18/3 (2020), 157-166
DOI: 10.5485/TMCS.2020.0489
tmcs@science.unideb.hu
http://tmcs.math.unideb.hu

Teaching Mathematics and Computer Science

Potential, actual and practical variations for teaching functions: cases study in China and France

LUXIZI ZHANG

Abstract. This contribution is based on two major hypotheses: task design is the core of teachers' work, and variation is the core of task design. Taking into account variation in task design has a profound theoretical foundation in China and France. Developing my PhD with two co-supervisors, in China and France, I wish to seize this opportunity for constructing an analytic model of "teaching mathematics through variation" making profit of this theoretical diversity. This model distinguishes between *potential variation* and *practical variation* and is based on the process of transforming potential variation into actual variation, and of using practical variation for rethinking potential variation. The design of this model is based both on theoretical networking, and on case studies, in France and China. In this contribution, we will focus on a critical aspect in the two cases, from potential to practical variation.

Key words: teaching through variation, task design, teachers' documentation work, model of variation, comparison.

MSC Subject Classification: 97-06.

Variation understood as the core of task design

The massive growth of resources, as well as the Internet potential, offer teachers new possibilities for selectively choosing tasks which address precisely their teaching goals rather than relying on the authority of an unknown author from the web (Watson & Thompson, 2015). Therefore, the role of task design, as part of teachers' documentation work (Gueudet & Trouche, 2009), is becoming more and more important. We assume that variation is the core of task design, as it could help teachers to choose tasks from a good number of resources and adapt the resources to their own classes.

Our PhD thesis is situated in between the variation traditions in China and France and aims to think fruitful interactions between them. In this paper, we focus on teachers' documentation work through the model of variation, selecting, in China vs. in France, task(s) chosen for introducing the notion of function, a topic that appears particularly rich for offering a large potential for variation, and selecting experienced teachers, able to take profit of students' feedback. With this contribution, we wish to contribute more particularly to the issue of frameworks and principles for task design through the lens of variation.

This paper describes and discusses variation from the following aspects: Variation in Chinese and French cultures and mathematics education; Modeling variation as grounding teachers' documentation work; Chinese case: potential variation embedded in series of tasks; French case: practical variation generated in the interaction between teachers and students in open tasks; making a simple comparison of two cases.

Variation in Chinese and French cultures and mathematics education

When we go through Chinese and French culture or tradition of mathematics education, it is not hard to find the idea of variation embedded inside.

Extending a lesson/exercise with variation practice can be traced back to Chinese traditional philosophy and culture, which could be rooted in the ancient "general consensus" developed from the system of Daoism and Confucianism. For example, one of the oldest classic Chinese texts from the Chinese I-ching theory (The Book of Changes), claims "abstracting invariant concepts from a varied situation" and "applying these invariant concepts to the varied situations" (Sun, 2011). Doubt and realization form the central theme of learning in Chinese culture, like Zhu Xi stressed on arousing skepticism and reflections of learners: "Reading books [learning] is to arouse doubt when one does not doubt and let those in doubt settle in the state of no doubt. This is how one grows" (Wong, 2004). Therefore, the master's guidance and initiation are of vital importance and variation could be helpful for arousing the students' doubt.

From the perspective of mathematics education in China, "teaching through variation" can also serve as a bridge between the "basic skills" and the development of higher-order thinking abilities (Zhang & Dai, 2004). According to Sun (2011), variation problems could be regarded as a curricula implementation tool encircled by the invariable "Two Basics" (namely, basic knowledge and basic skills, proposed by the Ministry of Education), which are characterized by "dispersed and progressive difficulties". Over the years, "Two Basics" has gradually refined, however the goals of mathematical education remain the same.

For the situation of France, we only take out the variation in French language as an obvious symbol of variation for a discussion. According to Leung (2017), language plays a crucial role in the culture since beliefs, ideas, etc., are constructed through the medium of language. French language is called as "fusional/inflectional language" (Humboldt, 1971), as there are relatively rich inflections, and the relationship between words is expressed through inflections. In French, verbs, adjectives, etc. can express different tenses, voices, and modes through their inflections (variations). Since all experiences (including the experience of mathematics learning) are mediated by language and language is the medium and tool for mathematics learning (Leung, 2017), we think that the idea of variation has been directly represented in French language which influence students' learning and teachers' teaching of mathematics.

When it comes to the variation in mathematics education in France, it is inevitable to mention French didactic research. There are three main theoretical pillars in French didactics: the theory of didactical situations due to Brousseau, the theory of conceptual fields due to Vergnaud, and the anthropological theory of the didactic that emerged from the theory of didactic transposition, due to Chevallard. (Blum et al., 2019). Steeped in these theories, Ghislaine Gueudet and Luc Trouche (2009) proposed the documentational approach to didactics (DAD) to study teachers' professional development. According to this approach, the process of developing the document works two ways: instrumentation (the affordances of the resources influence the teachers' practice) and instrumentalization (the dispositions and knowledge of the teacher guide the choices and transformation of resources, the teacher adapts to the resources). From the variation perspective, the former is varying teachers' knowledge and the latter is varying teachers' resources.

Modelling variation as grounding teacher's documentation work

In this section, we will introduce our preliminary model of variation from the perspective of teacher's documentation work.

As mentioned above, teaching through variation is a traditional and typical teaching method in China and the majority of Chinese teachers teach consciously or unconsciously through variation problems. Based on this phenomenon, Gu (Gu et al., 2004) conducted a systematic and in-depth experimental study and theoretical analysis on "BIANSHI jiaoxue" (teaching and learning with variation) and proposed two kinds of variation: "conceptual variation" (varying the connotation of a concept and its instantiations) and "procedure variation" (varying the way of solving a problem and metacognitive strategies). In fact, we can also find the relevant theoretical foundations in western approaches. For example, Marton's group has developed variation theory of learning, which also highlights the important role of invariance (similarity) in affording the discernment of differences (Gu et al., 2004). In France, the concept of didactical variable in Brousseau's theory of didactical situations is a key tool for organizing variation. Brousseau (2002) gives a definition of didactical variable as follows: A variable of a situation or a problem is called a didactical variable, when it can be modified by the teacher, and when its modifications (even slight ones) can significantly influence students' behavior and provoke different procedures or types of responses.

By studying and comparing several theories and cultural background, we propose a preliminary model of variation (Figure 1). This model aims to provide guidance to help teachers implement variation in class and reflect their own work after class. In this contribution, we introduce three notions structuring this model, potential, actual and practical variation, and illustrate their interest through excerpts of our case studies.

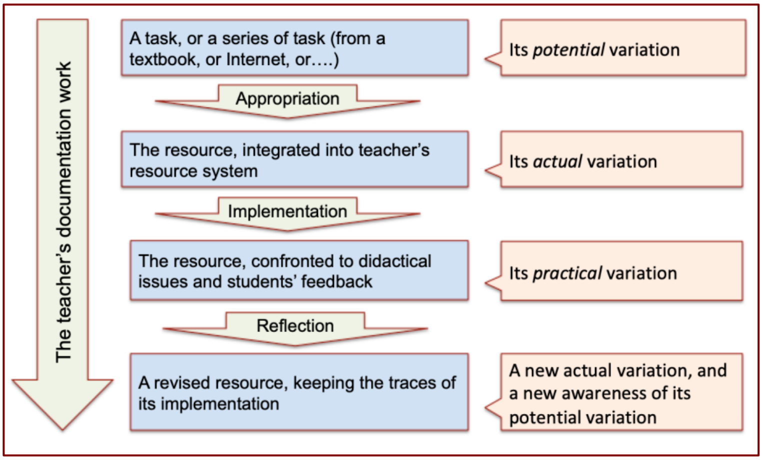


Figure 1. A preliminary model of variation

- Potential variation represents the possible changes offered by a given task, or series of tasks, meaning its possible conceptual and procedural variations. Determining potential variations supposes a priori analysis of the task, i.e. it falls under the responsibility of an expert;
- A teacher takes more or less profit of this potential variation for designing his/her resource, making explicit actual variation;
- Practical variation represents the changes occurring when implementing the resource in class, revealing to the teacher new aspects of potential variation, particularly from the point of view of didactical variables.

We believe that teachers' documentation work (Gueudet & Trouche, 2009) plays an important role to promote the design work from potential variation to practical variation and the reflection work from practical variation to a new actual variation or potential variation. Here, we regard teachers' documentation work as a thread that runs through the entire process of the model. What's more, to what extent can a potential variation be transformed into a practical variation is largely related to the teachers' design capacity. We try to use this model to analyze the data in the two following sections and to refine the model through case studies.

Chinese case: potential variation embedded in series of tasks

We choose an episode of one lesson on function to study in Chinese case. This episode is for introducing the notion of function to students in grade 10. The teacher chose a series of tasks (Figure 2) for supporting the students' work, interacting with the whole class during 20 minutes.

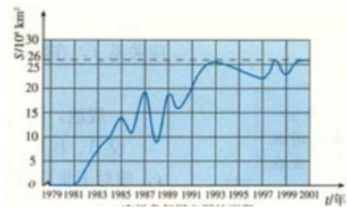
Two types of potential variation have been embedded in the task based on conceptual variation: i) Inducing concepts by varying visual and concrete instances: offering three contexts from daily life and making use of various representations of functions: such as a formula (example 1), a graph (example 2) or a table of values (example 3); ii) Highlighting essential features of the concepts through varying unessential features by providing a series of tasks: "thinking" is set at the end of three examples to provide an opportunity for students to reflect, compare and generalize the concept.

Regarding the actual variation, the teacher retained the series of tasks with their potential variations and added procedural variations to ensure students' learning coherence of comparing and reflecting tasks. First, the three examples were proposed one by one in class with the same two questions for each, one being "What are the ranges of the two variables? Try to represent them in set" and the other being "Describe the relationship between variables using the language of sets and correspondence". Second, she took profit of the technology. For example, she simulated the shell launching process with computer software and chose several specific times with the corresponding height to help students discover the correspondence relationship.

In this case, we may infer several potential variations embedded in the series of tasks and some procedural variations added by the teacher in the process of her adaptation work, both of them constituting the actual variation. However, the implementation is exactly what the teacher planned and there is no practical variation occurring in class. We think it is because the teacher is very experienced and has used this task for many years. Therefore, the cyclic way of teachers' documentation work has been developed in the past, leading to a well-structured lesson.

Example 1: A shell fired from a height of 845m felt down to the ground and hit the target after 26s. The height h (unit: m) of the shell from the ground varies with time t (unit: s): $h = 130t - 5t^2$.

Example 2: The ozone in the atmosphere has decreased rapidly due to the problem of ozone hole. The curve (right side) shows the change of the area of the ozone hole over Antarctica from 1979 to 2001.



Example 3: Engel's coefficient reflects people's quality of life. The lower the Engel's coefficient, the higher the quality of life. The change of the Engel's coefficient over time in the table below shows that since the "Eighth Five-Year" plan, the quality of life in China has changed significantly.

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Engels' coefficient	53,8	52,9	50,1	49,9	49,9	48,6	46,4	44,5	41,9	39,2	37,9

Thinking: Analyse, summarize the three examples above, what do they have in common?

Figure 2. The series of tasks (chosen by the teacher in the students' textbook, translation and adaptation by the author)

French case: practical variation generated in the interaction between teachers and students in open tasks

Unlike the teacher in the Chinese case choosing series of tasks, for introducing the chapter on the qualitative study of functions in grade 10, the teacher observed in France chose an open task (Figure 3), with which learners of different capabilities could interact, for three hours.

The potential variation can be illustrated here by procedural variation. Students need to use: (1) the notions of image and pre-image, of graph, and table of values; (2) the variation of a function: including representation by a graph, expression in algebra, using intervals, and the table of values; (3) the maximum and minimum of a function. The knowledge above is all embedded in the open task and the teacher can manipulate each of these elements as different kinds of variation according to her needs.

Taking a sheet of paper in A4 format, we want to build a box without lid, having the shape of a cuboid and the largest possible volume. What are its dimensions?

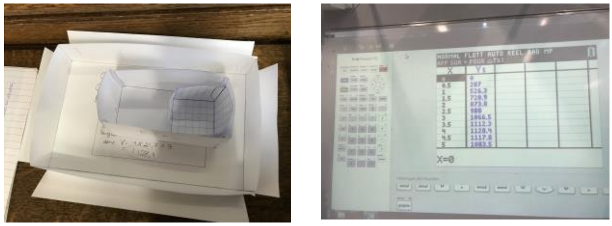


Figure 3. An open task (chosen by the teacher, translation and adaptation by the author) and pictures of students' work

Regarding actual variation, the teacher mainly adopted two strategies: (1) organizing student-centred classroom under teacher's guidance: firstly, the teacher asked the students to work in groups and submit a group report with their solution of the task. Then, she worked on these reports, classified the solutions, and decided their order of presentation (incorrect - partial correct - correct with some possible improvements - new method). The students' groups were asked to present their work following this order, the rest of students were asked to take notes and reflect on the others' solutions. At the end of this class, each student handed in his or her own report. (2) Transferring and dividing this open task into a lot of small closed tasks, aiming to work on different aspects of the notion of function, and by the time the teaching of this chapter is finished, the students will be able to solve this open task completely by themselves.

As for practical variation, besides the fact that the teacher classified the solutions and decided the order of presentation based on the group reports as mentioned, one of the students used a calculator to solve the task (see figure 3, right photo), which was beyond teacher's expectations, and appears as a critical didactical variable; after his presentation, the rest of the students expressed great interest in this solution. The teacher took this opportunity to let students have a preliminary understanding of using a calculator to solve this task, which is also required by the curriculum.

Teachers' work is very crucial in such open task case. In this case, the potential variation is very broad and it is necessary for the teacher to design specific variation and adaptions as actual variation according to her own teaching objective. Obviously, in this case, the teacher's documentation work is inseparable from practical variation, as she reflects, modifies, and adapts the task according to students' feedbacks and her own experience.

Discussion

Comparing the two cases above, the Chinese teacher and the French teacher use variation in a completely different way. Chinese teacher tends to choose well-organized task, which is targeted to her teaching objective, while French teacher tends to choose open task to extend the initiative of students as much as possible. Therefore, potential variations are clearly embedded in the task in the Chinese case. As for the French case, we observe that more practical variations are generated in the interaction between teacher and students. Time is also playing a critical role (much shorter in the first case), opening a more or less important time to students' initiatives.

The model showed here is a preliminary one, which needs to be improved. We hope that in the PhD thesis, we can present more contents about the context, the model and more data to support the model, about what happened in class and teachers' reaction, etc.

References

Blum, W., Artigue, M., Mariotti, M., Sträßer, R., & Van den Heuvel-Panhuizen, M. (2019). *European Traditions in Didactics of Mathematics.* ICME-13 Monographs. Cham: Springer.

Brousseau, G. (2002). *Theory of Didactical Situations in Mathematics*. Netherlands: Springer.

Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 309 – 347). Singapore: World Scientific.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, *71*(3), 199-218.

Humboldt, W. V. (1971). *Linguistic Variability and Intellectual Development*. Coral Gables, FL: University of Miami Press.

Leung, F. K. S. (2017). Making Sense of Mathematics Achievement in East Asia: Does Culture Really Matter? In Kaiser G. (Eds). *Proceedings of the 13th International Congress on Mathematical Education ICME-13 Monographs* (pp. 201-218). Springer, Cham.

Sun, X. (2011). “Variation problem” and their roles in the topic of fraction division in Chinese mathematics textbook examples. *Educational Studies in Mathematics*, *76*, 65-85.

Watson, A., & Thompson, D. R. (2015). Design issues related to text-based tasks. In Watson, A., & Ohtani, M. (Eds.), *Task Design In Mathematics Education-ICMI study 22* (pp. 143-190). Switzerland: Springer.

Wong, N. Y. (2004). The CHC learner’s phenomennon: its implications on mathematics education. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 309-347). Singapore: World Scientific.

Zhang, D. & Dai, Z. (2004). “Two basics”: Mathematics teaching approach and open-ended problem solving in China. *Regular lecture delivered at the 10th International Congress of Mathematics Education*, Copenhagen, Denmark, July.

LUXIZI ZHANG
ENS DE LYON, FRANCE; EAST CHINA NORMAL UNIVERSITY, CHINA
E-mail: zhangluxizi@outlook.com