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Teaching Mathematics and Computer Science

The tools for developing a spatial geometric approach

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Abstract. Tamás Varga writes about the use of tools: "The rational use of tools - the colored bars, the Dienes set, the logical set, the geoboard, and some other tools - is an element of our experiment that is important for all students, but especially for disadvantaged learners." (Varga T. 1977) The range of tools that can be used well in teaching has grown significantly over the years. This paper compares spatial geometric modeling kits. Tamás Varga uses the possibilities of the Babylon building set available in Hungary in the 1970s, collects space and flat geometry problems for this (Varga T. 1973). Similarly, structured kits with significantly more options have been developed later, e.g. ZomeTool and 4D Frame. These tools are regularly used in the programs of the International Experience Workshop (http://www.elmenymuhely.-hu/?lang=en). Teachers, schools that have become familiar with the versatile possibilities of these sets, use them often in the optional and regular classes. We recorded a lesson on video where secondary students worked with the 4D Frame kit. We make some comments and offer some thoughts on this lesson.

Keywords and phrases: Manipulatives, Platonic solids, plane and space geometry.

MSC Subject Classification: 97G40, 97D40.

Introduction

The main goal of our research team is to come up with a concept - based on the mathematics teaching and methodological traditions by Tamás Varga - which takes the last decade's scientific development and technological advancement into consideration.

Our paper would like to present how this goal is reflected in the usage of tools. Why did we base our lesson on building regular solids? For one, the building of regular solids and two-dimensional lattices are featured in Tamás Varga's book: "Játsszunk matematikát! 2. Tér és sík, valószínűség, logika, kombinatorika" („Let's play mathematics! 2. Space and plane, probability, logic, combinatorics), however not all of them can be built from the Babylon set, but can be built from the 4D Frame we used. Also, the topic allows us to extend in multiple directions, for example in mathematics, but we can also show the connections between different subjects.

The spatial skills, and the development of spatial skills (review of literature)

One of the main goals in the teaching of geometry is the development of spatial skills. The concept of spatial skills comes into sight in different intelligence theories, spatial intelligence appears as an independent and important component in each of them. Spatial skills are one of the most complex intelligence factors, which includes the establishment and detection of spatial relations, visualization, and spatial orientation (Herendiné Kónya E. 2007). „Research evidence has shown that spatial skills play a crucial role in developing expertise and success in science, technology, engineering, and mathematics (STEM) fields." (Ha, O., & Fang, N. 2018). Good spatial skills are basic competencies in many professions, such as architects, mechanical engineers, electrical engineers, decorators, doctors, and it is an essential concept in our everyday lives as well. A lot of research has gone into at what ages, with what efficiency can spatial skills be developed. "Most research has shown that the most efficient development comes during childhood, before the age of 10. A. Kárpáti and V. Gyebnár (1996) has come to a surprising result. They think visual education, in general, does not affect spatial abilities. It looks like spatial skills - representation of space and reading of such figures - are the visual semi-skills that can be improved by educational programs only to a small extent or not at all at certain ages." During our experiment, only the environmental culture program seemed to advance such skills. In this program students of ages 11 to 14 performed manual work. They made scale-modells, models and three-dimensional objects.” (Ekéné Abuczki M. 2014). We can say that traditional, manipulative visual aids are quite useful in the development of spatial skills. The students can pick them up, inspect them from various angles and disassemble-assemble them.

The importance of the use of tools in the teachings of mathematics can be supported by various theories, such as Bruner’s representation theory. The process of learning mathematics gets damaged if there is not enough time for illustrations, or for the timely discovery and the studying of relationships. The process of learning takes place too often on the symbolic level. The usage of well-chosen tools strengthens the involvement of material level thinking. According to Bruner’s (Bruner, J. S., Olver, R. R., & Greenfield, P. M. 1966) theory, the flexibility of transition between representative levels helps the development of creative thinking.

One achievement of the learning theory of Tamás Varga’s complex mathematical experiments was that he achieved the acquisition of knowledge from personal experience, in accordance with the results of developmental psychological research. He wanted to provide a sufficiently broad personal experience for every child - acquiring appropriate material, manual and mental activities for them -, so they can gather knowledge from generalizing and abstracting personal experiences with the teacher’s help and guidance (Klein S. 1980). This kind of learning comes with the need for many tools, significantly more and better organization, a different way of teaching and different ways of control methods than the usual frontal instruction. The usage of tools was thought of as very important by Varga Tamás as well, this is what he wrote regarding the topic: "The rational use of tools - the colored bars, the Dienes set, the logical set, the geoboard, and some other tools - is an element of our experiment that is important for all students, but especially for disadvantaged learners." (Varga T. 1977)

We can say, high schoolers’ spatial skills are usually unsatisfactory based on decades of experience in teaching high schoolers and in higher education. The mathematics curriculum has gone through some changes with the introduction of the National Core-curriculum: statistics have been brought in as a whole new chapter, and probability has been extended. This means there is less time for geometry, which affects the development of spatial skills in a negative way (Katona J. 2012). As spatial orientation and spatial skills are emphasized in the final examination, in higher education, in several professions and also in everyday navigation, we can say that the use of manipulatives is very important (of course along with computer modeling, which we will not be discussing in this article), as the best tool for developing spatial skills in primary- as well as high schools.

Geometric Modeling Kits

The Babylonian game

The Babylonian building set consists of balls with small holes and rods which fit into those holes. The holes are located in such a way, that three, four, six or eight-pointed stars can be built, as shown on the left side of Figure 1. This means that regular triangles, squares, hexagons, and octagons can be built from the set, but no other regular polygons. We can then create regular- and semi-regular grids from these regular polygons as shown on the right side of Figure 1. We can also see – this time on the bottom right side of Figure 1 – regular Platonic solids which we can build from the set, these are: the hexahedron (cube), the tetrahedron and the octahedron. We cannot build the icosahedron and the dodecahedron from the Babylonian game (Varga T., 1973).

About the ZomeTool modeling set

The inventors of the Zometool set: Marc Pelletier and Paul Hildebrandt. The inventor’s comments about the set: “Even though Nobel prize and Wolf prize winners – such as Linus Pauling or Richard Smalley (the discoverer of the C60 Buckminsterfuller giant molecule) and Dan Shechtman (discoverer of the quasi-crystals) – use/used the Zometool during their work, it was originally made for children. Why? It is because people learn new languages much faster and much easier during their childhood...”

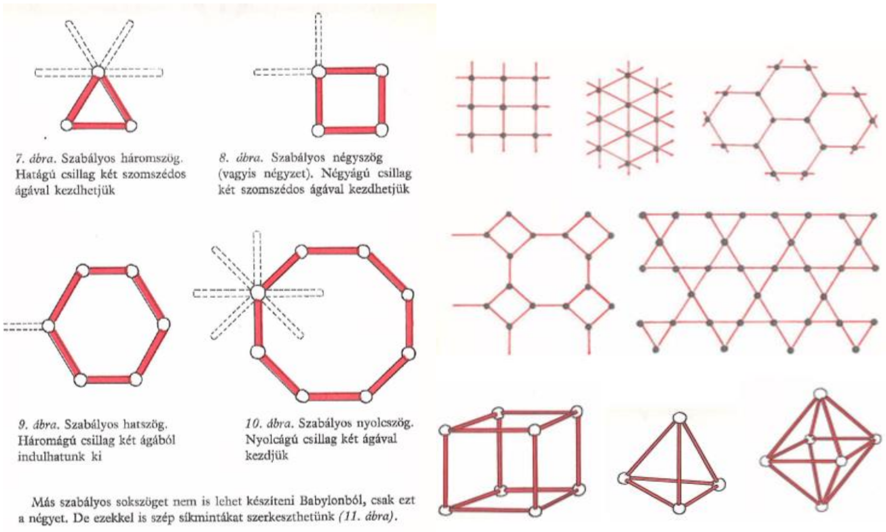

Figure 1. Figures from Tamás Varga's (1973) book: Játsszunk matematikát! 2. Tér és sík, valószínűség, logika, kombinatorika (Let's play mathematics! 2. Space and plane, probability, logic, combinatorics).

The Zome model's vertices are similar to a semi-regular body, the rhombicosidodecahedron, but are different in a way that they swapped out the square to a rectangle (the ratio of the rectangle's sides matches the ratio of the golden section), most probably so the end of the triangular rods can be sturdier, so it does not break so easily. The yellow rods are triangular, so they can be placed in the vertex's 20 triangular openings. The blue ones are rectangular, these can be placed in the vertex's 30 rectangular openings, the red ones are pentagon-shaped which we can place into 20 openings. The green rods are also pentagon-shaped, but we can place these in 5 different directions in the opening. This means that with the Zometool, we can build into 20+30+12+5*12=122 directions, however, we can find a stick on the opposite side pointing in the same direction, in every direction, which reduces the building potential to 61 spatial directions. There are different sizes in each color. The quotient of successive dimensions matches the ratio of the golden section.

In the Original Zome model, we could only build the cube, the icosahedron and the dodecahedron from the Platonian solids with the blue, red and yellow rods. One reason for bringing in the green rods was so we can build all the regular solids. We can only build the tetrahedron and octahedron from the green rods (Hart G. W. & Picciotto H. 2001), (Hildebrandt P. 2007), (Davis T. 2007). https://www.zometool.com/.

4DFrame for free imagination and infinite creativity

The 4Dframe's inventor, developer was Korean engineer Ho-gul Park. Inspiration for the model came from traditional Korean wooden architecture, where they built structures without the use of nails. We can use the 4Dframe in teaching STEAM[1], and in recent years the use of it became widespread.

The 4Dframe set consists of hollow tubes (similar to drinking straws) and connecting stars (bridges). The connectors' junctions spread from 1 to 10. The big benefit of the set is its flexibility, the tubes can be cut into preferred sizes and they can be connected with specific elements to make them longer if required, the connecting stars are freely bendable. Therefore the range of models that can be made is almost infinite. Naturally, all of the 5 Platonian solids can be built from the set, but we can also make bounded by curves - such as the torus or Klein-jug - using this set. We can also make bodies that require moving elements, such as a bicycle, a car, a windmill, but we can also make mechanisms for robots. (Park H. 2013) http://www.nordic4dframe.com/eng/

Constructing Platonic bodies from the 4Dframe

The lesson on video

Location: Tamási Béri Balogh Ádám Catholic Gymnasium
Date: 2018. 05. 25.
Participants: Class 11. NY, Grade 11 in the 4+8 system, 16-17-year-olds. They now

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[1] The complex thinking, referred to in international literature as the integration of "STEM" (Science, Technology, Engineering, Mathematics), is complemented by the tools of aesthetic and artistic education (Science, Technology, Engineering, ART & Mathematics), (Szabó I., Fenyvesi K., Stettner E. 2014).

have their „language year”, no new curriculum, the material of the last two academic years is reviewed, skill development (earlier this was the language year of the 4+8 classes, this system is no longer in place, it was moved to Grade 9).
The idea: Constructive proof, Witzlné, Ulrich Anikó, mathematics teacher
Students worked in groups of 3-5 (there were 6 groups) using 4Dframe.
Classroom: We used two **cameras**, one moved and the other fixed.

The chain of thought of the lesson, its logical structure

Firstly, the concept of a regular polygon was reviewed, then the concept of a regular polyhedron was reviewed. Why do we have exactly 5 regular solids? Let's get back to the regular polygons! Which ones can be used for tiling the plane?

Then the lesson continued with the following questions, while the students built the grids and polyhedrons step by step.

Let’s make a regular triangular grid!

- What kind of connecting elements should we use?
- How many triangle vertices meet on one point in the grid?
- What happens if we take out a triangle?
- What should we switch the six-way connecting element to?
- Let’s try building a body this way!
- Let’s examine the number of faces, vertices, and edges!

We call this body the **icosahedron**.

- Let’s take out 2 triangles that fit onto one vertex on the original triangular grid!
- Let’s switch out the connectors once again! What kind of connectors should we use now?
- How many faces are needed to build the solid? Let’s examine the number of vertices and edges!

We call this body the **octahedron.**

- Can the number of triangles be reduced further in the triangular grid?
- How far can we continue this procedure?
- What kind of connectors do we need this time?
- Let’s examine the number of faces vertices, and edges!

The name of the body is the **tetrahedron.**

- Let’s make a regular rectangular grid!
- The vertices of how many squares fit on a grid point?
- Let’s take one of them out!
- Let’s build the solid!

This is the **hexahedron**, also known as the **cube**.

- Can we continue this procedure?
- Can we build a solid using this procedure from a hexagonal grid?
- We are unable to cover the plane with regular pentagons, but we can move the 3 pentagons which fit onto one vertex in space.
- How is this possible?
- How many pentagons are needed to build a solid?
- Let’s examine the number of vertices and edges!

This solid is the **dodecahedron**.

The bell rang at this point, the discovery of the Euler theorem, and the interdisciplinary outlook remained for the next lesson.

Figure 2. Some snapshots from the lesson

Some notes for the lesson

The work of the groups was very different. There were groups who, after building the first two polyhedra, figured out how to proceed and were able to make the next solids even before the class discussion. There was a very lagging group who also built shapes that had nothing to do with the theme of the lesson. They could only build one icosahedron per group because few of the type five interfaces were available. Each group was able to make multiple copies of the other polyhedra. Although the seating arrangement and the distribution of tools inspired group work, students worked individually most of the time. The students were rather passive, this is most likely because this year was a level year from mathematics, without grading. The teacher went over the building of regular solids using the same methods with grade 5 students during an afternoon activity. The teacher had a much better experience during this activity, the students were a lot more active, more cooperative. The young ones modelled more before, they were more used to it.

Opportunities to move forward

After the students got to know the five regular polyhedrons and realized there can only be five, we can start discussions about topics in other subjects that are closely related to our everyday lives, and of course not only during classes.

We can tell them how and why ancient Greek philosophers identified regular polyhedrons with the four elements and the cosmos. We can talk about controversial archaeological research, which suggests the discovery of polyhedra among paleolithic finds. We can tell them how Kepler linked the structure of the solar system to the five regular bodies, why this theory was accepted for a long time, and whether it is scientifically substantiated. Meanwhile, the evolution of science, the change of scientific theories, and the relationship between science and pseudoscience may also come up. We can show pictures of tiny protozoans that take on the shape of Platonian bodies, viruses that have the shape of icosahedrons and dodecahedrons (in relation to biology), crystals (chemistry). Even gambling can be brought up, as the Egyptians and Romans used not only six-sided dice but numbered sided icosahedrons.

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