool guaranteeing the correctness of calculations. This is clear in the following two excerpts: (1) *Probably the task to be solved through the use of Matlab makes me more confident, since possibly I know the processes to be carried out, but it is possible to make mistakes on computations. This is especially true during the exam, since it is more likely that it happens when the emotions take over.*

(2) *I feel more confident of my solving process carried out by using Matlab since, when I am in anxiety and feel the time shrinking, I cannot reason on what I was writing and probably I may have miscalculated or at least have taken a longer road.*

Some students grasped that their feeling of more confidence arises from relieving themselves from worry of the correctness of calculations. This relief of the commitment seems to impact on the cognitive level in terms of understanding; moreover, this seems to induce a different approach to the discipline that is generally perceived as less difficult: *Through Matlab it is possible to have a degree of certainty of the reasoning and the results, also making verifications.*

The considered aspects are linked also to the view of mathematics, as shown in the following excerpt, since typically the students facing a traditional task worry of making wrong computations and this fact induces anxiety. This reveals their operational view of mathematics. Instead, the use of Matlab to solve the task fosters a structural view of mathematics, putting the focus on strategic aspects of the solving process: *I prefer to perform the exam by using Matlab, since it allows me to focus on the side of problem solving, which characterizes the Engineering Degree, without dwelling on unnecessary computations to be done by hand.*

Overall aspects

Some further aspects have been highlighted by the students' comments concerning

- the balance between the pros and cons of using Matlab to solve a task:

The students grasped the need of learning the language but also the advantages offered by the use of the software. This clearly emerges from the following excerpts: (1) *I think Matlab is better, since learning the language and guessing the utility of the routines takes little time. The automatic computations allow you to focus only on the solving strategy.*

(2) *Matlab favors the theoretical process, since the calculations are performed by the software, and it is not so hard to learn the software language.*

- links among different opportunities offered by Matlab:

  The various aspects highlighted in the previous subsections should not be considered as separated from each other. Indeed, there are some connections among them. For example, in the following excerpt the ease of monitoring perceived by the student seems to be allowed just by the clarity, readability and organization of the solving process: *In my opinion, Matlab makes the solving process clear, readable and easy to be monitored.*

Conclusion and further research

The outcomes of our analysis suggest that globally the students positively perceived the impact of the use of Matlab LiveScript on their possible shift to advanced mathematical thinking. This is testified by the general students' preference for the task to be performed by the use of Matlab and by the different shifts of attention emerging from their answers. As expected, the relief of the computation induced the students to focus on theoretical aspects, the solving process of the problem, the reasoning or its justification. This is in tune with other ongoing research, based on the analysis of the students' productions concerning the solving processes of the tasks (Albano et al., 2023). These shifts of attention seem to be the effect of three essential causes:

- the opportunity of viewing the solving process as a black-box, according to Sfard's encapsulation;
- the increased time at disposal due to the opportunity to transfer the computations to Matlab;
- the more confidence of the correctness of the computations.

We plan to investigate how much students' perception of confidence in Matlab adheres to their Matlab task performance with respect to the Moodle task. Furthermore, it would be interesting for us to investigate how different kinds of identified shifts can impact students' success in mathematics.

Moreover, some other aspects arose, such as a creative use of the software as a verification tool, possibly also in an examination test. Indeed, on the one hand, the students can check the computations, and, on the other hand, they can implement different methods to check the correctness of their solving process. In this way, the monitoring process shifts to a higher level.

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