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Teaching Mathematics and Computer Science

Visualisation in geometry education as a tool for teaching with better understanding

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Abstract. In primary and secondary geometry education, some problems exist with pupils' space thinking and understanding of geometric notions. Visualisation plays an important role in geometry education, and the development of pupils' visualisation skills can support their spatial imagination. The authors present their own thoughts on the potential of including visualisation in geometry education, based on the analysis of the Hungarian National Core Curriculum and Slovak National Curriculum. Tasks for visualisation are also found in international studies, for example the Programme for International Student Assessment (PISA). Augmented reality (AR) and other information and communication technology (ICT) tools bring new possibilities to develop geometric thinking and space imagination, and they also support mathematics education with better understanding.

Key words and phrases: spatial ability, Hungarian National Core Curriculum and Slovak National Curriculum, AR and ICT tools in mathematics education.

MSC Subject Classification: 97U10, 97G10.

Introduction

According Gutiérrez (2014), visual thinking is necessary in any area of mathematics, at all levels, especially in geometric contexts, and developing visualisation skills is important for students. The development of dynamic geometry software brings the possibility to represent geometrical concepts in new ways. Visual thinking has many applications not only in mathematics, but also in medicine, physics, chemistry, and other natural sciences. Weigand and Weth (2002) argue that students need adequate visual images of mathematical notions, since those images allow for a graphical representation of the main properties.

This art of teaching supports inquiry into the problem-solving process. Chapman (2003) observed that some teachers presented students with a problem to solve on their own. Teachers expected the group of students to analyse their process by considering, for example, why they used a particular method, why this method worked, and how they made sense of the problem and the solution. This was followed by group sharing and whole-class discussions. The process was repeated for a few problems to establish a model for solving problems. One teacher had students work in groups to solve the problem, with one student in each group assuming the role of an observer who “looks at the process that is going on to solve the problem”. Each student had a turn to be the observer. A whole-class sharing and comparison of findings followed each round of observations. This process eventually led to the development of a problem-solving model.

Geometry in Curriculum – the case of Slovakia and Hungary

The Innovated National Educational Programme for Primary Education Mathematics in Slovakia (see INEP, 2015) defines the following goals for geometry in primary education:

- Knowledge about basic geometric shapes;
- Creation of buildings from cubes;
- Plan of these buildings;
- Views from top, right, and front;
- Orientation in the plane and in the space;
- Length, surface, and volume.

The Slovak national curriculum for mathematics at the lower secondary level has the subject “Geometry and measurement” (see INES, 2015) wherein pupils observe and discover basic geometrical shapes. They study the shapes’ properties and learn to find the lengths of their segments, magnitudes of their angles, surfaces, areas, and volumes. Pupils solve positional and metrical tasks from real life, and spatial imagination plays an important role here. Important topics are constructions tasks with triangles and the Pythagorean theorem.

The National Curriculum for the upper secondary level (see INEU, 2015) also includes the subject “Geometry and measurement”. Pupils observe and discover plane and spatial geometric figures as well as their properties. They learn to find the lengths of those figures’ segments, magnitudes of their angles, surfaces, areas, and volumes. Pupils solve positional and metrical tasks connected to reality; spatial imagination plays an important role here, and new topics are using the history of mathematics, mathematics and society, Euclid, Billiard games, and other applications.

With regard to the Hungarian National Core Curriculum (see NAT, 2013), the syllabuses were created in the first half of the 2010s for textbooks used at the time. The subject of geometry is fairly diffused in the 4 years of lower primary school. During these 4 years, only 1–1 and 2–3 consecutive lessons are planned to examine the topic.

In the first year, the topic “Geometric bodies and plane figures” appears in May, at the end of the school year. This topic is relatively close to the section named “playing with the mirror”.

During the second school year, “Geometric bodies and plane figures” is also divided into multiple parts. At the beginning of the year, “Let’s build from cubes” introduces the modelling of geometric bodies. The identification of simple plane figures then occurs in the second half of the year, and there are also two lessons on cuboids and quadrilaterals (see also NATC, 2012).

Many teachers at secondary schools usually use different sequences from the ones offered in textbooks, mainly because if they start the year with a material similar to the previous year’s topics, then that makes the repetition at the beginning of the year more important. Sometimes they could not finish the material in some groups of students with weaker abilities, and a portion of the previous topics consequently moved to the beginning of the following year. This situation had some technical reasons: Most of the students borrowed their textbooks from the school library and had to return them at the end of the year to allow other students to access them.

According to the OECD (2018), geometry thinking is observed in PISA studies. One thematic area is "Relationships within and among geometrical objects in two and three dimensions". Here, we understand static relationships such as algebraic connections among elements of figures (e.g. the Pythagorean theorem as defining the relationship between the lengths of the sides of a right triangle), relative position, similarity and congruence, and dynamic relationships involving transformation and motion of objects, as well as correspondences between two- and three-dimensional objects. Another topic is "Measurement", which includes quantifying the properties of shapes and objects, such as angle measurements, distance, length, perimeter, perimeter, area, and volume. This topic corresponds to the "Geometry and measurement" topic in the Slovak National Curriculum for Mathematics (compare with INES, 2015 and INEU, 2015).

Augmented reality opportunities in developing a geometric approach

Augmented reality (AR) is a growing field of technology where real life is modified and enhanced by computer-generated sights and sounds. The most common use of AR can be seen through mobile apps. For example, if an individual points his or her device's camera at something that an AR app recognises, the app will generate a 3D animation or video superimposed over whatever is on the camera's screen. The effect makes the computer-generated item appear as if it is really there.

First, GeoGebra 3D AR may be one of the best AR tools used, thanks to its free access and multiplatform with a large social base. GeoGebra 3D AR allows one to choose from pre-created tasks and display arbitrary shapes in extended reality (Figure 1):

Figure 1. GeoGebra 3D AR

GeoGebra 3D AR's scope is almost limitless at any level of public education. Even in primary school age groups, it is worth improving problem-solving computation through real-life examples:

- exploring the surface area of a rectangular prism;
- exploring volume;
- finding the diagonal of a rectangular prism;
- modifying a triangular prism;
- unwrapping a cylinder;
- studying the anatomy of a right circular cone;
- exploring an octahedron net;
- exploring a sphere as a locus.

IKEA Place is a useful and free AR application that includes many options for improving space skills (Figure 2).

Figure 2. Ikea Place application

The application allows children to arrange a room or apartment of a certain size in real time with real-size elements. This allows them to lay out, for example, a 2-meter by 3-meter room in such a way that includes a bed, table, television, and wardrobe. It also has a development factor of high creativity.

The AR app for MeasureKit – AR Ruler Tape (see Figure 3) – is also useful and offers many options.

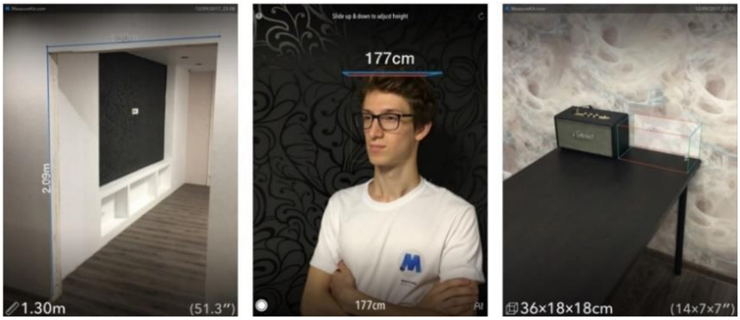

Figure 3. MeasureKit – AR Ruler Tape

Leveraging Apple's new ARKit technology, MeasureKit contains the following measuring AR tools:

- Ruler – measure straight lines on any surface, such as a desk or wall.

- Magnetometer – measure magnetic field force around one's device.
- Trajectory – measure by "drawing" (moving one's device) in the real world.
- Face Mesh – check different attributes and export a 3D model of one's face (available on iPhone X).
- Marker Pin – measure distance from device camera to fixed points in space.
- Angles – measure corners.
- Person Height – measure how tall someone is.
- Cube – visualise how big something is.
- Level – check if something is horizontal or vertical.

Chaining Mode in Ruler Tool allows a user to quickly link multiple rulers together in one measuring session and measure the floor area enclosed by a chain – this is useful for measuring an area of a room. MeasureKit features an advanced internal algorithm for world understanding using ARKit. This allows the app to offer a smooth and accurate measuring experience, including on walls and other vertical planes. While most similar AR ruler apps are limited to measuring accurately only on a desk and floor, MeasureKit enables users to easily measure a painting on a wall, for example. In addition, in the Geometry-AR app, computing tasks can be performed quickly in AR (Figure 4.):

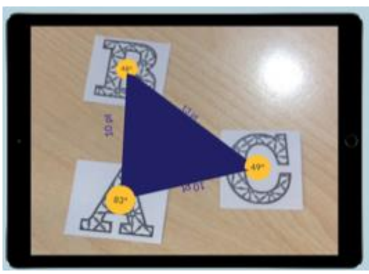

Figure 4. Geometry-Augmented Reality application

The webpage http://www.quivervision.com/ offers some basic instructions for using models of platonic solids to engage in print-colour-play. Pupils must print and colour in the given worksheets on their own, and the app will then bring their worksheets to life. For example, based on a coloured (patterned) net of Platonic solids, the app folds the net into 3D shape. It is magical for pupils (see Figure 5):

Figure 5. Quivervision Platonic Solids

Based on educational experience, the use of AR technology will be worthwhile to complement our traditional development tools. In addition, it has a significantly positive impact on students' motivation.

Conclusions

Spatial ability is one part of geometrical thinking. Using AR and other information and communication technology (ICT) tools in the educational process of mathematics (both quantitatively and qualitatively) makes a more effective development of geometric thinking possible, rather than the development taking place on traditional devices exclusively (Fuchsová, Koreňová, 2019). The above-mentioned tools are some easy ways in which to motivate students. Those tools also make the acquisition of different problem-solving strategies easier. Moreover, it is possible to support the development of students' analogical cognitive processes at the primary and secondary level.

These applications also allow one to work with different geometric shapes and solids, and they help to explain the properties of those geometric figures (see Partová, Marcinek, Žilková and Kopáčová, 2013). Finally, exchanging experiences in education is possible through the use of different AR applications in a wider international context.

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