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The role of representations constructed by students in learning how to solve the transportation problem

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Abstract. The purpose of the research presented in this paper was to study the role of concrete and table representations created by students in learning how to solve an optimization problem called the *transportation problem.* This topic was learned in collaborative groups using table representations suggested by teachers in 2021. In 2022, the researchers decided to enrich the students' learning environment with concrete objects and urged the students to use them to present the problem to be solved. The students did it successfully and, to be able to record it in their notebooks, they constructed a table representation by themselves without any help from their teacher. After that, they managed to solve the problem by manipulating the objects. At the same time, each step in the solution was presented with changes in the table. The students were assessed before (pre-test) and after collaborative learning (test) in both academic years. The pre-test results were similar, but the test results were better in 2022. Therefore, it can be concluded that using concrete and table representations constructed by students in learning how to solve transportation problems makes collaborative learning more constructivist and more effective than when they use only table representations suggested by their teachers.

Key words and phrases: representation, constructivism; mathematics, problem-solving, transportation problem.

MSC Subject Classification: 97M10, 97M40.

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Introduction

According to Binkley et al. (2012), learning to learn, problem-solving, collaboration, and communication are some of the skills that should be developed during education because of their crucial importance in preparing students for their future life. To develop these skills in everyone as much as possible, it is necessary to improve existing ways of teaching and introduce more effective ones. During the research presented in this paper, the researchers established an optimal learning environment that empowered students to construct concrete and table representations, helping them learn how to solve a mathematical optimization problem known as *transportation problem.* In a transportation problem, goods are transported from a set of warehouses (e.g., bread factory) to a set of stores (e.g., bakery) subject to supply and demand limits. The transportation is organized optimally, meaning that the total cost of the transportation is minimal. It is a special class of linear programming problem (Ford & Fulkerson, 1962), in which the optimal value of the linear objective function is to be determined subject to linear constraints. As such, the transportation problem can be solved by techniques of linear programming. The problem is solved numerically with many routes, but for a small number of routes, the simplex method can be used. There are some techniques introduced specially for solving transportation problems. One of these techniques consists of two phases. In the first phase, the initial basic feasible solution is determined by the minimum cost, northwest, or Vogel methods. In the second phase, the MODI method determines the optimal solution. These methods are based on matrix notation and require knowledge of matrix operations. Matrices can be represented in the form of a rectangular array, i.e., with a transportation table (Rashid et al., 2021). The schematic description of the transportation problem by the directed graph is given in Figure 1.

In Figure 1, $A_1, A_2, \ldots, A_m$ $(m \in N)$ denote the warehouses, while stores are denoted by $B_1, B_2, \ldots, B_n$ $(n \in N)$. The cost per unit distributed from the warehouse A_i $(i = 1, 2, \ldots, m)$ to the store B_j $(j = 1, 2, \ldots, n)$ is given by c_{ij}. The goal of the above problem can be expressed as determining the quantity of x_{ij} units distributed from warehouse A_i to store B_j $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ so that supplies will be consumed and demands satisfied at an overall minimal cost.

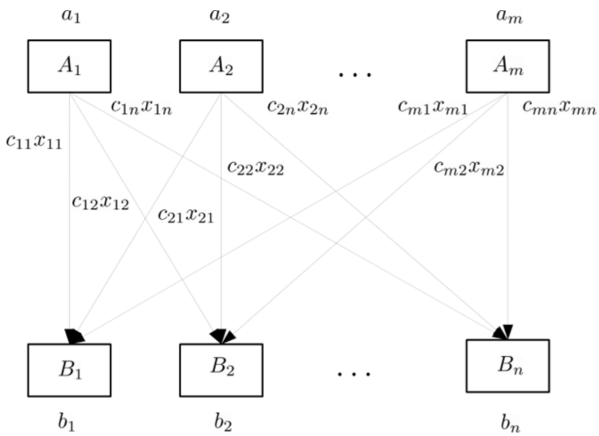

Figure 1. Schematic description of the transportation problem

The corresponding linear programming problem can be written as follows:

$$\min F = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \dots, m,$$

$$\sum_{1=1}^{n} x_{ij} = b_j, \quad j = 1, 2, \dots, n,$$

$$x_{ij} \geq 0, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n.$$

The minimum cost method uses shipping costs to determine an initial basic feasible solution. The algorithm of the method is as follows. First, the variable x_{ij} with the smallest shipping cost is located. The largest possible value is assigned to variable x_{ij}. This value is the minimum of a_i and b_j. After that, row i and column j are crossed out and the supply or demand of the non-crossed-out row or column is reduced by the value of x_{ij}. The next route with the minimum shipping cost is chosen among the ones which do not belong to the crossed-out row or column. The procedure is repeated until all capacities are exhausted and all demands are satisfied.

One of the authors of this paper has been teaching transportation problems using matrices and tables for ten years (in the winter semester of the academic year) to the third-year students of the study program Technical Communication Management at Subotica Tech – College of Applied Sciences, within the course named Theory of Decision Making. Annually, he is faced with the fact that students have a lot of learning difficulties while learning to solve this problem. To make learning easier for students, the researcher decided to use collaborative learning three years ago, and in addition, this year, the students were urged to create representations of the problem together and to use them to solve the problem. In this paper, the authors compare and present the results of teaching this topic in the academic years 2022 and 2021. The researchers got the idea for using real objects as representations for goods of transportation problem during their participation in PUNTE project, where the use of the Poly-Universe game family in teacher training was examined in the academic year 2021. Jaaska and Aaltonen's (2022) report on the use of game-based learning methods was also inspiring for researchers. Some other game-based learning methods are presented in Anastasiadis et al. (2018), Cheung & Ng (2021). The Poly-Universe game family provides an equal opportunity for children of different ages and mental and emotional maturity levels to develop their personalities (Dárdai et al., 2018). The Poly-Universe game family is based on three base forms: circle, triangle, and square. Each element is coloured in four colors: red, yellow, blue, and green as shown in Figure 2. Base color is the color of the central part of the given basic element.

Figure 2. Basic elements of the Poly-Universe game family with the base color red

The researchers based the organization of the students' collaborative learning on constructivism.

Among the theories of learning, constructivism has a prominent place. According to this theory, the students are actively involved in the process of learning. They construct their knowledge in their own way and do not get it from somebody else. As Bordner (1986) emphasizes: "Knowledge is constructed in the mind of the learner." When learners explore their learning environment, the concrete world and the learner mutually influence each other, and the learner gets a lot of experience. In that way, the learner makes sense of some new concepts and tries to assimilate them into his/her existing knowledge. Sometimes he/she must adjust his/her knowledge to accommodate the new concepts. This way, the learner enriches his/her knowledge. During the learning process, the learner communicates and collaborates with other people. Through this interaction, the learner can be exposed to quite different points of view, which can influence his/her learning process in such a way that the learner changes his/her mind. Learners direct and control their own learning process and they are responsible for it, see Bordner (1986), Naylor & Keogh (1999), Taber (2011), Sjoberg (2010), Iran-Nejad (1995). Teachers prepare learning environments to make learning easier for their students. They urge students' exploration of the learning environment; promote discussions with one another, and encourage students to ask questions of the teacher and other students. This process encourages students to explain their thoughts and experiences, which can help them to realize that there are some contradictions to their knowledge (Tobin & Tippins, 1993; Brooks & Brooks, 1993; Dogru & Kalender, 2007). In Good & Brophy (1994), authors emphasize that the most qualitative social interaction arises while learning in small groups. Because of that, learning in small groups, constructivism becomes a base for developing new learning approaches called collaborative and cooperative learning.

Collaborative learning means that students organized in groups solve a problem or complete a task together (Laal & Ghodsi, 2012). There are some important properties of collaborative learning that are not mentioned in constructivism. In addition to being responsible for their learning, students are also responsible for the learning of other members of their group. Working together, group members help, support, and inspire one another. During collaborative learning, learners build their knowledge together (Koschmann, 1996). The leading role in the structuring of the learning environment for learning how to solve transportation problems has representations.

Something can be called a representation of something else when it stands for that other thing (Duval, 2006). Duval emphasizes that: "...representations can also be signs and their complex associations, which are produced according to rules, and which allow the description of a system, a process, a set of phenomena" (p. 104). Representation can be constructed of real objects, something presented on paper, or ideas in an individual's mind (Janvier, 1987). Depending on where the representations were created, two main types of representations are distinguished: external and internal. The first ones were formed and exist in a human environment. Internal representations in the form of mental imagery were constructed and stored in one's mind as a part of his/her knowledge (Zhang, 1997). Zhang further states that through memorizing external representation, it can be converted into an internal representation and vice versa: internal representation can be converted by externalization into external representation. Very often the internal representation is not the same as the external one (Haciomeroglu et al., 2010). According to Bruner (1966), there are three types of internal representations of knowledge: enactive, iconic, and symbolic. Enactive representations are created through action, iconic are constructed by using pictures and images, and symbolic representations are formed when the learner uses symbols. He suggests that in the first phase of learning, students should use concrete objects and only after that they should use pictures, images, and symbols. Miura (2001) emphasizes that there are two types of representations: instructional representations and cognitive representations. Instructional representations are those which are used by teachers to make learning easier for their students. On the other hand, cognitive representations are built up by students themselves while learning mathematical notions or solving problems. Palmer (1978) says that important relationships between objects in the represented world are presented by relationships between corresponding objects in the representing world. Samsuddin and Retnawati (2018) state: "Representation served as a bridge connecting the abstract mathematics concept with daily life context." Switching from one representation to another has crucial importance in learning mathematics, according to Duval (2006). Different representations highlight different characteristics of the thing being represented. As representations are used in communication and reasoning, students should learn how to make and interpret them (Greeno & Hall, 1997). Mainali (2019) says that students should know different ways of representation because some problems can be easily solved by using appropriate representations. A lot of researchers agree that using representations can help the learning process (Kilpatrick et al., 2001; Greeno & Hall, 1997; Goldin & Shteingold, 2001). Researchers document their exploration of employing concrete representations in higher education within the following research papers: Chan & Chan (2023); Hunt et al. (2011); Stankov (2014).

Research questions

The solving process of the transportation problem was collaboratively learned using table representations given by the teacher in 2021. Given that Bruner (1966) suggests the use of concrete representations before any other types, and Duval (2006) emphasizes the importance of employing multiple representations and transitioning between them in mathematics learning, in 2022, the researchers opted to enhance the learning environment compared to previous years by incorporating concrete objects. They hoped that students would utilize these concrete objects as representations. The researchers planned to compare these two types of learning how to solve transportation problems and sought to answer the following research questions:

- Is a collaborative learning environment created for learning how to solve transportation problems, including concrete objects (potentially serving as representation), more aligned with constructivist principles compared to collaborative learning environments that lack tangible objects but involve students using instructional tabular representations?
- Does using concrete and table representations constructed by students in learning how to solve transportation problems make collaborative learning more effective than when they use only table representations suggested by their teachers?

Methods

The research was conducted at Subotica Tech – College of Applied Sciences in Subotica, Republic of Serbia. Students of the study program Technical Communication Management who attended the third-year course Theory of Decision Making in the academic year 2021 and 2022 participated in the research. These students all approved their participation in the research. In the academic year 2021, fifteen students attended the course, while in the academic year 2022, seven students attended. The experimental group (EG) was formed of all the students from 2022 and seven corresponding students from 2021 formed the control group (CG). Each student from EG had his/her correspondent pair in the control group. The EG-CG student pairs were formed in accordance with the pre-test results of the students. With a pre-test, the teacher assessed the mathematical bases needed for solving the transportation problem. The EG-CG pairs were chosen in such a way that the differences between points on the pre-test of each corresponding pair were less than three points. It means that the level of knowledge needed for solving transportation problems was not significantly different.

Based on pre-test results, the CG was divided into three four-member heterogeneous groups and one group of three students. The explanation of the transportation problem with mathematical formulation and matrix notation was given to students in written form. The minimal cost algorithm for determining the initial feasible solution to the transportation problem was also presented in written form in one example. The example was as follows:

- *Two bread factories make the daily bread in a city. The capacity of the bread factory A_1 on a daily basis is* 14 *bread boxes, while the capacity of A_2 is* 6 *bread boxes. The bread is delivered to the three bakeries B_1, B_2 and B_3 in the city. The demand of bakery B_1, B_2 and B_3 is* 6, 4, *and* 10 *bread boxes, respectively. The transportation costs per bread box from A_1 to B_1, B_2 and B_3 are, respectively,* 2, 8, *and* 4 *money units. From A_2 to B_1, B_2 and B_3 are, respectively,* 6, 4, *and* 2 *money units. Determine the initial basic feasible solution of the given transportation problem.*

The detailed explanation of the solution process of the given transportation problem example was presented in written form, where instructional tabular representations were given. The less detailed version of the explanation is as follows:

The given transportation problem was also presented as a directed graph shown in Figure 3:

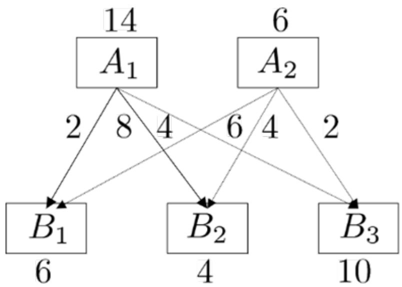

Figure 3. The graph form of the transportation problem

The transportation problem from the example can be written in the form of Table 1.

	B_1	B_2	B_3	Supply
A_1	2	8	4	14
A_2	6	4	2	6
Demand	6	4	10	

Table 1. The table representation of the transportation problem

The minimum cost per bread box is 2 money units. Since the transportation cost of one bread box from A_1 to B_1 and from A_2 to B_3 is both 2 money units, one can choose arbitrarily the distribution plan. If one chooses to satisfy the demand of B_1, the rest of the algorithm is as follows. After transporting 6 bread boxes from A_1 to B_1, one obtains distribution plan displayed in Table 2. Since the demand of B_1 is satisfied, that column can be neglected.

	B_1	B_2	B_3	Supply
A_1	$\boxed{2}$ 6	8	4	8
A_2	6	4	2	6
Demand	0	4	10	

Table 2. Distribution plan after the first step

In the second step, the demand of B_3 can be satisfied as shown in Table 3:

	B_1	B_2	B_3	Supply
A_1	$\boxed{2}$ 6	8	4	8
A_2	6	4	$\boxed{2}$ 6	0
Demand	0	4	4	

Table 3. Distribution plan after the second step

Since the supply of A_2 is canceled out, from now on, the bread will be transported only from A_1. The following minimal transportation cost per unit is 4, so the rest demand of B_3 will be satisfied. In this way, the distribution plan given in Table 4 is obtained. After that, the demand of B_2 will be satisfied by obtaining a distribution plan which cancels out the supply of both bread factories A_1 and A_2, and satisfies the demand of all bakeries. Table 5 represents this distribution and is the initial basic feasible solution to the transportation problem.

	B_1	B_2	B_3	Supply
A_1	$\boxed{2}$ 6	8	$\boxed{4}$ 4	4
A_2	6	4	$\boxed{2}$ 6	0
Demand	0	4	0	

Table 4. Distribution plan after the third step

	B_1	B_2	B_3	Supply
A_1	$\boxed{2}$ 6	$\boxed{8}$ 4	$\boxed{4}$ 4	0
A_2	6	4	$\boxed{2}$ 6	0
Demand	0	0	0	

Table 5. Distribution plan after the fourth step

From the distribution plan in Table 5, the initial basic feasible solution can be written in matrix form

$$X = \begin{bmatrix} 6 & 4 & 4 \\ 0 & 0 & 6 \end{bmatrix}.$$

The total cost of transportation given in money units is

$$F(X) = 2 \cdot 6 + 8 \cdot 64 + 4 \cdot 64 + 2 \cdot 66 = 72.$$

The students of CG had two classes to study the given material on the transportation problem. After that, the students practiced solving five transportation problems across three classes. The students collaborated. One student read the problem, two tried to solve it, and explained to the professor the reasoning of the algorithm steps. In some groups, there were students who did not understand the algorithm, but other students from the group tried to explain them.

Based on pre-test results, the EG was divided into one four-member and one three-member heterogeneous group. Instead of a detailed explanation of the transportation method, they got the instruction that it is expedient to start the transport by filling the route with the lowest transportation cost first, and then progressively filling lower-cost cells with available supply until all demand has been met. The same example, given to CG, was given to EG, but the teacher did not explain the solution process. Instead, the teacher motivated the students to construct a concrete representation of the transportation problem given in the example. During the learning, both collaborative groups of EG were audio recorded and the researchers took pictures of students' representations and made notes of them. In this paper, only the thinking and reasoning of the four-member group will be presented. The name of the students of this group will be denoted by Student A, Student B, Student C, and Student D. Since Student A and Student B attended in the summer semester of the academic year 2021 the course named Applied Project – PUNTE, and within this course they used the Poly-Universe game family for concrete representations, they had an idea about representing data given in the example with the Poly-Universe game family. At first, Student B tried to represent data only by various basic forms and colors. The students agreed that they would denote bread factory and its daily capacity of 14 bread boxes by the triangle with a red basic color. Student C realized that in one package of the Poly-Universe, there are only six triangles with red basic color, so they cannot limit the representation to elements with one basic color. Student D explained: "This way of representation is good for this case with three bread factories and two bakeries, but for problems with more than three bread factories and more than two bakeries is not good, because they cannot be represented with this set." Student A persuaded other students to construct a representational system, which will help to solve the given transportation problem. So, they gave up using a unique color for the notation of one bread factory or bakery. The idea of Student A was to distinguish bread factories and bakeries with the combination of basic shapes and colors. He/She said: "Let us denote 14 bread boxes produced in the first bread factory with 14 red, blue, and green triangles, while the bread boxes produced in the second bread factory with six yellow triangles. For the demand of the first bakery, we can use six blue circles. For the demand of the second bakery, we can use four yellow circles, while for the demand of the third bakery, we can use ten green or red circles." Student A arranged these elements in one row, as we can see in Figure 4. Then Student D said: "This is too complicated. Let's find something simpler."

After some brainstorming, students agreed that besides shapes and color, the position of elements may also help in simplifying the representation. Student B added: "We can show the demand of the bakeries." Student A suggested: "It is not necessary to use only one row. Let us make more rows." After some manipulations with elements and discussion, students proposed the following concrete representation shown in Figure 5.

The bread factories are denoted by triangle form elements with two different basic colors. The three bakeries are denoted by circle-shaped elements with

Figure 4. The first representation of the transportation problem

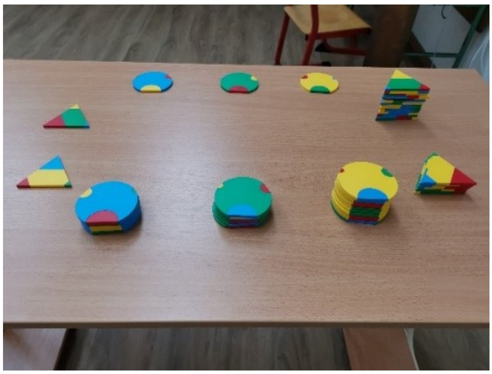

Figure 5. The second representation of the transportation problem

different basic colors. The supply capacity of the bread factories is denoted by triangle-shaped elements. Since the supply capacity of A_1 is 14, students denoted this quantity by stacking 14 pieces of triangles. These triangles do not have to have the same basic color as the representation of A_1. Similarly, the supply capacity of the bread factory A_2 is represented by stacking 6 triangle elements. Since the bakeries are represented by circles, their demand is also represented by stacking circle-shaped elements. The demand of B_1 is represented by stacking six pieces of circles. The demand of other bakeries is represented in a similar way.

Student B noted: "And where are transport costs?" The solution to this crucial problem came from Student C: "Write down the costs of transportation on pieces of paper. For example, if the cost from A_1 to B_1 is 2 money units, then in the crossing of the imagined row of A_1 and column of B_1 students will write the transportation cost of 2 money units on a label." Following his instructions, the students got the representation shown in Figure 6.

Figure 6. Concrete representation of the transportation problem

Student D, who was from the beginning sceptical about using Poly-Universe as the representation of the transportation problem, argued: "We cannot put Poly-Universe game family in the pocket and take it to the exam!" Student C said: "Could we write it down somehow?" After short silence, Student A screamed: "Look! As we have rows and columns, we in fact have a table!" The students draw the table from Figure 7:

	B_1	B_2	B_3	
A_1	2	8	4	14
A_2	6	4	2	6
	6	4	10	

Figure 7. Tabular representation of the transportation problem

After constructing a concrete representation of the given transportation problem in the described way, according to teachers' instruction, the students considered the minimal cost of transport, which was 2. Since there were two possibilities for the transportation of one bread box with this cost, the students wanted to choose the bread factory from which they could transport as many bread boxes as possible. Unfortunately, in both cases, six bread boxes could be transported from bread factories. After some discussion, Student C suggested: “Let us choose the first bread factory.” Then he moved six representations of bread boxes to the first bakery. This means that the demand of B_1 is satisfied, and the supply capacity of A_1 is decreased by six bread boxes. Student B suggested: “Let us take six circles and the same quantity of triangles representing the supply of the first bread factory.” Since the initial supply capacity was 14 bread boxes, the current supply capacity is 8. Student D said: “Let us denote the transported quantity of bread boxes by stacking six squares in the intersection of the imagined row of the first bread factory and column of the first bakery.” Every change in the element number in the concrete representation implies crossing out the previous number of elements in the appropriate position in the tabular representation and writing down the new number of elements in that position. Since the demand of B_1 is canceled out, the column of B_1 is neglected in the rest of the algorithm. After that, students transported bread boxes from the bread factory from where they could transport for minimal cost, and that was 2. So, they transported six bread boxes from A_2 to B_3. Now, the supply of A_2 is canceled out, while the demand of the B_3 is decreased by six.

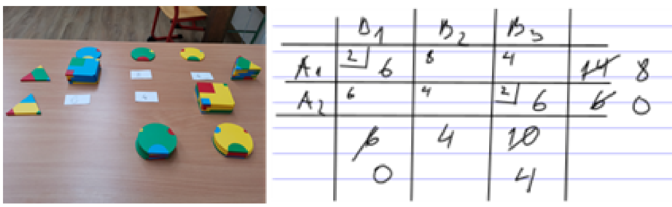

Figure 8. Representation of the transportation problem with Poly-Universe and table

Following this procedure, students got the final solution shown in Figure 9:

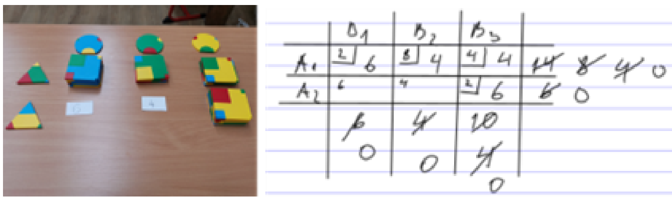

Figure 9. The final step of the minimal cost method

For solving this example, students needed 3 classes. After solving this example, students solved five more examples across two more classes. For the fifth example, students did not use concrete representations for solving, but only tabular, since they were able to solve the problem without concrete representations and they wanted to solve the last problem faster. Compared to students from CG, students from EG *needed fewer classes to solve the other five examples*. In total, both groups learned and practiced determining the initial basic feasible solution to the transportation problem for five classes. The experimental group was slower at the beginning, since they needed to construct the concrete representation of the given transportation problem.

The need to make notes about the concrete representation urged students to create a new form of representation: the table representation, which can be easily drawn in their notebooks. This step held paramount significance in solving the transportation problem, since it allowed them to obtain a tabular representation without prior expos. While the students of CG learned the method of tabular representation from their teachers, the students of EG *created the tabular representation by themselves*. Therefore, we can say that the learning of EG was more constructivist than the learning of CG.

While the students of CG represented the minimal cost method by modifying the numbers in the table representation of the transportation problem, the students of EG represented the minimal cost method by manipulations of Poly-Universe elements and after each step, they wrote down the appropriate modifications in their tabular representation. Even though students of EG used two representing worlds (concrete and tabular), *they needed one class less to solve the remaining five transportation problems* than the students of CG. It appears that students' learning of the minimal cost method in the EG was more effective than in the CG.

Results

Results of the assessment

Three weeks after learning about the transportation problem, students of both the EG and the CG were assessed. The assessment test consisted of two transportation problems. On each problem, students could collect a maximum of ten points. Students had forty-five minutes to solve the test. The obtained points on the assessment test for seven experimental-control group pairs are given in Table 6:

	Pair I	Pair II	Pair III	Pair IV	Pair V	Pair VI	Pair VII
Experimental group	9	19	20	20	11	13	19
Control group	8	9	17	13	7	10	13

Table 6. Assessment test results of the EG and CG for solving two problems

The average of the assessment test points of EG students is 15.86, while the average of the assessment test points of CG students is 11.00.

Since *EG students scored higher* than the other group in each pair of students, it is likely that they learned the minimal cost method better.

Answers to posted questions

The students of the experimental group were required to answer in writing the following two questions regarding the use of representations:

- *Is the use of real objects useful for learning how to solve transportation problems? Justify your answer.*
- *Should the learning of this topic have been taught only using table representations? Justify your answer.*

All students answered that the objects are useful for learning this topic and the topic should be taught using both types of representations.

We quote four typical explanations:

- "By tiles, we can play out the transportation of goods from the warehouse to the store."
- "With the help of objects, it is easier to imagine how many goods are currently in certain stores."
- "I don't think I would have remembered to draw the table if we hadn't placed the Poly-Universe elements to stand like in the table."
- "Solving the task with objects is fun like a board game."

The use of real objects is useful for learning how to solve transportation problems because, by using concrete representations, students can follow the story from the text of the problem and really move the objects that represent bread boxes. It seems that this is easier for students than modifying symbolic representations of the number of bread boxes.

From the third answer, it follows that the learning of this topic should not be taught only using table representations, because the students would not be able to create them without concrete representations.

Conclusions

As the first step of solving transportation problems, the concrete representations can be easily constructed by students, because they reflect the concrete properties described in the text of the transportation problem. The concrete representations enable students to represent warehouses and stores as real objects, and the transportation of goods as the real movement of concrete objects. Using concrete representations help students create a tabular representation of the transportation problem on their own, without any help from their teachers. All our students from EG agreed that using objects is helpful for learning transportation problems, and that the topic should be taught using both concrete and symbolic representations. Moreover, they wrote that they could not have constructed a tabular representation of the transportation problem without using concrete representations. Therefore, we can conclude that:

- Collaborative learning environment created for learning how to solve transportation problems, including concrete objects (potentially serving as representation), is more aligned with constructivist principles compared to collaborative learning environments that lack tangible objects but involve students using instructional tabular representations.

The students of CG solved the first transportation problem in two classes, but the students of EG took three classes to solve the same problem. For the other five problems, the students of CG needed three classes, while the students of EG solved them in only two classes. EG students outperformed the CG in every student pair comparison of the assessment test results. As a result of the analysis, we can state that:

- Using concrete and table representations constructed by students in learning how to solve transportation problems make collaborative learning more effective than when they use only table representations suggested by their teachers.

Future research possibilities:

Future research could extend this experiment by increasing the sample size of both CG and EG, which would enhance the statistical power and precision of the analysis.

References

Anastasiadis, T., Lampropoulos, G., & Siakas, K. (2018). Digital game-based learning and serious games in education. *International Journal of Advances in Scientific Research and Engineering*, *4*(12), 139–144.

Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., Miller-Ricci, M., & Rumble, M. (2012). Defining twenty-first century skills. In P. Griffin, B. McGaw, & E. Care (Eds.), *Assessment and Teaching of 21st Century Skills* (pp. 17–76). Springer.

Bordner, G. M. (1986). Constructivism: A theory of knowledge. *Journal of Chemical Education*, *63*(10), 873–878.

Brooks, J. G., & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classrooms.* Association for Supervision and Curriculum Development (ASCD).

Bruner, J. S. (1966). *Toward a theory of instruction.* Belknap Press.

Chan, J. W. Y., & Chan, W. W. L. (2023). Examining the learning effects of concrete and abstract materials among university students using a two-dimensional approach. *British Journal of Educational Psychology*, *93*(4), 1053–1071.

Cheung, S. Y., & Ng, K. Y. (2021). Application of the educational game to enhance student learning. *Frontiers in Education*, *6*:623793.

Dárdai, Zs. et al. (2018). Poly-Universe in School Education (PUSE) Erasmus+ Project 2017-1-HU01-KA201-035938. `http://www.poly-universe.com/wp-content/uploads/PUSE_STUDY_EN.pdf`

Doğru, M., & Kalender, S. (2007). Applying the subject "Cell" through constructivist approach during science lessons and the teacher's view. *International Journal of Environmental & Science Education*, *2*(1), 3–13

Duval, R. A. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1-2), 103–131.

Ford, L. R., & Fulkerson, D. R. (1962). *Flows in networks.* Princeton University Press.

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco, & F. R. Curcio (Eds.), *The roles of representation in school mathematics: 2001 yearbook* (pp. 1–23). National Council of Teachers of Mathematics.

Good, T., & Brophy, J. (1994). *Looking in classrooms.* Harper Collins College Publishers.

Greeno, J. G., & Hall, R. P. (1997). Practising representation: Learning with and about representation forms. *Phi Delta Kappan*, *79*(5), 361–367.

Haciomeroglu, E. S., Aspinwall, L., & Presmeg, N. C. (2010). Contrasting cases of calculus students' understanding of derivative graphs. *Mathematical Thinking and Learning.* *12*(2), 152–176.

Hunt, A. W., Nipper, K. L, & Nash, L. E. (2011). Virtual vs. concrete manipulatives in mathematics teacher education: Is one type more effective than the other? *Current Issues in Middle Level Education.* *16*(2), 1–6.

Iran-Nejad, A. (1995). Constructivism as substitute for memorization in learning: Meaning is created by learner. *Education.* *116*(1), 16–32.

Jääskä, E., & Aaltonen, K. (2022). Teachers' experiences of using game-based learning methods in project management higher education. *Project Leadership and Society*, *3*:100041.

Janvier, C. (1987). *Representation and understanding: The notion of function as an example.* Lawrence Erlbaum.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics.* National Academy Press.

Koschmann, T. (Ed.) (1996). *CSCL: Theory and practice of an emerging paradigm.* Lawrence Erlbaum.

Laal, M., & Ghodsi, S. M. (2012). Benefits of collaborative learning. *Procedia - Social and Behavioral Sciences*, *31*, 486–490.

Mainali, B. (2019). Investigating the relationships between preferences, gender, task difficulty, and high school students' geometry performance. *International Journal of Research in Education and Science (IJRES).* *5*(1), 224–236.

Miura, I. T. (2001). The influence of language on mathematical representations. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics: 2001 yearbook* (pp. 53–62). National Council of Teachers of Mathematics.

Naylor, S., & Keogh, B. (1999). Constructivism in classroom: Theory into practice. *Journal of Science Teacher Education*, *10*(2), 93–106.

Palmer, S. E. (1978). Fundamental aspects of cognitive representation. In E. Rosch, & B. Lloyd (Eds.), *Cognition and categorization* (pp. 259–303). Lawrence Erlbaum.

Rashid, F., Khan, A. R., & Uddin, M. S. (2021). Mixed constraints cost minimization transportation problem: An effective algorithmic approach. *American Journal of Operational Research. 11*(1) 1–7.

Samsuddin, A. F., & Retnawati, H. (2018). Mathematical representation: The roles, challenges and implication on instruction. *Journal of Physics: Conference Series. 1097*:012152, pp. 8.

Sjøberg, S. (2010). Constructivism and learning. In P. Peterson, E. Baker, & B. McGaw (Eds.), *International encyclopedia of education* (pp. 485–495). Elsevier.

Stankov, G. (2014). Játszva tanuljuk a lineáris egyenletek megoldását mérlegelvvel. In J. Szitányi (Ed.), *Fókuszban a tanuló: Tanulmányok Szendrei Julianna emlékére* (PP. 112–115). ELTE Tanító- és Óvóképző Kar.

Taber, K. S. (2011). Constructivism as educational theory: Contingency in learning, and optimally guided instruction. In J. Hassaskhah (Ed.), *Educational theory* (pp. 39–61). Nova Science Publishers.

Tobin K., & Tippins, D. (1993). Constructivism as a referent for teaching and learning. In K. Tobin (Ed.), The practice of constructivism in science education (pp. 3–21). AAAS Press.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science*, *21*(2), 179–217.

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