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Teaching Mathematics and Computer Science

Freudenthal fantasy on the bus, an American adaptation

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Abstract. In the 1960's two mathematicians, Hans Freudenthal in the Netherlands and Tamás Varga in Hungary, had argued that people learn mathematics by being actively involved and investigating realistic mathematical problems. Their method lives on in today's teaching and learning through the various components of cooperative and active learning, by taking ownership in learning, and learning through student dialogue. The goal is to create a welcoming classroom atmosphere in which play takes the front seat. One such scenario is visiting various (animal) stations at the zoo by bus (illustrated by pictures). Passengers are getting on and off the bus at each station (illustrated by arrows), which is modeled on the open number line. This adapted and modified action research was carried out with 5-yearl-old children in public schools of Staten Island, NY in 2019.

Key words and phrases: active learning, real-life mathematical problems, Freudenthal method, dinamic problem-solving, problems in context.

MSC Subject Classification: 97D40, 97F20, 97F30.

Theory and Methods

The research presented in this paper was adapted from the original one carried out at the Freudenthal Institute (Streefland, 1991, p. 80-81). It was modified to better illustrate the real-life situation of children taking a school bus trip to the zoo. This adapted and modified action research was carried out by student teachers at the College of Staten Island, CUNY (City University of New York). These student teachers were interacting with 5-year-old children at Public Schools 45 and 57 on Staten Island, NY, during the Spring and Fall semesters of 2019.

Our research question was the following. If the children learn through play, acting out a real-life scenario, allowing them to choose different animal stations, the number of kids getting on/off the bus, and by constructing their own solutions, is the quality of their learning affected in a positive way, is their understanding of addition and subtraction better?

Similar examples in various formats and lengths were described, for ex. in Gravemeijer (1994), Jirotková, Kloboučková, & Hejný, (2013), and Wubbels, Korthagen, & Broekman, (1997). Our adaptation, however, is unique regarding the type of the bus (school bus), the destination (zoo), and the type of the route (field trip). The reason we chose this particular adaptation is because children are usually excited about going on field trips by school bus, they are eager to share their knowledge about animals, therefore are naturally curious and involved about the different “animal stations” they are creating and “visiting”.

“[…] sometimes we can immediately connect something we are told to what we know already, and the thought becomes our own. But notice that if we really understand what we have been told, we make new connections for ourselves. We are now the master of these new connections and can express them our own way. If we cannot make these new connections for ourselves, we do not really grasp what we have been told.” (Duckworth, 2006, p. 18)

In the past what was taught in schools was static, fixed, rigid mathematics, invented and developed by famous mathematicians over hundreds and thousands of years. These ideas, rules, formulas, algorithms, proofs, discovered by renowned mathematicians had to be learned, memorized, and recalled from memory, then applied in order to solve math problems. This way every math problem was a new/different math problem.

This type of learning and memorization could have been done for a short period of time, until the next exam or test. However, since the newly learned material was not connected to previous knowledge, it was easy to forget a tiny part of the formulas/rules, therefore they could not be applied. If the memorization was not completely correct, the entire solution collapsed and was not working. These unsuccessful experiences led to math anxiety and the feeling of disliking math.

Instead of this, we create a student-centered dynamic environment, where the children construct their own knowledge. These little mathematicians love what they are doing, are successful, therefore will be able to solve problems in our fast-changing world and will be open to new challenges.

In other words, the goal is to create a welcoming classroom atmosphere in which play takes the front seat. In this free learning environment, the children consider everyone's opinion while constructing the solution, and they are allowed to make mistakes. They listen to each other's solutions, building on them, combining other solutions with their own by working in teams. This becomes a lifestyle for them in their problem-solving environment. Step by step, they are correcting their own mistakes, thus refining their solutions. Their final solution is written down on a large chart paper or poster so that it can be presented and discussed in two different ways: inside the group and in front of the whole class. Then the large chart papers are displayed in the hall, where they can be seen by other classes, during parent-teacher conferences and professional development participants. Thus, the children can learn from the opinion of their peers from other classes, this way opening their way of thinking to the community. Another opportunity for communicating their ideas is called the Mathematics–Science Fair, where the kids explain their solutions to other students and parents. After such a Mathematics-Science Fair the kids get a deeper understanding of their methods and solutions. By answering questions, they rethink their solutions, refine them, and due to their natural curiosity, they search for other, better solutions.

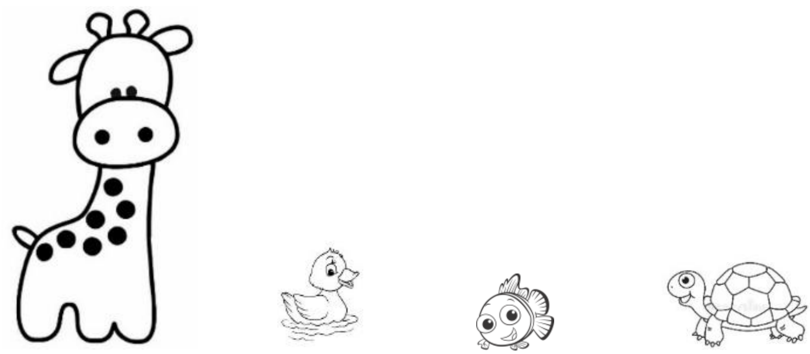
Because the children handled the problem in many different ways at several levels, they build self-confidence and become mathematical thinkers. This is in correlation with what was happening for thousands of years, generation after generation, when people used the solution of their previous generation and refined it.

The goal of learning is not the product, is not to get the correct answer, but the process, to own and understand the solution constructed by themselves. Learning is not linear, it is bumpy. Children learn in different ways, for example one is good in problem solving, the other in recognizing real-life situations, the third one has very good spatial sense, and so on. The teachers and teacher candidates cannot enter the classroom with their only solution to teach. They have to know many different solutions to be able to stretch the children's thinking in order to facilitate their learning. This needs far more preparation on the teacher's and teacher candidate's part, but the children will have much better understanding of problems, leading to successful experiences.

The Activity

One real-life scenario is visiting various (animal) stations at the zoo by a school bus (illustrated by pictures). Passengers are getting on and off the bus at each station (illustrated by arrows), which is modeled on the open number line.

First, the kids are playing in the classroom by going on an imaginary field trip to the zoo, they construct a map of the zoo, and create different stations. They exchange ideas about the animals, arguing why they should include a specific animal. Each of them provides good reasons based on their previous knowledge and experience. For example, Sara wants to include a giraffe, since she really likes how tall they are. After that they search for the 10 animals they could potentially include, then they come to agreement about which 4 animals to feature. In this process they learn lots of facts about animals, and - more importantly - they learn how to compromise and come to a decision which works for the entire group. Here the teacher did not use his/her authority to decide which animals to incorporate. The kids were allowed to choose the 4 animals they wanted to include, these were those they learned most about and loved the most:



They printed the colored picture of these 4 animals and positioned them at 4 different places in the classroom. They made the decision together regarding the number of kids waiting at each station. Then they placed the corresponding number of identical teddy bears at each station.

In the meantime, the assistant teacher chose a kid, Peter, after which they both left the classroom. They both agreed that today Peter will be the school bus driver, and a bus driver cap was put on Peter’s head. Peter knocked on the door of the classroom. The kids shouted:

“Come in!”

“Today I’ll be the school bus driver,” said Peter. “Would you like to come to the zoo?”

“Yes!!!”

Peter chose three kids who – they agreed - were already on the bus. This meant one of the kids putting her hands on the shoulders of the bus driver, the second kid putting his hands on the shoulders of the first kid, and so on. Thus, they formed a chain: the bus driver was first, followed by the kids on the bus.

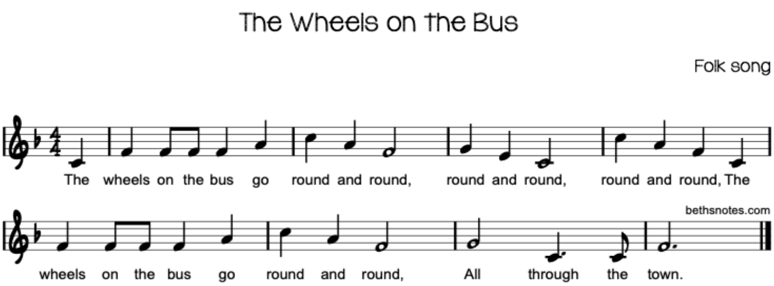
“Which station would you like to see first?” asked the driver.

“The giraffe!”

“Why?”

The kids told Peter the characteristics based on which they chose the giraffe.

“Good,” - said Peter, the school bus driver, and they went to the giraffe station, while the kids were singing their favorite song:



When they arrived at the giraffe station, Peter asked:

“How many kids are waiting here to get on the bus?”

“Two,” said the class.

The two kids got on the bus by one of them placing her hands on the shoulders of the last kid in the chain already formed, followed by the second kid imitating her.

"Let's go to the next station. Which would you like it to be?" asked Peter.

"The ducks, because we really want to see the little ducklings!" replied the class.

When they arrived at the ducks station, four kids got on the bus by mimicking the other kids "on the bus", i.e. elongating the chain already formed. As soon as they started their next leg of the trip to the fish station, one of the kids started to have a stomachache, so he and his friend got off the bus to rest on a bench. This was done by two kids getting out of the chain. At the last station, five kids who already saw the turtles, got on the bus by following the pattern in the chain. Finally, at the door of the classroom, Peter said:

"We have arrived back to school."

In the next phase of the play, the teacher gave the kids a large piece of paper, which was placed on the floor in front of the blackboard. The teacher also provided plastic animals, colored pencils/pens and she offered the kids to draw what they were playing about, starting with the map:

"Design the map of the zoo!" she asked the children.

The kids draw the map and with the help of the plastic animals, they modeled what they did in real life. They put the corresponding number of teddy bears at each station: 3 at the school, 2 at the giraffe station, 4 at the ducks station, 2 a little further from the fish station, since they got off the bus, and finally 5 at the turtle station. When this was done, the map was complete.

Then the teacher asked the kids if they want to play again, now using the map, and by choosing a different bus driver.

"Do you want to go on an imaginary trip to the zoo? Come sit on the rug around the map! Can we start playing?"

"Who would like to be the bus driver? "

"Me, me!" Maria said happily.

As the kids were playing, the school bus started the trip on the map from the school with three teddy bears. At the giraffe station two more bears got on the bus. They continued to play the same game at each animal station, until the bus got back to the school. In the meantime the kids were also drawing on their clipboard their own map and the bus going from station to station.

Next, the teacher said:

"Now we will be young mathematicians. We'll draw what we just played, using arrows. Mathematicians call this arrow language. The rules are as follows: if the teddy bears get on the bus, the arrow is pointing to the right, if they get off the bus, the arrow is pointing to the left. The length of the arrow is as many units as the number of teddy bears getting on/off the bus."

"Now, draw the arrows on your clipboard!"

After the kids draw their arrows based on their previous play, the teacher said:

"Very good! Who would like to share his/her drawing with us?"

Using the projection lamp, the entire class can see the drawing and the child sharing his/her drawing can explain what he/she did.

The teacher should invite a simple, less sophisticated solution first, which would be understood by the majority of the class. Then the teacher should call for a more elegant, sophisticated solution. Based on the kids' understanding of the previous solution, they will grasp the more sophisticated one. They will have a better understanding compared to the opposite situation, where they would have seen the more advanced solution first, then the simple solution second.

In this sequence, first, the kids are part of a playful, active, special treat. Second, they are playing the same game with teddy bears on the map. Next, parts of the play on the map are transformed into arrow language. This learning by understanding internalizes the arrow language and is connected with real life. The children understand the meaning of the direction and length of the arrow. After they understand this, the teacher invites them for a new constructivist game.

"You know what? Now we are going to create a number line. This number line is ours. The number line is a line, so that if we want more, we go to the right, if we want less, we go to the left. The number of units we go to the right corresponds the number of kids getting on the bus, the number of units we go to the left corresponds the number of kids getting off the bus."

"Now you are going to create the number line. I'll be writing what you are saying. You'll be the little mathematicians, you tell me what to do."

"Let's draw an open line. Do you remember how many kids were on the bus originally? "

"Three," remembered the kids.

“How many got on the bus at the giraffe station?” asked the teacher.

“Two,” said the children.

“If two kids got **on** the bus, we jump 2 units to the **right** on the line. Where are we?”

“At five.”

They continued to follow the steps of the play. Instead of the teacher some of the kids took turns drawing on the number line. Finally, they concluded that 12 children are going back to school.

Conclusion and Discussion

Based on our research described above, we could conclude the following. If the children learn through active play, connected to a real-life situation, have freedom of choosing different elements of the play, and construct their own solutions, their learning is positively affected, their understanding of addition and subtraction is better.

“According to Freudenthal, mathematics must be connected to reality, stay close to children and be relevant to society in order to be of human value.” (Heuvel-Panhuizen, 1996, p. 10, and Freudenthal, 1977).

Similarly our project follows one idea from the introduction of Tamás Varga’s dissertation “Their aim is gaining individual mathematical experiences based on manipulative activities, which play a significant role especially in the lower grades; later drawings, written symbols become more and more the means of gaining individual experiences.” (Varga, 1975, p. 14)

During the course of the active research, while children were playing, we were also reinforced that “children should have sufficient time to inquire, explore, discuss, and revisit ideas so that they can build deep conceptual understanding”. (Rowan & Bourne, 2001, p. 14)

“It is impossible to talk about mathematizing without talking about modelling. […] At the heart of modelling is number sense – the representation of number relationships. As children construct mental maps of these relationships, they are developing powerful tools with which to mathematize their own lived worlds.” (Fosnot & Dolk, 2001, p. 95)

Mathematics is not a closed subject, but open, can be improved upon by anyone. The teachers and parents facilitate the children’s thinking with lots of knowledge and preparation, so that in this free learning environment the little mathematicians love what they are doing. If they love it, they do it, and if they do it, they learn it. This knowledge is theirs, they own it. Thus, by developing their own way of thinking, they become confident in solving problems. As little mathematicians, they grow up in a mathematical community, where they are proud of their achievements, they are confident about their abilities in solving problems, therefore they love solving problems. This student-centered environment builds mathematicians whose success is the community's success.

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