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Connections between discovery learning through the Pósa Method and the secondary school leaving examination in three Hungarian mathematics classrooms

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Abstract. The Pósa Method is a guided discovery learning method that has been used in Hungarian education in the form of extracurricular activities for "gifted" mathematics students. A four-year experiment implemented the method in three more "average" classrooms. This article reports on the relationship between the Pósa Method and the standardized secondary school leaving mathematics exam (Matura Exam in short) in Hungary. Data consists of students' survey responses, teacher interviews, and exam results from the three Hungarian classrooms who took part in the four-year experiment. We identify aspects of the Pósa Method that can benefit and hinder exam performance. In addition, we find that learning through the Pósa Method for the four years of high school has adequately prepared students for the exam.

Key words and phrases: Pósa Method, Matura Exam, guided discovery, discovery learning, standardized assessment, school leaving mathematics exam.

MSC Subject Classification: 97D44, 97D54, 97D64.

Introduction

Developed in the late 1980s by Lajos Pósa, the Pósa Method is a pedagogical approach based on discovery learning to nurture advanced students' mathematics skills. Pósa developed problem threads that by building upon previous knowledge

and experiences, encourage students to notice connections between seemingly disparate areas of mathematics (Katona & Szűcs, 2017). Doing so, Pósa believed, would encourage students to think autonomously and critically about mathematics with the primary goal of making pupils happy while thinking about the subject (Győri & Juhász, 2017). While the Pósa Method has primarily been used in extracurricular activities, there remains the question of whether Pósa's desired outcomes can be experienced by a wider population of students. Our review of the literature indicates that research in this area is sparse (Daunt et al., 2021). As such, the Content Pedagogy Research Program of the Hungarian Academy of Sciences began an experiment in 2017 which supports the implementation of the Pósa Method in public secondary school settings. Our results report some of the findings of this experiment.

In the public secondary school setting in Hungary, students attend high school for four years studying mathematics with the same teacher, and in order to graduate from high school, students must pass the standardized Hungarian secondary school leaving examination (Matura Exam in short). This exam has important implications for those planning to attend university as their performance has a strong influence on their placement. The three groups who participated in the experiment studied mathematics exclusively through the Pósa Method for the four years of high school, and took the standardized Matura Exam in 2021. Due to disruptions of the COVID-19 pandemic during the 2020/2021 academic year, this mixed method research project analyzes the first three years of student/teacher experiences through observations, interviews, and questionnaires collected several months prior to the exam. Then exam results are analyzed.

Answering the question of what aspects of the Pósa Method could benefit or hinder students on their mathematics secondary school leaving Matura Examination, this paper examines both student and teacher reflections on the Pósa Method and their thoughts about the exam. The paper begins with a literature review containing some key features of the Pósa Method, Hungarian mathematics education, and the Matura Exam. Next, we discuss our motivation, research questions, and methodology used for data collection. Finally, we present our findings and conclude with suggestions for future work.

Literature review

The Pósa Method

The Pósa Method is a pedagogical approach that utilizes many tenets from discovery learning, and in particular, guided discovery. Discovery learning is a process in which a student "draws upon past knowledge and experience to infer underlying strategies and gain an understanding of concepts" (Honomichl & Chen, 2012, p. 615). Different interpretations vary in how much autonomy is given to students, how important the development of inquiry competences or scientific ideas are, and how much importance is given to real-life questions (Artigue & Blomhøj, 2013). Though discovery learning typically focuses on the student as an active participant, guided discovery assumes that the teacher plays a very important leading role. Honomichl and Chen (2012) compare guided discovery to Vytgosky's idea of scaffolding, in that "guidance that is dynamic and responsive to the learner's current state of experience and ability, with inexperienced learners receiving greater guidance or supervision and experienced learners receiving less intervention" (pp. 615–616). Guided discovery has a long history in Hungary through Tamás Varga's Complex Mathematics Education reform of which Pósa was also part of (Gosztonyi et al., 2018). In addition, the Pósa Method has two main goals: to make students enjoy the mathematical thinking process, and to teach students to think independently and creatively (Győri & Juhász, 2017; Katona & Szűcs, 2017).

There are several core qualities of the method that promote these goals. According to Győri and Juhász (2017), these are the following. The most important principle is that students be given adequate time to think through problems and attempt a solution. In particular, students should spend a lot of their time thinking independently. In fact, the Pósa Method has specific rules to ensure this happens. One is that collaboration can only occur if students have problems with the same task and have exhausted their own problem-solving. The other is that once students have solved a problem, they cannot share their solution or help their classmates. This ensures that fast problem-solvers do not discourage their peers, and that students are able to develop their own critical thinking process. Another important component of the method is the freedom to make mistakes. This is facilitated by instructors' treatment of students as colleagues, or equal partners, and encouraging the intellectual value of learning from mistakes. In alignment with the freedom to make mistakes, students are encouraged to ask questions. In particular, instructors are tasked with teaching students to ask "good" questions, as

conceptual and relevant follow-up questions. This helps students gain a deeper understanding of the material, making learning both interesting and exciting.

The structure of the instructional material plays a large role in achieving the goals of the Pósa Method. Instructors implementing the Pósa Method use problem threads (sets of problems) consisting of "conventional areas of mathematics (e.g. algebra, geometry)" and "fundamentally important ideas and less obvious connections" (Győri & Juhász, 2017, p. 100), allowing students to learn new ideas while also bridging connections between what they have previously learned. According to Katona and Szűcs (2017), "one aspect of this connection is that the problems may be, to a different extent, built on each other, meaning that the anticipated or regular solution to one problem (problem A) needs certain ideas, methods, types of mental representations, etc., or in general, any separable element of thinking, that can be made more easily available for students by thinking on and solving another problem (problem B), or even some other problems" (p. 20). Students are presented with several problems simultaneously, from separate or intersecting threads, allowing them to choose which problems they want to prioritize. This provides students with autonomy and independence, and further promotes their enjoyment of mathematics.

According to Győri and Juhász (2017), "less gifted children find it more difficult to concentrate for extended periods, and they are quicker to regard it as a failure on their part if they are not able to solve a problem" (p. 96). Given these challenges, the Pósa Method was originally designed for "gifted" students and implemented in extracurricular summer camps, colloquially known as "Pósa camps". Given the success and expansion of these programs, a natural question arises: Can the Pósa Method be implemented in an "average" classroom setting? On a mission to answer this question, the Content Pedagogy Research Program of the Hungarian Academy of Sciences supported a four-year experiment supporting the use of the Pósa Method in three public secondary classrooms.

Hungarian mathematics education and the Matura Exam

In order to understand the context of the research, it is important to mention some facts about Hungarian mathematics education in public secondary school settings in which the Pósa Method was implemented. In public primary and secondary schools in Hungary, there is one mathematics subject throughout the twelve years in which the different areas of mathematics (e.g., algebra, geometry, and so on) reappear in each year with deeper content. The exact content and

output requirements are determined by the Hungarian National Core Curriculum. It is further specified by a framework curriculum approved by the minister of education, which is then turned into local curricula compiled by the schools.

Upon the completion of high school, students take an exit exam which also functions as a college entrance exam (Matura Exam). Our description of the Matura Exam is drawn from Csapodi's (2017) evaluation of the exam. This exam is based on standardized requirements for Hungarian schools and is organized at both an intermediate and higher level. In the school year 2020/2021, 92.5% of students took the intermediate and 7.5% the higher level mathematics Matura Exam in Hungary. The written intermediate level exam is 180 minutes long and is divided into two parts: a 45 minutes part which contains twelve short 2-3-point questions and a second part with six longer questions for 12-17 points, adding up to a total of 100 points. From the last three questions of the second part, the student only has to choose two. The grading of the intermediate exam is the following: grade 5 from 80%, grade 4 from 60%, grade 3 from 40%, and grade 2 from 25%. If the result of the written part is between 12 and 25%, the student can take an oral exam for 50 points, which is then added to the points acquired in the written part. During the oral intermediate level exam, the student is given three simpler problems to solve and three theoretical questions. In contrast to the intermediate exam that students write in their own schools, the higher exam is organized by the Educational Authority and can affect college placement. The written higher level exam is 240 minutes long and contains nine longer problems. Among the last five problems, the student has to choose only four. The higher level oral exam is compulsory and gives the students a topic and asks them to provide a definition, verify a theorem, solve a problem, and demonstrate an application. The sum of the points in the written part is 115 and in the oral part is 35, adding up to a total of 150 points. The grading of the higher level is the following: grade 5 from 60%, grade 4 from 47%, grade 3 from 33%, and grade 2 from 25%. Due to the COVID-19 pandemic, in both school years of 2019/2020 and 2020/2021, oral exams were canceled and grades were given based only on the results of the written parts. As students of the experiment completed high school in the school year 2020/2021, they also took only written exams.

Methods

Research motivation

The Pósa Method has garnered a favorable reputation from mathematicians and mathematics educators alike. It is espoused for its collaborative, immersive, and engaging properties. These qualities have been shown to augment both student enjoyment and appreciation of mathematics (Daunt et al., 2021). Given its increasing pervasiveness in Hungarian culture, there are questions about where the Pósa Method can be applied and to what effect. A natural starting point is to investigate its relationship with the all important Matura Exam. This information could provide both educators and policy-makers insight into settings where the Pósa Method can have the most utility.

Research questions

While the Pósa Method has been traditionally used for strong mathematics students in Hungary, the question remains if more typical students can also benefit. By collecting data from three more "average" Hungarian classrooms, the current study hopes to add to this body of work by investigating the relationship between the Matura Exam and the Pósa Method. The study was guided by the following research questions: (1) What aspects of the Pósa Method could benefit students on the Hungarian Secondary School Leaving Matura Examination? (2) What aspects of the Pósa Method could hinder students on the Hungarian Secondary School Leaving Matura Examination? (3) How does performance on the Hungarian Secondary School Leaving Matura Examination compare between those who have been taught with the Pósa Method to those taught traditionally?

Setting and participants

This study was part of a four-year experiment through the MTA-Rényi Research Group on Discovery Learning in Mathematics of the Content Pedagogy Research Program of the Hungarian Academy of Sciences that seeks to investigate the implementation of the Pósa Method in public school settings. The experiment commenced in 2017 and concluded at the time of this study (2021), culminating in the Matura Exam. Studies have looked at the experiment from different perspectives (Kile et al., 2018; Matzal et al., 2020; Daunt et al., 2021).

For this reason, the setting and participants are identical to Daunt et al. (2021): "Our team investigated three grade 12 classrooms which participated in the four-year experiment. These classrooms will be referred to as Classrooms 1, 2, 3 with Instructors 1, 2, 3, respectively. Classroom 1 is in a bilingual vocational school with a chemistry pathway, and Classrooms 2 and 3 are located at the same general secondary school, which is considered one of the top ten high schools in Hungary. This school hosts special math classes, too, but Classrooms 2 and 3 are not specialized in math. In grade 9 and 10 these classrooms had three math lessons a week, but in grade 10 the vast majority of the class decided to choose to take the advanced level high school leaving Matura Exam, thus they have five lessons a week in grade 11 and 12. Due to the fact that students in Classroom 1 study math in English, they have four math lessons through all the four years of high school. Classroom 1 has 16 students, Classroom 2 has 14 students, and Classroom 3 has 16 students. The experiment was led by one of Pósa's former students, who recruited two colleagues, a fellow Pósa camp instructor and his former university teacher trainee, both trained in implementing the Pósa Method. As part of the experiment, these three instructors collaborated to design a curriculum which utilizes the Pósa Method. Instructors modify appropriate material based on student needs and abilities." We note that students in this experiment were taught exclusively by the Pósa Method, as opposed to more traditional instruction. As the experiment took place in the public secondary school setting, the output requirements set by the National Core Curriculum had to be fulfilled in the given number of lessons determined by the framework curriculum. The three instructors teaching the groups had to ask for permission to diverge from the schools' local curricula and cover the required material in a different manner following the tenets of the Pósa Method. Based on students' further plans, Table 1 shows the number of students choosing the intermediate versus the higher level of the Matura Exam.

Level of Matura Exam	Intermediate	Higher
Classroom 1	15	1
Classroom 2	8	6
Classroom 3	4	12
Sum	27	19

Table 1. Number of students taking the different levels of the Matura Exam

Research instruments

This paper uses the data gathered by the same research instruments described in Daunt et al. (2021): "A mixed methods research design with a concurrent embedded approach was used to address the research questions. A concurrent embedded approach, 'has a primary method that guides the project and a secondary database that provides a supporting role in the procedures' (Creswell, 2009, Ch. 10)." Our primary method was the qualitative component (teacher interviews and open-ended survey questions for students) which investigates the properties of the Pósa Method that can benefit/hinder exam performance. Quantitative data (exam results) was used as a secondary source to access performance and compare exam results from previous years. A description of the research instruments is given below.

"Our team interviewed each of the instructors in the respective classrooms. Each interview was conducted over Zoom and lasted approximately one hour. Interview questions discussed include the delivery of the Pósa Method, teachers' expectations, reflections about the experiment, and the Matura Exam." In this paper, we report our findings on the Matura Exam, while the other aspects are discussed elsewhere (Daunt et al. 2021). "Our interview with Instructor 1, who also served as a member of the research team, was done as a pilot interview. Appropriate changes were made to the interview protocol, including modifying ambiguous statements and reordering questions. Interviews of Instructors 1 and 2 were given in English while the interview from Instructor 3 was conducted in Hungarian with a translator and subsequently transcribed to English" (Daunt et al., 2021).

In addition to explicitly asking teachers about the ways in which the Pósa Method can benefit or hinder exam performance, we also asked teachers to reflect on their own views of the exam such as whether it accurately reflects mathematical aptitude. Given their experience with the Pósa Method and familiarity with the Matura Exam, teacher interviews gave us deep insight into the relationship between these two entities.

"The student survey was created using Google forms and distributed to students through their instructors. Our survey was modeled after Kile et al. (2018), as they surveyed the same group of students, and we wanted to be able to compare their answers from 2018 to 2021. Their survey consists of six constructs: background, discovery learning expectations, reasons for participation, workload, interests/math preferences, and grades. The constructs in our survey include: background, Pósa Method, interests/math preferences, Matura Exam, and remote

learning" (Daunt et al., 2021). The results in this paper report on the findings from the Matura Exam section, while other aspects are reported in Daunt et al. (2021). Given that students were preparing for the Matura Exam at the time of completing the survey, questions in this section considered students' views about the exam, future goals, and how prepared they felt. In addition, we asked students to reflect on some of the salient aspects of their experience that could potentially benefit or hinder their exam performance. While several questions used a Likert scale, most were open-ended to have a more in-depth understanding of students' beliefs.

Our study sought to investigate the relationship between the Matura Exam and the Pósa Method. However, through teacher interviews, we found that every class had experienced remote learning for the majority of the 2020/2021 academic year. As a result, a "Remote Learning" construct was created to determine how disruptive the online format was to the implementation of the Pósa Method and preparation for the Matura Exam.

Data analysis

Our data required the use of both quantitative and qualitative analysis. We utilized grounded theory methods (Corbin & Strauss, 2015) to analyze the interviews and open-ended survey questions. Initial coding was done after each interview. These initial codes were used to analyze the open-ended survey questions. The constant comparative method (Corbin & Strauss, 2015) was applied to create new, more elaborate codes in which new instances could be placed in already existing categories or help modify a current description. The use of both open-ended survey questions and teacher interviews helped triangulate our data. In addition, member checks were performed via a presentation and discussion of the findings with the teachers interviewed in this study.

In student questionnaires for questions involving a Likert scale, we report frequencies, measures of central tendency (mean and mode), and variability (standard deviation). In connection with the analysis of the Matura Exam results, the Hungarian national average and the average of schools in Budapest in the respective exam levels (intermediate and higher) are presented. As schools do not keep an official record of all of their exam results, we could only get the exam results of some previous classes to see how much the exam results of the classes in the study differ from the results of the schools' classes of the same tracks from earlier years.

Results and discussion

As mentioned in our methods, Hungarian schools had been primarily online for the final year of this experiment. Before addressing our research questions, we want to consider the impact that online learning had on teachers' implementation of the Pósa Method and students' preparation for the exam. Through open-ended survey responses, 11 of the 44 students (25%) reported that remote learning had little to no effect on their preparation for the Matura Exam. Interestingly, 13 of the 44 students (30%) indicated that remote learning helped their preparation, usually because of the increased independence and additional practice time. However, 17 of the 44 students (39%) reported that remote learning made preparation more difficult, typically due to decrease in motivation. While each teacher acknowledged that the situation brought certain challenges, they emphasized that the "basics" or "major components" of the Pósa Method had not changed.

The findings from this study are presented by research questions. Namely, in the first section, we use data from student responses and teacher interviews to the benefits and drawbacks of the Pósa Method as it relates to the Matura Exam. Next, we report the exam results for students involved in the experiment and compare them to other schools and the same schools' previous years.

Benefits of the Pósa Method on the Matura Exam

While students prepared for the exam at the time of the study, we asked them to reflect on how well they thought the Pósa Method prepared them for the exam. Specifically, students were asked the extent to which they agreed with the statement "The last 4 years have prepared me for the exam". Responses to this question used a five-point Likert scale from "I completely disagree" (1) to "I completely agree" (5). The overall results from this question are presented in Figure 1. We note that an overwhelming majority of students, 36 of the 44 responses (81.8%), believed their previous experience helped prepare them for the Matura Exam.

Following this question, students were asked why they felt this way. We present the findings from the thematic analysis of this open-response question and teacher interviews. Three aspects of the Pósa Method emerged that participants believed could benefit exam performance.

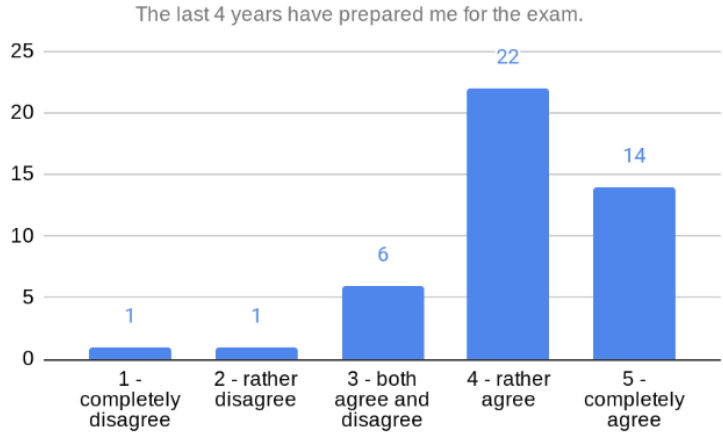

Figure 1. Student Response to "The last 4 years have prepared me for this exam"

Recurrence of Topics

According to Győri and Juhász (2017), "pupils can personally experience that in mathematics everything is connected with everything" (p. 14). Indeed, a key facet of the Pósa Method is that topics are constantly being integrated and reinforced over time. Commonly, different ideas are developed simultaneously. The following student testimony and teacher comment indicate that the recurrence of topics can be seen as an advantage. "We didn't deal with a topic just once, it came up in the problems over and over again. And it wasn't that we learned something once in 9th grade, but we dealt with it again and again" (Student). "The other thing, which I think helps is that in the traditional method [...] they've learned geometry, and then they move on to algebra, and then they move on to this or that. And when you have the exam, these questions are all mixed. And in the Pósa Method, students are used to having questions from all different fields all the time" (Instructor 1).

The student believes that the recurrence of topics keeps previously discussed material relevant, so as not to forget. Instructor 1 makes the point that the segmented presentation of content, commonly used in traditional instruction, can make it difficult for students to determine the topic(s) associated with a particular question. In contrast, students experienced with the Pósa Method may

fare better in making these delineations. Identifying this information is important because it could inform students on the strategies and techniques to apply.

"Understanding" Problems

In his seminal book "How to solve it", Pólya (1945) identifies four steps in mathematical problem solving, the first being "understanding the problem". This includes understanding the words in the problem, thinking about what is being asked to show, creating a diagram/picture to aid in understanding, etc. Through their four-year experience, students have grown accustomed to "understanding" problems. A representative student response is provided below: "Because we really understood the problems, instead of looking at them as a schema, so it is easier to think of the solution from a practical point of view." (Student) This student interprets "understanding" as more than just following steps of a procedure. Instead, the Pósa Method helped cultivate a deeper understanding and ability to think of solutions more "practically". As a result, students feel better prepared to tackle a variety of problems. Antithetically, some teachers see "understanding" problems as a drawback because, in their view, the exam rewards students that memorize procedures. We will discuss this matter below.

Attitude

One of the key goals of the Pósa Method is to make students happy thinking about mathematics. Students become versed in several types of inquiry like experimentation, questioning assumptions or hypotheses, and looking at problems from different perspectives. These forms of investigation seem to provide students with the confidence they need to approach many tasks. For instance, Instructors 1 and 3 explain: "If you are learning through discovery learning, you have this attitude, that whatever problem you face, you try to solve it, you try to find the solution, you are not giving up, you look for different approaches, I think that it helps in this exam as well" (Instructor 1). "Another experience they had in the Pósa Method is that they're not afraid to try, experiment, look at a problem from different points of view, not give up, and so on. And I think that this attitude is also something that can help because in the exam you get lots of different problems that you have to solve, and if you have this attitude, it helps" (Instructor 3).

Research has shown a positive correlation between self-confidence and exam performance (Guy et. al, 2015). Indeed, our previous results show students in this experiment have great confidence in themselves as learners and problem solvers

(Daunt et al. 2021). As a result, the teachers believe that the Pósa Method can help provide students with this psychological advantage.

Drawbacks of the Pósa Method on the Matura Exam

While the majority of students believed the Pósa Method helped prepare them for the exam, there were several responses that differed. We looked closely at these responses to determine what aspects of the Pósa Method might hinder exam performance. The following themes were observed.

Time

As mentioned in the literature review, students are given limited time to complete the exam. This raised some points of concern for students: "At the exam you won't have a lot of time to think about the problems, you have to know the solutions quickly" (Student). The most important principle in the implementation of the Pósa Method is giving students enough time to think about solutions to mathematical problems (Győri & Juhász, 2017). This enables them to think creatively and critically. According to our survey, students spent a considerable amount of time working/thinking about problems outside of class (Daunt et al., 2021). However, students do not have the luxury of time on the Matura Exam. The topic of time was also mentioned by some of the teachers. For instance, Instructor 1 mentions: "There are things in this exam that you have to learn: how to use different formulas [...] that you wouldn't have the time to figure out yourself in a discovery way" (Instructor 1). Like the student comment, this teacher acknowledges that in an exam context students need to apply specific knowledge to specific questions. This is in stark contrast to students' experience with the Pósa Method when most of their knowledge was constructed from a "discovery way" which inevitably takes more time.

Tenuous Relationship

Despite empirical findings that praise the benefits of guided discovery, some students questioned if their performance on the Matura Exam would reflect their mathematical ability. Two representative responses are below: "I have the knowledge, but I have to practice a lot to be able to solve the typical exam problems more skillfully" (Student). "From the point of view of my math knowledge, I agree [that the last 4 years have prepared me for the exam], I just have to get used to the format of the exam, what the things are I have to write down when solving a problem" (Student). Both of these students indicate they have the confidence in their

mathematical knowledge, but question how this would translate to exam performance. In particular, they admit they need practice solving "typical problems" and knowing "things to write down when solving a problem". These students feel the Matura Exam benefits those who memorize problems and procedures, in direct contrast to the Pósa Method, which prides itself on creativity and discovery learning.

The tenuous relationship between the Matura Exam and the Pósa Method was also echoed by instructors during the interviews. For instance, several months prior to the Matura Exam, Instructor 2 mentioned he was "training" students to answer exam questions: "I started to train them to solve these kinds of problems [of the Matura Exam]. So in the first three years, we didn't have problems which were intentionally similar to exam problems. But nowadays, in the last three months [...], we have very similar problems" (Instructor 2).

In addition, Instructor 2 admitted in the months leading up to the exam he "didn't use proper Pósa Method". Other instructors made similar remarks when asked about exam preparation, namely a more pointed focus on typical exam questions and in some cases memorization, which does not seem in spirit of the Pósa Method.

Exam results

In terms of the level of the exam, from the 46 students in the three groups 27 students (59%) took the Matura Exam in mathematics on the intermediate level, and 19 students (41%) chose the higher level. The fact that more than a third of the students chose the higher level exam, a ratio far above the national 7.5%, shows that the sample is unique from many points of view. However, this ratio is consistent with the numbers of students of the respective schools' earlier classes of the same tracks. Thus, it is not a result of the experiment, but intrinsic to the schools.

Students of the experiment took their Matura Exam May 4, 2021. Link to their exact exam problems can be found in the Appendix. In connection with the intermediate level, out of 100 possible points the country average of the May 4, 2021 exam is 51.2 points, while the average of general (grammar) secondary schools in Budapest is 63.1 points. Classroom 2 and 3, both considered general (grammar) schools, performed significantly higher with both averaging 88.25 points, but not very different from the results of similar classes of the school from previous years. The country average for mathematics in English is 68.5 points, which can be compared to the results of Classroom 1, in which the average is

75.5 points, for which is also true that these exam results are higher than the national average but consistent with the school's earlier classes of the same type. 962 students took the mathematics Matura Exam in English on May 4, 2021.

Due to the COVID-19 pandemic in 2020 and 2021, the higher level Matura Exam did not contain oral exams, so students' grades were calculated from the 115 points of their written exam. The country average is 72.5 points (63%), while the average of general (grammar) secondary schools in Budapest is 82.6 points (72%). The average of students taking the higher level exam in Classroom 2 is 89 points (77%), while in Classroom 3, it is 78.7 points (68%). (In Classroom 1, only one student took the higher level exam with 91 points (79%) result.) These results are significantly higher than the country average, but not very different from the average of general (grammar) schools in Budapest, nor the schools' own classes from earlier years. We conclude that exam results of students studying exclusively through the Pósa Method are similar to students who study traditionally at similar classes of the respective schools.

How well students perform at an exam is affected by several factors. One of them is the importance of that exam for their future studies. As the survey inquired about students' further plans, responses of the students indicate that roughly 86% of students intend to pursue a STEM career, which includes both university and vocational technical training, and for 87% of them the mathematics exam results count in their further education. The following table shows students survey responses in connection with their plans before the exam, and how they continued their studies after the exam in September 2021.

	plan in April 2021	started in September 2021
Vocational technical training	12 (26.1%)	14 (30.4%)
University of their first choice	32 (69.6%)	26 (56.5%)
University of their second choice	0	3 (6.5%)
University of their third choice	0	2 (4.3%)
University of their fourth choice	0	1 (2.3%)
no information	2 (4.3%)	0

Table 2. Number of students according to their plans and where they continued their studies after the experiment

Conclusion

The overarching goal of this project is to evaluate the efficacy of the Pósa Method in more typical math classrooms in relation to the Matura Exam. Our results indicate that both teachers and students felt that the Pósa Method provided adequate preparation for the exam. Although there were concerns about the time limit, proper practice, and memorization of problems, the Pósa Method equipped students with the necessary mathematical knowledge and problem-solving skills needed to succeed. In addition, their exam results are not significantly different from results of similar classes, which makes it possible for them to pursue their planned further studies. Our findings suggest that the Pósa Method should not be viewed as an impediment for success on the Matura Exam. This should give comfort to educators that they can implement discovery-based pedagogies, like the Pósa Method, without sacrificing standardized test performance.

Our findings suggest several directions for further research. One limitation of this project is that the Matura Exam was administered shortly after the student surveys, which restricted the questions asked and information collected. It would be interesting to distribute a post-exam survey about how students thought the Pósa Method directly affected their exam preparation and performance possibly in light of their further studies. Similar to Csapodi and Koncz (2016), future work might also investigate how students performed on different questions and topics on the exam. Having this information could help teachers know where to supplement or modify the Pósa Method for optimal exam performance. Though this study suggests hopeful outcomes from students learning with the Pósa Method, we suggest more research that investigates the relationship between different aspects of discovery learning and standardized exams.

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Appendix

Problems of the Mathematics Matura Exam administered May 4, 2021

Intermediate mathematics Matura Exam (in Hungarian):
https://dload-oktatas.educatio.hu/erettsegi/feladatok_2021tavasz_kozep/k_mat_21maj_fl.pdf

Intermediate mathematics in English Matura Exam:
https://dload-oktatas.educatio.hu/erettsegi/feladatok_2021tavasz_kozep/k_matang_21maj_fl.pdf

Higher level mathematics Matura Exam (in Hungarian):
https://dload-oktatas.educatio.hu/erettsegi/feladatok_2021tavasz_emelt/e_mat_21maj_fl.pdf

Higher level mathematics in English Matura Exam:
https://dload-oktatas.educatio.hu/erettsegi/feladatok_2021tavasz_emelt/e_matang_21maj_fl.pdf

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