Teaching fractions at elementary level in the light of Hungarian mathematics textbooks in Romania

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Abstract. According to the new curriculum in Romania, fractions are introduced in the second grade. The present study analyses Hungarian elementary mathematics textbooks on the topic of fractions focusing on the types of tasks in the textbooks, the significance of representations and the proportion of word problems. Additionally, the paper presents a questionnaire-based research on teachers' opinion regarding the adequacy and sufficiency of the digital materials and exercises related to fractions in the textbooks.

Key words and phrases: teaching of mathematics, teaching of fractions, mathematics textbooks, e-learning.

MSC Subject Classification: 97F40, 97F80, 97U20, 97U50.

Introduction

Fractions constitute an important part of the elementary curriculum. Children often encounter fractions of a quantity in everyday life, thus teaching the concept is essential. The concept of fractions is first introduced by dividing concrete objects into equal parts, followed by dividing plane figures into equal parts by folding, and finally by drawings, shading, or colouring. (Olosz & Olosz, 1999)

The concept of factions is introduced in the second grade, after teaching division, however, a more comprehensive study takes place in the third and fourth-grades. (Centrul Naţional de Politici şi Evaluare în Educaţie, 2023; Manuale Digitale, 2023). From a methodological point of view, it is the various representations

of fractions and solving word problems that facilitate a more accurate understanding of the concept.

Our previous research (Baranyai & Stark, 2011, 2012) focused on the content of the new curriculum and new mathematics textbooks published in Romania in 2011. We interviewed teachers and students regarding the textbooks. We found that the textbooks contained many grammatical errors and few exercises. Over the past 12 years, new textbooks have been published, and we were eager to find out to what extent these have improved. The current study focuses on the teaching of one specific topic, namely fractions.

According to Czeglédy and Kovács (2008), when analysing mathematics textbooks, six aspects should be considered:

I. Is a textbook necessary for teaching mathematics?

II. Usability

III. Structure, content, requirements

IV. Expectations

V. Design

VI. Evaluation

The aim of the current research is to assess how elementary mathematics textbooks interpret the concept of fractions and what types of fraction activities they contain. Thus, the focus is exclusively on the third aspect from those six listed above.

The mathematics curriculum (Centrul Naţional de Politici şi Evaluare în Educaţie, 2023) and the approved mathematics textbooks for elementary grades are available on the website of the Ministry of Education (Manuale Digitale, 2023). These documents show that textbooks fully adhere to the curriculum requirements regarding the teaching of concepts, however, unfortunately, they do not contain sufficient practice material, a problem (Kojanitz, 2008). In the 2020-2021 school year, online education mandated due to the pandemic highlighted the importance of e-textbooks and online learning materials. In our research conducted among university students (Baranyai & Debrenti, 2020), respondents stated that although they did not prefer online education, electronic learning materials and books were more accessible and helped them prepare for exams.

When it comes to science textbooks, an important question is how authors can align scientific knowledge to educational goals (Kojanitz, 2008). According to Koljanitz, textbooks place too little emphasis on teaching problem-solving.

He argues that textbooks should contain more real-world problems and focus more on problem-solving skills.

Solving word problems is a complex task in which students, relying on the text provided, try to find the right operation(s) to solve the problem. Textbooks and workbooks generally contain a large number of closed problems but few open-ended tasks that draw on real-world context and develop children's problem-solving skills (Ambrus, 2018).

Another study (Dıvrık & Pılten, 2019) analysed elementary mathematics textbooks, interviewing 120 teachers on textbooks used in Kazakhstan. It was found that 68.33% of teachers had a negative opinion, while 31.67% had a positive opinion about the mathematics textbooks in use. The majority of teachers find mathematical concepts presented in the textbooks complicated. Respondents claim that there are not enough examples, activities, and exercises in the textbooks. Nevertheless, teachers consider textbooks to be the main sources of information and an important guide for lesson plans.

Research method

One of the aims of this research was to examine how fractions are addressed in Hungarian elementary mathematics textbooks in Romania. The textbooks were analysed in terms of their content, taking into account curriculum requirements. In addition to the content analysis, a questionnaire-based research was conducted among teachers, in which they evaluated issues related to the teaching of fractions, the quality and quantity of digital content and presented their attitudes towards the textbooks. The questionnaire-based research was conducted during the first semester of the 2021-2022 school year.

The content analysis part of the research discusses the current elementary mathematics curriculum and the mathematics textbooks approved by the Ministry of Education for classes where the medium of instruction is Hungarian.

The other part of the research is based on a self-designed online questionnaire, which consists of 18 closed-ended and 3 open-ended questions.

The questionnaire aimed to investigate teachers' perception of the presentation of fractions in the mathematics textbooks they use. As the literature states the importance of real-world problems, the questions focused on the quantity of word problems, the quantity of activities in general, and the quantity of digital content in the textbooks. The open-ended questions investigated how teachers teach fractions, what visual aids they use, how they represent fractions, and what

additional resources they use besides textbooks when it comes to the teaching of fractions.

The questionnaire was completed by 60 teachers, all of them women.

The majority of the teachers surveyed, 47 (75.8%), have a university degree, 8 have a master's degree, three have a college degree, and two individuals graduated from teacher training colleges.

The age distribution of the sample was as follows: most teachers (48.3%) were aged between 41 and 50, 16.7% over 50, and the remaining 30% consisted of teachers between the age of 20 and 40.

Teaching experience	Percentage	Frequency
Less than 2 years	10%	6
Between 2-5 years	6.7%	4
Between 6-10 years	5%	3
Between 11-20 years	15%	9
More than 20 years	63.3%	38
Total	100%	60

Table 1. Distribution of the sample by years of experience

Table 1 shows the distribution of the sample by years of experience. The vast majority of teachers (63.3%) have more than 20 years of experience in the field of education.

35 of the teachers surveyed work in urban areas, while 25 teachers work in rural areas, which corresponds to 58.3% and 41.7%, respectively.

Results

Based on the content analysis and the questionnaire, the following hypotheses were formulated.

Hypotheses

1. Elementary textbooks contain a limited number of digital tasks related to fractions.

2. Chapters in textbooks related to fractions contain fewer word problems and more tasks requiring representation.

3. The majority of teachers surveyed used digital textbooks during online education.

4. Teachers feel that there is a need for more tasks related to fractions in textbooks.

5. A significant portion of teachers would find it useful to supplement textbooks with digital tasks and content.

In what follows, we provide an introduction to the elementary school curriculum related to fractions.

The concept of fractions first appears in the second grade after learning division. Children are introduced to the concepts of halves and quarters, as well as equivalent fractions.

In the third grade, they learn about proper fractions with denominators of 10 or less, as well as unit fractions. They divide wholes into halves, thirds, quarters, ..., tenths, and draw representations of fractions. The curriculum prescribes the use of terms related to fractions: numerator, denominator, and fraction line. Arranging and comparing fractions with the same denominator is also included in the requirements.

In the fourth grade, children become acquainted with fractions with denominators less than 10, 10 and 100, and they draw representations of a tenth and a hundredth.

Fraction classification is also introduced, using terms such as proper fraction, unit fraction and improper fraction. Addition and subtraction operations are performed on fractions having the same denominator.

Table 2 summarises the data from the textbooks that were examined during the research.

It is shown that out of the eight mathematics textbooks, only one textbook (Grade 2 textbook by CD Press) includes specific digital content, while the remaining textbooks are only available as PDF files.

There are a total of five Grade 2 textbooks available on the official website of the Ministry of Education. These are published by CD Press, Aramis and Ed. Didactica si Pedagogica. Similarly to the mathematics textbooks for other grades, Grade 2 textbooks are authored by Romanian authors, and only the textbook by CD Press provides digital content. This textbook also includes a video on fractions that demonstrates the concepts of halves and quarters using plane figures. (Dumitrescu et al., 2015; Bădescu & Radu, 2015; Bălan et al., 2016; Piţilă & Mihailescu, 2015)

Grade	Textbook title	Authors	Publisher	Online practice
Grade 2	Matematika és környezetismeret II. osztály [*Mathematics and Environmental Science Textbook Grade 2*]	Iliana Dumitrescu, Nicoleta Ciobanu, Alina Carmen Birta	CD Press	yes
Grade 2	Matematika és környezetismeret II. osztály [*Mathematics and Environmental Science Textbook Grade 2*]	Anina Badescu, Mihaela Ada Radu	Aramis	no
Grade 2	Matematika és környezetismeret II. osztály [*Mathematics and Environmental Science Textbook Grade 2*]	Mihaela Ada Radu, Rodica Chiran, Olga Pirifita	Ed. Didactica si Pedagogica	no
Grade 2	Matematika és környezetismeret II. osztály [*Mathematics and Environmental Science Textbook Grade 2*]	Constanta Balan, Corina Andrei, Cristina Voinea, Nicoleta Stan	Ed. Didactica si Pedagogica	no
Grade 2	Matematika és környezetismeret II. osztály [*Mathematics and Environmental Science Textbook Grade 2*]	Tudosa Pitilila, Cleopatra Mihailescu	Ed. Didactica si Pedagogica	no
Grade 3	Matematika III. osztály [*Mathematics Grade 3*]	Mihaela Ada Radu, Rodica Chiran	Aramis	no
Grade 4	Matematika IV. osztály [*Mathematics Grade 4*]	Mihaela Ada Radu, Rodica Chiran	Aramis	no
Grade 4	Matematika IV. osztály [*Mathematics Grade 4*]	Mariana Mogos	Art	no

Table 2. Data on the examined mathematics textbooks

Unfortunately, the Grade 3 textbook published by Aramis does not contain any digital material. Similarly, the two Grade 4 textbooks published by Art and Aramis also lack digital content related to fractions. (Radu & Chiran, 2016a, 2016b; Mogoş, 2016)

Based on the above, it can be concluded that the first hypothesis is confirmed, i.e., elementary mathematics textbooks contain a limited number of digital tasks related to fractions.

The second hypothesis focused on the proportion of examined textbooks that contain tasks requiring representation versus word problems in relation to fractions.

Textbook	Number of tasks involving representation	Word problems	Tasks involving operations	Other type of tasks	Total
Grade 2 Aramis	2 (33.2%)	1 (16.6%)	2 (33.2%)	1 (16.2%)	6 (100%)
Grade 3 Aramis	14 (35%)	13 (32.5%)	1 (2.5%)	12 (30%)	40 (100%)
Grade 4 Aramis	18 (39.13%)	13 (28.26%)	2 (4.35%)	13 (28.26%)	46 (100%)
Average percentage	35.77%	19.2%	13.35%	24.82%	100%

Table 3. Classification of tasks in the examined textbooks

Table 3 above shows that the second hypothesis has also been confirmed, as the percentage of word problems (19.2%) is lower than the percentage of representation tasks (35.77%). Among the textbooks examined, the Grade 3 textbook contains the highest percentage of word problems related to fractions. However, it should be noted that the Grade 4 textbook contains a variety of tasks, and due to the curriculum, it includes more operations related to fractions.

In what follows, let us examine the hypotheses related to the questionnaire-based research.

The third hypothesis pertained to the proportion of teachers who used the digital version of the mathematics textbook during the online teaching period in the 2020-2021 academic year.

Results show that 35 (59.33%) of the respondents used the digital textbooks, while 20 (33.33%) did not. 5 teachers did not respond to this question. 28 of the respondents (46.66%) stated that they used the digital textbook several times a week.

The questionnaire also inquired about other materials teachers used during online instruction. Responses indicated that 24 teachers (40%) used self-designed online assignments, 23 teachers (38.33%) used assignments found on the Internet, and 13 teachers (21.66%) used workbooks during online teaching.

Since the vast majority of respondents (59.33%) reported that they used digital textbooks during online teaching, the third hypothesis has been confirmed.

The table below presents the statements regarding the teaching of fractions that teachers were asked to evaluate on a scale of 1-5 (1 – disagree; 5 – strongly agree).

	Statements	Mean (1-5 scale)	Standard deviation
1	I can allocate sufficient time for teaching fractions.	3.19	1.06
2	Children encounter the concept of fractions in everyday life.	3.22	1.059
3	Mathematics textbooks provide an adequate amount of practice related to fractions.	4.50	0.834
4	Word problems are predominant in mathematics textbooks when it comes to fractions.	2.50	0.911
5	Mathematics textbooks contain an adequate number of visuals related to fractions.	2.88	0.885
6	I see a need to supplement textbooks with digital content on fractions.	2.57	0.810
7	Children can fully master the concepts of fractions required by the curriculum by the end of grade 4.	4.53	0.853
8	Teaching fractions posed challenges during online teaching.	3.45	0.428

Table 4. Means and standard deviation of the responses to the statements on teaching fractions

As shown in Table 4 above, teachers feel that textbooks contain sufficient tasks related to fractions (mean 4.5). However, when examining the fifth statement, we can see that teachers moderately agree that there is a need for more visual representation related to fractions, i.e., the fourth hypothesis is only partially confirmed.

Furthermore, a significant number of teachers would find it useful to supplement textbooks with digital content, as indicated by the mean for statement 6. Thus, the fifth hypothesis is partially confirmed.

We wanted to find out whether there were any differences in the responses to the statements when grouping the data according to years of experience. Table 5 shows a difference, the highest mean was obtained for teachers with more years of experience. The difference is statistically significant (p = 0.035).

	N	Mean	Std. deviation	Std. error	95% Confidence Interval for Mean		Min.	Max.
					Lower Bound	Upper Bound		
Less than 2 years	6	2,33	0,816	0,333	1,48	3,17	1	3
From 2 to 5 years	4	2,50	0,577	0,289	1,58	3,42	2	3
From 6 to 10 years	3	1,67	0,577	0,333	0,23	3,10	1	2
From 11 to 20 years	9	2,11	0,601	0,200	1,65	2,57	1	3
Over 20 years old	38	2,79	0,811	0,132	2,52	3,06	1	4
Total	60	2,57	0,810	0,105	2,36	2,78	1	4

Table 5. Distribution by years of experience for the statement "I see a need to supplement textbooks with digital content"

Responses to the statements were also analysed based on the location of teaching. The ANOVA test reported two significant differences for statements 5 ("Mathematics textbooks contain an adequate number of visuals related to fractions", p = 0.04) and 8 ("Teaching fractions posed challenges during online teaching", p = 0.019).

Open-ended questions revealed that most teachers, in addition to mathematics textbooks, use workbooks, self-designed online exercises, and tasks found on the internet. Regarding visual aids, the majority of teachers surveyed said they use pizzas, apples, fruits, cards, puzzles, and chocolate. As regards representing fractions, teachers provided the following answers: segments, collaring, shading, and geometric shapes.

Summary

The aim of this research was to investigate the teaching of fractions at elementary school level, in the light of the curriculum and mathematics textbooks, as well as from teachers' perspective. The content analysis revealed that current Hungarian mathematics textbooks teach fractions through a variety of tasks, most

of which emphasise the representation of fractions, with only a few word problems and very little digital content.

The survey, which contains the responses of 60 teachers, also focused on teaching fractions.

The hypotheses formulated in the research were either confirmed or partially confirmed.

The teachers interviewed consider that there is a need for more tasks on fractions and that digital content would greatly assist their work, especially during online teaching. The questionnaire also included three open-ended questions on the visual aids teachers use for presenting fractions and the preferred modes of representation. The responses abounded in creative ideas, indicating that teachers find it important to use a variety of ways to represent and illustrate fractions and fractions of an amount.

In addition to the hypotheses, we would also like to highlight the importance teachers place on the use of realistic tasks in textbooks (this statement achieved a mean of 3.22).

Further research is intended on interviewing more teachers on the subject matter and also examining the findings according to teachers' level of competence.

Additionally, a summary of the answers and ideas received for the open-ended questions would serve as a useful collection for future teachers.

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