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Teaching Mathematics and Computer Science

Mathematical Laboratory: Semiotic mediation and cultural artefacts in the mathematics classroom

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Abstract. Aim of this presentation is to summarize the influence of Tamas Varga on the Italian research and practice concerning didactics of mathematics since the 70s of the 20th centuries. While being in Budapest for the Conference I noticed that this influence was not known by most Hungarian mathematics educators. I guess that also in Italy, only the teacher educators of my generation know Varga's influence on the teaching and learning of mathematics in primary school. Hence I start from a brief summary of development of mathematics curriculum in Italy (mainly in primary school) in the last decades of the 20th century. I focus some elements that may be connected with Varga's influence and, later, some recent development of them.

Key words and phrases: Varga's influence, semiotic mediation, cultural artefacts.

MSC Subject Classification: 97G20, 97-U6, 97A40.

Introduction

Discussions about school curriculum started in Italy after the unification of the country in 1860. Then from time to time (1860, 1867, 1888, 1905, 1923, 1945, 1955, 1985, 2007, 2012) school reforms were designed and issued, with more and more references to the international trends, especially when the foundation of the International Commission on Mathematical Instruction (Rome, 1908, Menghini et al., 2008) allowed to introduce an international perspective into the national process. There is no time enough here to reconstruct the whole process, not even with limited reference to the field of mathematics. Anyway, also the choices for mathematics (arithmetics and geometry) mirrored general pedagogical choices, addressing what has to be considered positive, appreciable and valuable. In other words, students must become „better" in some sense. These choices may be illustrated showing, for instance, the different instructions about using manipulatives in subsequent versions of the mathematics curriculum. In the list of the furniture and materials required in a classroom the slavonic abacus appears in the Italian programs 1923. Yet in the past documents the abacus was not always mentioned as a useful tool. For instance, in 1860 the official instructions read

The calculation in mind will have to be done on the concrete numbers with the help of the so-called abacus or other mechanical tool.

whilst a few decades later the situation was completely different.

Reprehensible is the custom of suggesting, in mental calculation, gimmicks and mechanical means, which, rather than helping, suppress reflection and reasoning. The work of the intellect must be independent and safe, that is to say, proceed according to the reasons for the composition and decomposition of the numbers (Italian Special Instructions, published in 1894).

At the end of the second world war (1945), a strong reform was realized after twenty years of fascism, but the most popular reform was issued in 1955. This reform remained in force for thirty years and had a significant influence on teaching practice even longer. In that period, thanks to the more and more frequent contacts between Italian mathematics educators and the international community, there was, in some parts of Italy, some innovation in the teaching practice. Publishers and producers of teaching aids proposed to schools books and materials inspired by the so called "New math" trend, but there was no official endorsement of new math of the Ministry of Education.

The path towards the "new" programs for elementary school

In the period, between 1955 and 1985, some books by Zoltan Dienes were translated into Italian and Dienes himself was invited by some institutions to give lectures for primary school teachers.

In the same years also Tamas Varga started to visit Italy from time to time, giving lectures and seminars in different parts of the country. A strong collaboration was realized with Morin Centre (Paderno, in northern Italy)[i] and with a group of mathematics teachers in Rome. The second collaboration was started by impulse of Michele Pellerey, a member then Vice President and President of the CIEAEM (the International Commission for the Study and Improvement of Mathematics Teaching) (Cannizzaro, 1979).

Many scholars from Eastern Europe were paid attention in Italy (e.g. Hungary, Československo, Poland, Soviet Union) especially for early grades mathematics education. As to my knowledge there is no detailed international study to explain this phenomenon: surely among Italian mathematics educators (and, more generally, among Italian educators) there were less ideological prejudices against marxist ideas about education: rather there was a strong interest for the marxist philosophical foundation of education where the contributions of authors such as Vygotsky, Gramsci and others were emphasized (see, for instance, in Italian, Bertoni Iovine, 1965; Manacorda, 1971; Baldacci, 2017).

Varga's influence was especially strong on the RICME (Rinnovamento del Curricolo Matematico Elementare - Innovation of Mathematics Elementary Curriculum) project, that was realized in Rome, under the leadership of Michele Pellerey in the 70s of the past century. In RICME (1979) there is this reconstruction of the process (p. 7 ff.). In short, in 1974 a seminar was held by Varga in Rome, where Hungarian OPI (*Orszagos Pedagogiai Intézet*) project was introduced. The project OPI had been extended in Hungary to the whole primary education. The success of the seminar addressing teachers and educators was very large and the idea of preparing an Italian curriculum for primary school was suggested. The project was funded by CNR (National Council for Research) in 1975. The process was very complex. Some parts of RICME materials were "simpy"

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[i] Morin Centre has a very rich library and publishes from 1970 the most important Italian journal about Mathematics and Science Education, that was a precursor of the STEM (or STEAM): *L'insegnamento della matematica e delle scienze integrate*. Every year the centre organizes a seminar for teachers,

translated from the Hungarian OPI (e.g. combinatorics and probability), but some parts were written anew (e. g. geometry) whilst some parts were reduced (e.g. the function of some teaching aids, such as Multibase Arithmetic Blocks). According to the scholars in charge of the RICME project, it was not possible to simply translate the OPI project, without considering some Italian traditions, such as the geometric activities (Cannizzaro et al., 1981) carried out by some Italian scholars (e.g. Emma Castelnuovo) in connection with art (e.g. perspective) and the basic activities in early "computer science" (what is now known as *coding*, e.g. Fasano et al., 1983; Fasano, 1988). Mathematics, and therefore Mathematics Education, are fields of knowledge that are culturally determined (i.e. that develop within a culture in dependence of its set of customs, beliefs and values). In this viewpoint, both Mathematics and Mathematics Education have a set of cultural implicit assumptions, such as the signs to be used and the way to use them, the language and the ways of representing it, habits of behavior, beliefs about the nature and the scopes of education (Mellone et al. in press). All these aspects are so deeply intertwined with and rooted in the cultural context that they are almost invisible to the members of the culture itself: we refer to them with the word "unthoughts", namely things people of a certain culture take for granted until they meet with a different culture. Hence, what is culturally determined – as are didactic practices – is so linked to several cultural beliefs, customs and habits, that a simple translation of a didactic practice or a content from one cultural context to another one could be not sufficient to make it work in the new context. A more complex action, then, is necessary. This process was actually activated in the RICME project.

The Italian group for RICME remained in connection with Varga. In Italy the Klein's book (1975) on the evaluation of the OPI mathematics project was published with emphasis not only on the mathematical contents but also on creativity, flexibility, a positive attitude towards mathematics and motivation.

The RICME project was designed and tested for a few years by some dozens of elementary school teachers between 1975 and 1980 (Pellerey 1978, 1991). Materials for teachers were published at the beginning of the 80s (Progetto RICME, 1979).

The RICME project was very influential in the process of innovation designed in some places where the limits of the 1955 Programs started to become evident. For instance (Bartolini Bussi, Franchi, Lancellotti, Malara, 1982), as a result of action research with a large number of elementary mathematics teachers, published a comparison of four different innovation projects about arithmetics coming from Italy or other countries (Nuffield, Canton Ticino Project, Dienes and RICME projects).

In the early 80s a national committee for the “new” programs for primary school was appointed by the Italian Ministry of Education. Michele Pellerey entered in that committee and gave values to the ideas developed in the RICME project, where the influence of Varga had been strong.

The “new” primary school programs (1985)

In 1985 the „new” primary school programs were issued after a long debate involving, at least for mathematics, the whole community of mathematics educators and many mathematics teachers. In some sense, this text, had to fill the gap between the previous institutional text (1955) and the innovations realized in schools in the thirty years between, including the discussions around some more ambitious projects (such as RICME). Actually when the “new” programs for elementary school were issued (1985) the text took the distance from the practice of “New math” that had been tested in some schools without any official document. A short excerpt of the text shows the need to connect the school practice to the new aims:

The teaching of mathematics in elementary school has for a long time been conditioned by the need to provide the child with indispensable tools for practical activities. With the expansion of education, it was possible to focus more decisively on educational objectives. In this situation, which offered a wider design freedom, the teaching of mathematics, in almost all countries of the world, was oriented towards the direct acquisition of mathematical concepts and structures and promoted an intense experimentation activity also in Italy. The vast experience gained has however shown that it is not possible to reach mathematical abstraction without following a long itinerary that connects the observation of reality, the activity of mathematics, the resolution of problems, the conquest of the first levels of formalization. The most recent didactic research, through a careful analysis of the cognitive processes in which the learning of mathematics is articulated, has revealed its great complexity, the gradualness of growth and non-univocal lines of development. In this context, it has been found that the algorithms (that is, the ordered procedures) of calculation and the study of geometric figures also have a formative value far beyond the practical uses that once justified their inclusion in the programs (Nuovi Programmi Didattici per la scuola Primaria, 1985, p. 23, translated by the author).

Beside some innovations carried on in Italy, the influence of scholars such as Freudenthal and Varga was evident. For instance, Freudenthal (1973) influence appeared in the multifaced approach to numbers to complement the most popular cardinal approach introduced by Piager. The introduction of natural numbers hinted at different approaches: perceptual approach (subitizing), recursive approach (counting), cardinal approach (sets), ordinal approach (number line), measuring approach.

Varga's influence was evident in the introduction of some early elements of combinatorics, probability and statistics that were new for the Italian mathematics teachers.

Significant educational importance must also be recognized in concepts, principles and skills connected with the statistical representation of facts, phenomena and processes and with the elaboration of judgments and forecasts in conditions of uncertainty. The introduction of the first elements of probability, which can find a place at the end of the elementary course, has the aim of preparing an intuitive ground in the child on which it is possible, at a later stage, to base the rational analysis of situations of uncertainty.

The classic definition of probability - as the ratio between the number of favorable cases and the number of possible cases in symmetrical random situations - cannot be taken as a starting point, but is rather the arrival point of a well-graded activity.

The release of the new programs was followed by a strong effort of in service teacher education, that involved the whole community of mathematics educators all over the country. A detailed list of reference is in Cannizzaro, (2019).

The 21th century: the Mathematics Laboratory

At the beginning of the 21st century, a committee chaired by Ferdinando Arzarello and appointed by the UMI-CIIM (Italian Mathematical Union, Italian Commission on Mathematical Instruction) prepared a Curriculum for Mathematics for primary and secondary schools. Some volumes were produced and are available (in Italian) in the UMI-CIIM website (https://www.umi-ciim.it). A synthesis of these documents (in English) was prepared for the ICME10 (2004).

In the methodological part a special emphasis was given to the Mathematical Laboratory. A webpage (in Italian) on the Mathematical Laboratory was created in the institutional website of UMI-CIIM for all the Italian mathematics teachers from primary to secondary schools.

A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments). We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students). It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning cannot be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities (Matematica 2003, p. 26, translated by the author).

Different examples of instruments are offered in Matematica 2003: e. g. "poor" materials, mathematical machines, dynamic geometry software, original sources from the history of mathematics. Hence, in this vision, a Mathematical Laboratory is neither reduced to modelling nor to experiments; moreover it does not contain only computers. It is rather related to meaning construction, hence to genuine experiences of mathematical reasoning (e. g. formulation of definitions, production of conjectures, and construction of proofs). It aims at exploring the cultural values of mathematics by means of historical sources and problems.

In the quoted documents some methodological indications are offered, mentioning the importance of mathematical discussion and, generally, of semiotic activity, that are carefully described in the document produced for ICME10. The document does not contain specific references, but the description hints at the theoretical frames developed at the beginning of the 21st century by research groups chaired by Bartolini Bussi (Bartolini Bussi & Mariotti, 2008) and Arzarello (2006).

An example of Mathematical Laboratory in primary school

The introduction of concrete teaching aids is a tradition of primary school. Varga too, as was evident in the exhibition in Budapest, collected a lot of materials and also invented some. Yet the challenge is to orient the practical activity towards the construction of mathematical meaning. This means to exploit the concrete exploration to construct mathematical reasoning.

I present here an experiment that exploits some ideas typical of the Italian tradition, where art and geometry were connected to each other. The experiment was realized in a 3rd grade classroom (Bartolini Bussi, 2017). In the experiment some historical sources (by Piero della Francesca) were read and cardboard perspectographs, one for each student, were constructed. A perspectograph is a traditional tool used in the painters' workshops as from the Renaissance period.

Using a perspectograph may be only a practical activity or a concrete game (Figure 1). In this figure a 6 year old boy visiting an exhibition draws a puppet (a cat) by transparency. His comments are: *I am not good at drawing, but with this gadget I have drawn a beautiful cat.* This is a beautiful experience but it is not geometry yet.

Figure 1. Drawing a puppet by transparency: an exciting experience.

Now, let's move to the experiment in the 3rd grade (Rosi, 2016). The 3rd graders used a cardboard perspectograph to draw (by transparency) a chessboard. The cardboard perspectographs can be folded and assembled (Figure 2) and then opened to become flat (Figure 3). In this case, however, the drawing is not completed: only the vertices of the chessboard are drawn as they appear on the transparent sheet, and later the extremes of the images of the lines of the grid of the chessboard. As a result, only points (not lines) are drawn on the transparent sheet. Then after opening the perspectograph the basic points are connected by ruler and felt pen. This procedure hints at a geometric task. The mathematical meaning may be summarized as follows: *a straight line of the chessboard is represented (projected) on the transparent sheet as a straight line. Hence it is enough to connect the extremes of the line to draw the whole line.*

The experience with the cardboard perspectograph, as documented in Rosi (2016), showed a rich evidence of metaphors and narrative thinking, as the students put together the reading of historical sources, the handling of a „new” (but ancient) artefact, the astonishing outcomes of their activity, an original way of using ruler and felt pen. They were also able to explore the cultural aspects of their experience, visiting an important exhibition on Piero della Francesca, one of the „fathers” of the Italian perspective (Camerota et al., 2015, Figure 4).

Figure 2. Drawing some basic points with a perspectograph

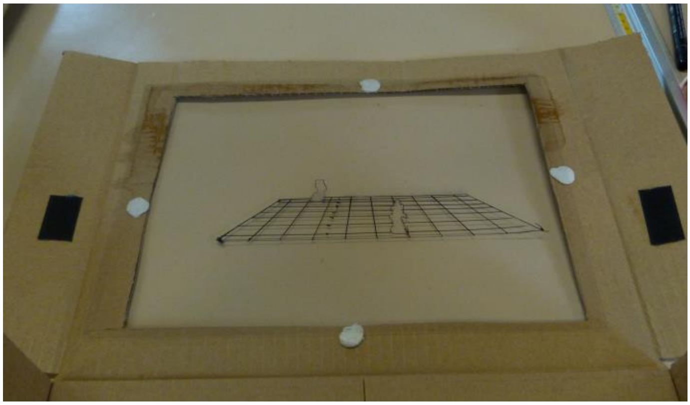

Figure 3. Connecting the basic points in the open perspectograph.

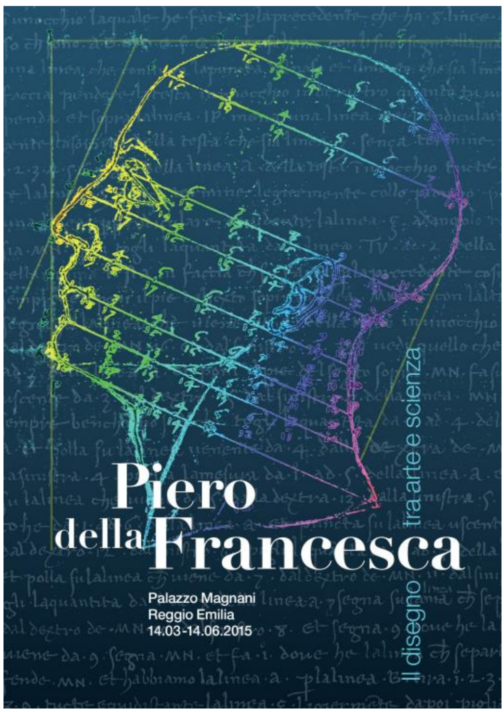

Figure. 4. The cover of the catalogue of the exhibition on Piero della Francesca (Camerota et al. 2015).

Concluding remarks

This experiment is an example of STEAM (Science, Technology, Engineering, Art and Mathematics, Contini & Manera, 2019). This trend has received much attention in the last decades in Europe and US. Experiments in STEAM realized in my Department within the MANIS centre (http://www.manis.unimore.it/site/home.html) have allowed to exploit the power of metaphors and narratives in science and mathematics teaching.

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One of the most active member of the International Scientific Committee of Manis is from Hungary: Zoltan Kövecses, Eötvös Loránd University, Budapest (Ungheria). I plan to discuss with Zoltan his relationship, if any, to Tamas Varga.

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