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Teaching Mathematics and Computer Science

Is it possible to develop some elements of metacognition in a Mathematics classroom environment?

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Abstract. In an earlier exploratory survey, we investigated the metacognitive activities of 9th grade students, and found that they have only limited experience in the "looking back" phase of the problem solving process. This paper presents the results of a teaching experiment focusing on ninth-grade students' metacognitive activities in the process of solving several open-ended geometry problems. We conclude that promoting students' metacognitive abilities makes their problem solving process more effective.

Key words and phrases: metacognition, problem solving, open problem.

MSC Subject Classification: 97D50, 97G40.

Introduction

Metacognition and problem-solving thinking are closely related. Our research focuses on this relation.

In an earlier exploratory survey (Kiss, 2019), we investigated the metacognitive activities of ninth-grade students, and found that due to a limited experience in the "looking back" phase of the problem-solving process, the participating students stopped their work as soon as a result was found, without considering whether this result is realistic or correct.

In this paper, we report students' solutions from the tests connected to a new teaching experiment among ninth grade students and summarize the experience.

Theoretical background

Nowadays, the concept of metacognition is widely used by several researchers. Their starting point is Flavell's pioneer definition.

"Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete [problem solving] goal or objective." (Flavell, 1976, p. 232)

To observe these issues in practice, it is recommended to determine, and examine students' metacognitive strategies – i.e.,*"... metacognitive strategies such as planning, to regulate the learning process"* (Hattie & Donoghue, 2016, p. 1). In our research, we emphasize regulation.

Polya (1957) defined a general model that aims to promote a more successful process of problem solving. Polya's model includes several metacognitive elements. The four main stages of his model are: (1) understanding the problem (2) devising a plan (3) carrying out the plan (4) looking back (Polya, 1957, pp. 16-17). Polya notes that it is impossible to examine deeply the problem solver mental activity because of its complexity. However, a problem solver's collected work can provide some indication with regard to the involved mental processes. (Polya, 1981)

Schoenfeld (1992) claims that researchers agree that the process of solving problem involves the following five aspects of cognition: (1) The knowledge base (2) Problem solving strategies (3) Monitoring and control (4) Beliefs and affects (5) Practices. In our research, we concentrate on the aspect of belief. Schoenfeld writes in his study that students usually believe that in mathematics problem-solving requires a ready-made method that offers an answer. *„As a result of holding such beliefs, students may not even attempt problems for which they have no ready method, or may curtail their efforts after only a few minutes without success."* (p. 27)

Based on Pehkonen's idea (1997), we differentiate between several types of open tasks - depending on whether their starting or goal situation is not exactly explained. In our research, we deal with problems whose starting situation is not clarified, but their goal situation is clear.

In view of these studies, we have elaborated a teaching experiment for an average mathematics class aimed at helping ninth grade students develop metacognitive abilities. We intended to promote students' ability to handle mathematical problems in a wider perspective and in a more conscious way. Moreover, another aim of this study was attempting to change the belief that *„[m]athematics problems have one and only one right answer."* (Schoenfeld, 1992, p. 359). We also tried to motivate students to think about mathematical problems that involve multiple answers or seemingly contradictory situations. The following geometry problem illustrates the notion of a multiple case problem: One angle of an isosceles triangle is 54°. What are the other two angles? Here the problem-situation allows for two cases (i.e., two correct answers) depending on the position of the given angle: (1) 54° and 72°, (2) 63° and 63°. If the given angle is e.g. 95°, then one of the cases is contradictory.

Research question

During the action research we focused on having the students (1) understand the problem, (2) look for more than one answer, (3) check for answers that are not valid for the given problem- situation, and (4) review the applied solution method.

The following two questions related to problem solving were considered in this study:

1. Do students realize that is the presented problems may have more than one correct answer?
2. Do students identify contradictory answers, and if they do, how do they handle them?

Methodology

The analysis of the teaching experiment is based on students' pre-, second, post- and delayed tests, as well as on notes made during and right after the experimental lessons.

The process started with a pre-test on 11 March 2019. This was followed by 11 lessons; in the fifth lesson, a second test was written. Then came a post-test, and at last a delayed test was given one and a half months after the post-test. Figure 1 shows the timeline of tests.

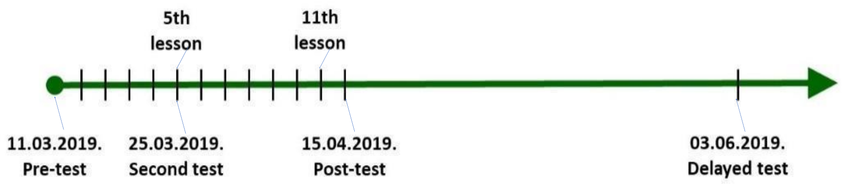

Figure 1. Timeline of tests

The teaching experiment took place in a 9th grade class of 29 boys with an average level of performance in mathematics, in a secondary technical school. The content of mathematics taught in the class was based on the standard curriculum. Students had three weekly lessons and the first author taught the 11 experimental lessons during the period of one month. The lessons dealt with the standard geometry curriculum including triangles, quadrilaterals, polygons and circles.

In each of the 11 lessons, students dealt with at least one of the two types of problems. (Sometimes the two approaches appeared in the same problem, where one of the answers was contradictory.) The two types were built in the lessons in a natural manner – i.e., the experimental problems were not mentioned or emphasized as such. Most of the time, the class worked as a whole guided by the teacher and the solutions were written on the board by a student or by the teacher. Occasionally, the homework included these two types of problems as well. The lessons (student work and remarks, teacher actions and our comments about the problems) were documented at the end of each lesson.

The test problems were discussed and solved in the following lessons. The level of difficulty and the content of the test problems were similar to the problems given during the experimental lessons.

The problems

The tests included both standard curriculum, and experimental (i.e., multiple, and contradictory case) problems – with a focus on the two types of experimental problems.

As a result, we analyzed two out of five pre-test problems (Figure 2 and 3), one out of two problems from the second test (Figure 4), one out of three post-test problems (Figure 5), and the problem from the delayed test (Figure 4). We briefly describe these problems and their content analysis.

Figure 2 presents a multiple case problem from the pre-test, where are four different answers depending on the order of the children's location.

Anna, Béla, Cili, and Dani are standing along a line. Béla stands 5 m from Anna, Cili stands 3 m from Béla, Dani stands 1 m from Cili. How far can Dani be from Anna?

Figure 2. The four children problem

Figure 3 presents a contradictory case problem from the pre-test. There are three connections concerning a given triangle's interior and exterior angles ($\alpha + \beta + \gamma = 180°$; $\alpha' + \beta' + \gamma' = 360°$; $\alpha + \alpha' = 180°$). The table can be filled in if one uses two of those connections, but with the help of the third one, the answer can be checked. The table relates to two different triangles, where the second row showing a non-existent triangle (contradictory case).

Complete the table. (α, β, γ are the interior angles of a triangle, while α',β',γ' the corresponding exterior angles)

α	β	γ	α'	β'	γ'
20°	*50°*				*70°*
	80°		*110°*	*120°*	

Figure 3. Problem of interior and exterior angles

The height problem is a multiple and also a contradictory case problem from the second and the delayed tests (Figure 4). In this problem, two different cases have to be taken into consideration, since the text does not tell which measure belongs to the triangle's base and which measure belongs to its legs. However, one of the two cases is contradictory, and it can be proven by using the triangle inequality theorem.

The length-measures of two sides of an isosceles triangle are 6 cm and 16 cm. What is the length-measure of the triangle's height to its base?

Figure 4. The height problem

The kite problem, like the previous one, is a multiple and also a contradictory case problem that was set in the posttest (Figure 5). In this problem, three different cases have to be taken into consideration, since the correspondence between angles and their measure is not given. However, one of the cases is contradictory, and it can be proven using the theorem about the sum of the triangle's interior angles.

The measures of two interior angles of a kite are 70° and 150°. What are the measures of the other two interior angles of the kite?

Figure 5. The kite problem

Findings

In view of the pre-test results, we concluded that average-performance mathematics students are not accustomed to geometry open problems and with handling contradictions.

In the four children problem (Figure 2), out of 27 students, only 2 students indicated all four possible cases, and all except one student identified only one case. One student dealt with all the four cases, but at a later stage, he crossed out three of them and left only the "expected" case. We note that at this stage, our students were not accustomed to this kind of task.

In the problem of interior and exterior angles (Figure 3), out of the 10 students who filled in correctly the first line of the table 9 students gave an answer in the second line of the table which corresponds to two connections out of the three. There was only one single student who realized the "mistake", indicated it but did not explain it. This student replaced an item with another one in order to avoid contradiction. Another student seemed to guess that something is not correct based on his deletion, but at the end, he did not realize that data of this triangle is contradictory ($80° + 120° \neq 180°$). (Figure 6) It was interesting that 5 students used a so called "halving technique". That is if the measures of two interior or two exterior angles were missing, their sum was halved, and the missing angles got those measures.

α	β	γ	α'	β'	γ'
20°	50°	110°	160°	130°	70°
70°	80°	30°	110°	120°	130°

Figure 6. A student's answer to the problem of interior and exterior angles

These findings are quite similar to the ones in other classes with an average performance in mathematics. In their study, Kovács and Kónya (2019) analyzed expert and novice problem solvers of different ages, and concluded that *„… expert-novice differences are not caused primarily by age, but by special training in mathematical problem solving."* (Kovács & Kónya, 2019, p. 256)

The analysis of students' answers to the second, post, and delayed tests can be attributed to our intervention. We use the following categories and subcategories (Figure 7):

(A) Attempt to solve a contradictory case

(B) Solve and identify a contradictory case (Note that the (B) is a subcategory of (A).)

(C) Find more than one solution in a "multiple case" problem

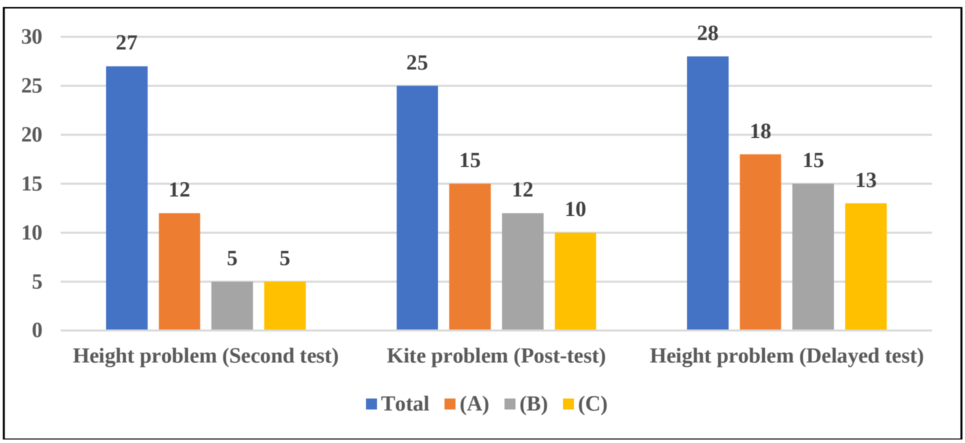

Figure 7. Comparing some results of second, post, and delayed tests

It can be seen in Figure 7 that in the height problem in the second test 12 students attempted to solve the contradictory case, and 5 out of these identified the contradiction, and gave more than one correct answer. A deeper analysis of the work of the 5 students shows that 2 of them started the solution with the existing triangle (16 cm, 16 cm, 6 cm), while the others with the non-existent one (6 cm, 6 cm, 16 cm). In the first case, we can be sure that the students were aware that there are multiple answers to the problem, while in the second case it is conceivable that they were only looking for another triangle because they believe that each math problem must have a right answer (see Schoenfeld, 1992). We note that there were 5 students, who did not use the Pythagorean Theorem correctly, and this could possibly prevent them from realizing the contradiction. For example, one student wrote down the Pythagorean equation correctly, got a negative length, and crossed out the negative sign (as if there cannot be a problem without an answer).

In the kite problem, 15 students attempted to solve the contradictory case and 12 out of them identified the contradiction. 10 students dealt with multiple cases, and all of them belonged to the previously mentioned 12 students. 7 students out of 10 looked for another case after giving a correct answer This means that the 7 students working with more cases did so out of awareness to the possibility of more than one case, and not because they have noticed a contradictory case first (like the other 3 students).

In the delayed test 18 students dealt with the contradictory case, and 15 students realized the contradiction. 13 students provided more than one solution multiple cases, and all of these belonged to the previously mentioned 15 students. 5 students out of 13 looked for another case after giving a correct answer.

We were also interested in whether students justified an identified contradictory case. Among the solutions given to the height or the kite problems, we identified only a few solutions, where some trace of explanation appeared. In the second test only one student, whereas in each of the post, and delayed tests 5 students gave some explanations for a contradictory case. Figure 8 presents an example of explaining a contradictory case in the delayed test by providing a drawing and noting that there is "no solution".

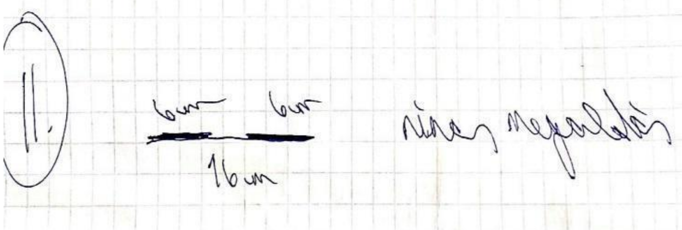

Figure 8. Justification of a contradictory case

Conclusion

Our findings support Schoenfeld's claim that many students believe that „*mathematics problems have one and only one right answer*" and that this belief is difficult to change. In response to our research question, we report that 10 / 13 out of 25 / 28 students, were able to work with multiple case problems at the stage of the post / delayed tests (Code (C) in Figure 7), whereas initially, only one student identified more than one case in the four children problem. In figure 7 code (B) shows that 12 / 15 students realized the contradiction (these numbers include all the students mentioned above), while in the problem of exterior and interior angles one student indicated it. Explaining why an answer is contradictory already seemed a more difficult task for these 9th-grade students. Nevertheless, 2 / 5 students were able to give some verbal or iconic explanation at the end of the teaching experiment; the others were satisfied with the short answer "such a deltoid/triangle doesn't exist". In general, we detected some improvement in the students' work on both multiple, and contradictory case problems. The question is why the level of success was rather low. After analyzing the tests' results and the teacher's notes taken after the experimental lessons, we can present some pedagogical implications as well. Although this study is related to a particular topic and its main purpose is not content knowledge, we claim that content knowledge has a significant influence on students' rate of success. This conclusion is based on our detection of a lack of basic factual information. We assume that some students' current level of development did not allow an improvement at this stage. Furthermore, we note that the quality of student work on these types of problems varied.

We also claim that the students' level of success depends on teaching as well. In order to obtain a change in their approach and attitude, most (9th grade) students need assistance, and we believe that if this is provided, the desired goals can be achieved. Although our data may be altered by certain circumstances, we can report on a considerable improvement with regard to this aspect of geometry learning, with a regular class and within a relatively short period of time.

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