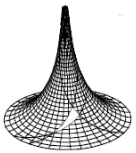


18/3 (2020), 11-28
DOI: 10.5485/TMCS.2020.0475
tmcs@science.unideb.hu
http://tmcs.math.unideb.hu

Teaching
Mathematics and
Computer Science

Tamás Varga's reform movement and the Hungarian Guided Discovery approach

KATALIN GOSZTONYI

Abstract. This paper presents Tamás Varga's work focusing especially on the Hungarian Complex Mathematics Education reform project led by him between 1963 and 1978 and the underlying conception on mathematics education named "Guided Discovery approach". In the first part, I describe Varga's career. In the second part, I situate his reform project in its international and national historical context, including the international "New Math" movement and the "Guided Discovery" teaching tradition, something which is embedded in Hungarian mathematical culture. In the third part, I propose a didactic analysis of Varga's conception on mathematics education, underlining especially certain of its characteristics which can be related to Inquiry Based Mathematics Education. Finally I briefly discuss Varga's legacy today.

Key words and phrases: Tamás Varga, Guided Discovery approach, Inquiry Based Mathematics Education, history of mathematics education, curricular reform.

MSC Subject Classification: 97-03, 97B20, 97D20, 97D40, 97D50.

Introduction[1]

Tamás Varga is one of the main characters of the history of Hungarian mathematics education. The Hungarian community of mathematics education keeps his memory alive in the name of a competition for middle school students, of a prize and a conference for teachers among other things. The main reason for this active commemoration is the reform project called Complex Mathematics Education, led by Varga between 1963 and 1978, considered as one of the milestones in the history of mathematics education in Hungary, leading to a national curriculum in 1978 and influencing mathematics education in various ways until today.

The reform offered not only a new curriculum, but a new approach of mathematics education with new and newly structured content, resources and teaching practices. Varga developed a coherent underlying conception, focused on students' mathematical activity, problem solving and inquiry, harmonizing mathematical coherence with playfulness and students' creativity. This conception, named "felfedeztető matematikaoktatás" ("teaching mathematics by Guided Discovery"), remains relevant in many aspects in the mirror of current efforts concerning the development and implementation of Inquiry Based Mathematics Education around the world.

Varga was also one of the most active Hungarian participants in the international community: he brought to Hungary the lessons of various international researches and experiences in the 1960s and '70s, and was internationally recognized for his work, especially in the domains of teaching logic, combinatorics and probability from the early grades on.

In this paper, I will first present Varga's career, focusing especially on the various contacts and effects influencing the development of his reform project and the underlying approach. In the second part, I will discuss the historical context in which his reform movement was developed, including its political, institutional, mathematical and educational background. I will underline how this reform was a synthesis of influences coming from the international New Math movement and of a "heuristic" epistemology of mathematics represented by a Hungarian mathematical community. In the third part of the article, I will analyse Varga's reform and the underlying conception from a didactical point

[1] Some parts of this article appeared earlier in a paper presented at the ICMI Study 24: School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities conference (Gosztonyi et al., 2018).

of view, emphasising especially its original elements and the aspects which connect it to current reflections on Inquiry Based Mathematics Education. I will end the article with a short reflection on Varga's legacy today.

Tamás Varga's life and career

Tamás Varga (Kunszentmiklós, 3 Nov. 1919 - Budapest, 1 Nov. 1987) was born as the second of seven children to a Kalvinist priest, Tamás Vargha. The family provided a dynamic intellectual milieu: several of his siblings become also well-known intellectuals, namely the novel-writer and journalist Domokos Varga and Balázs Vargha, a writer, historian of literature and anthropologist of games[2]. They all recognize the influence of their parents on their later work, for example that of the logical and linguistic games played with their father (Balogh, 2014; Dancs, 2016). The three brothers were also in good contact with the Kalvinist educator, linguist and writer Sándor Karácsony – we will come back to his influence on Varga's work later.

Tamás Varga married the psychologist Ágnes Binét, who was a student of Piaget, and close collaborator of Ferenc Mérei, a particularly influential Hungarian psychologist in the second half of the 20th century, specialized in child psychology[3]. These relationships contributed to the psychological background of Varga's work.

Varga, after his graduation at the University of Sciences of Budapest (the later Eötvös Loránd University) as a mathematics and physics teacher, was awarded a scholarship to spend 18 month at Scuola Normale Superiore in Pisa, Italy (Szendrei, 2017). He started to teach mathematics at the secondary school of Kunszentmiklós in 1945, but very early, in 1947 he was invited to the Educational Ministry, later to the Pedagogical Institute to work on new curricula and textbooks. From 1951, he worked in mathematics teacher education at the Eötvös Loránd University. From 1955 on, he also regularly took in charge high school mathematics teaching in one class (Pálfalvi, 2019).

The most important stimulation to start experimentations aiming the renewal of mathematics curricula came from a series of lectures given by Paul Dienes in Budapest in 1960, and a UNESCO symposium on mathematics education organized in Budapest in

[2] Tamás and Domokos abandoned the „h" from the spelling of their names.

[3] See e.g. their common work (Mérei & Binét, 1970).

1962 (Hungarian National Commission for UNESCO, 1963; Pálfalvi, 2019). Varga was charged, with W. Servais, to edit a book based on this conference (Servais & Varga, 1971)[4]. After a short experimentation in 1961, Varga started the "Complex Mathematics Education" project in 1963. Starting with two first grade classes in a primary school, the project grew progressively and attained hundreds of classes from grade 1 to 8 (covering all levels of compulsory education) all over the country. In the beginning of the 1970s, the project was chosen by a ministerial committee as the basis of the new curriculum. A provisional and optional version of the curriculum was introduced in 1974, and the obligatory version in 1978, in the framework of a general curricular reform.

From 1967, Varga was moved to the National Pedagogical Institute were he lead the team responsible for the development of the reform project. The group closely collaborated with a group hosted in the Mathematics Research Institute, responsible for the development of the curricula for high school classes specialized in mathematics. Varga defended his doctoral thesis based on the reform project in 1975, and he also obtained several national prizes and recognitions for his work.

From the second half of the 1960s, Varga developed an international career in the frame of the emerging mathematics educational research community. He published and was regularly invited both in the countries of the "Eastern" and the "Western" blocs, including Poland, the USSR, France, Italy, Canada and the USA among others. He also took in charge responsibilities in various international organisations: he was for example a member of the Editorial Board of *Educational Studies in Mathematics* and Vice Chair of CIEAEM (Szendrei, 2017).

The "Complex Mathematics Education" reform in its historical context

Political and institutional context

In the period in question, Hungary was a socialist country, under the influence of the USSR. However, the reform started after an important political turn. In the 1950s, the hardest period of the dictatorship led to the revolution in 1956 and the following

[4] About the relationship of Varga and Servais, see (De Bock, 2020).

retribution, but a consolidation began from 1962. The 1960s and '70s are the period of softer authoritarianism with restricted oppression, some liberalization of the communist system and some opening toward the Western world (Romsics, 1999). Although we don't have proof, it is likely that the possibility to organize an international UNESCO conference in Hungary in 1962, and to start experiments inspired by this conference are related to this political turn.

The frames of the educational system in which this reform arrived were established since 1946. Compulsory education was provided by the 8-grade single-structure "basic schools", comprising elementary (grade 1-4) and lower secondary (grade 5-8) education. Upper secondary education was provided by general and vocational secondary schools. During the 1950s and '60s, the regulation of the educational system was extremely centralized, with detailed curricular instructions. Soviet influence and the communist ideology were quite apparent in instructions as well as in teaching materials of this period. From the late 1960s however, a slow liberalization was launched (Báthory, 2001): the influence of the ideology was pushed into the background, pedagogical and psychological considerations were taken into account, differentiation as well as teachers' autonomy and liberty were emphasized. This turn plays a crucial role in the preparation of the 1978 reform, and – as we will see below –, Varga's project can be considered as a pioneer in this sense.

Thus, contrarily to some other reforms of the New Math era[5] which fitted in the frame of a unification process of the educational system, like the creation of the "collège unique" in France (d'Enfert & Kahn, 2011), the Hungarian movement was set out from a unified system and fitted into a liberalization process. The impact of this context is perceptible on many aspects of Varga's reform, even in less evident characteristics as the role of mathematical language: while the above mentioned French "Mathématique Moderne" reform insisted on the unifying role of a formal mathematical language, Varga's Complex Mathematics Education reform emphasized the importance of working with the diversity of students' personal expressions.

[5] The term „New Math" is originally used in the US, but become wide-spread describing also the international movement of mathematics educational reforms of the 1950s, '60s and '70s. (Karp & Schubring, 2014, p. 198)

The Hungarian reform in the context of the international New Math movement

The international New Math movement is often considered as being developed in the context of the cold war's scientific and technological competition. Based on this (only partly grounded) assumption, it would be an obvious hypothesis that the New Math was a Western movement, without relevant contributions from the "Eastern bloc" or with two parallel movements in the two "blocs". However, the Hungarian reform is a good example illustrating that it is not the case. Varga always declared being influenced by the New Math; from the 1960s, he actively participated in the work of different international organizations of the movement like the UNESCO or the CIEAEM, and was invited to and published in various „Western" countries, as we saw earlier. According to his doctoral thesis (T. Varga, 1975) as well as his colleagues' memories expressed during interviews, Eastern influence was much less important on his work, although he also published in several countries of the Eastern bloc; his only important partner from these countries was Krygowska, the leader of the Polish reform – also an active and recognized actor on the international mathematics educational scene of the 1960s and 1970s.

Many impact of the New Math movement can be observed on the Hungarian reform: the introduction of a coherent subject named "Mathematics" instead of "Arithmetic and Measurement"; new mathematical domains introduced in early ages like sets or logic; the reference to Piaget's psychology and Dienes's mathematical games; the important role of manipulative tools etc. However, Varga was also critical about some aspects of the New Math reforms, especially with the excessive emphasis on mathematical formalism.

Epistemology of Mathematics: a Hungarian "heuristic" tradition

When his colleagues evoke Varga's reform movement, they usually underline that, while being inspired by the New Math, it was also a specifically Hungarian conception fitting into the local traditions of teaching mathematics by discovery. This tradition exists indeed in the teaching of young mathematical talents[6] and goes back at least to the beginning of the 20st century. Varga seems to adapt this approach for mass education, and extend it for the teaching of young children.

[6] Nowadays its most important representative is L. Pósa. See http://agondolkodasorome.hu/en/ and (Artigue et al., 2020)

Varga himself was in intensive personal contact with some representative mathematicians of this tradition (L. Kalmár, R. Péter, A. Rényi, J. Surányi among others) since the 1940s; and they all supported, more or less actively, Varga's later reform movement. These mathematicians, together with well-known thinkers like G. Pólya or I. Lakatos, represent a quite coherent, "heuristic" epistemology of mathematics, which is closely related to questions of mathematics education and published mostly in texts popularizing mathematics and lectures about mathematics education (Gosztonyi, 2016).

They see mathematics as a constantly developing creation of the human mind, this development being guided by series of problems. According to them, the source of mathematics is intuition and experience; mathematical activity is basically dialogical and teaching mathematics is a joint activity of the students and of the teacher, where the teacher acts as an aid in students' rediscovery of mathematics. They discourage early and excessive formalism, seeing formal language also as a result of a development. They describe mathematics as a creative activity close to playing and to the arts.

Beyond the "heuristic" epistemology, the influence of Hungarian mathematicians can also be observed on the mathematical content of the reform. Varga was particularly active in the domains of logic, the main research domain of L. Kalmár and R. Péter, in probability, a mathematical domain introduced in Hungary by A. Rényi, and in combinatorics, a particularly well developed domain in Hungarian mathematical research.

Pedagogical and psychological background: a complex situation

The pedagogical and psychological background of the reform is quite complicated to reconstitute. The reference to Piaget is obvious, but not the only influence on Varga's conception. His wife, Á. Binét and the above mentioned F. Mérei, playing a leading role in the Hungarian reception of Piaget, could have themselves exerted important influence, their view on the development of children diverging partially from that of Piaget (Pléh, 2010).

Varga refers to some soviet pedagogues too, but only a few times in his politically relevant writings: so, it is difficult to know if these are real or only politically motivated references. However, Vygotsky is almost missing from his references, although Varga's conception shows some similarities with Vygotsky's socio-constructivism.

The socio-constructivist approach, as well as the importance of visual intuition in the learning process can be inspired by views of the above mentioned S. Karácsony too, who

was also in contact with most of the mathematicians mentioned, and apparently influential on their views on mathematics education (Gurka, 2004; Máté, 2006; Szabó, 2013). According to Varga's colleagues and family, Karácsony had a great influence on him – but he couldn't be mentioned in Varga's writings, again because of political reasons. In summary, pedagogical and psychological influences seem to be quite complex and their more detailed identification would need further research.

The impact and the reception of the reform

Varga's experimental project was accompanied by a multidimensional psychological impact analysis which showed the experimentation successful in various senses (Klein, 1980). The students acquired the same mathematical knowledge than in ordinary classrooms, completed with the new elements of the reform curriculum, while among other things, their capacities of learning, their creativity, and their attitude towards mathematics improved. This is also the subjective opinion of Varga's colleagues[7]: while the new approach was disseminated on voluntary basis and by personal contact with the leaders of the experimentation, the dissemination was generally successful, teachers and students were satisfied. Problems started to arise in the 1970s, after that the project became the official basis of the prospective new curriculum. In that period, the number of participating classes grew exponentially, teachers were obliged to participate and personal contact between the teachers and the leaders wasn't possible anymore. These conditions generated failures and resistance.

In this period, similarly to many other reforms of the New Math era, Varga's Complex Mathematics Education reform provoked vivid public debates. It was finally followed by an important correction in 1985. Varga's former colleagues interpret this as a failure, and they consider the obligatory introduction of the reform as the main reason of its rejection. According to them, Varga's approach should have been disseminated progressively in the frame of a bottom-up process, as it happened during the (generally successful) experimentations – but this kind of slow diffusion wasn't politically supported. While a narrow circle of teachers (mostly colleagues of Varga and their disciples) continued to follow the *Guided Discovery* approach with success, the majority of Hungarian teachers didn't really adopt it, or integrated only partial elements of the approach in their practice.

[7] Based on interviews with M Halmos, Cs. Kovács, E. Csahóczi, E. Neményi, L. Pálmay and E. Deák made in 2013.

Despite that, Varga's work remained influential in Hungarian mathematics education in several senses until today, as we will see below.

The main characteristics of Varga's conception on mathematics education

Although Tamás Varga published several papers in Hungarian and also in international journals and books, he mostly focused on the practical realisation of his reform project, and he never offered a comprehensive conceptual description of his approach. In the reconstruction of his conception, beyond the above mentioned articles, the reform's written sources, especially teacher's guides were taken into account. In addition, interviews were made with Varga's colleagues as well as observations of his collaborators' and disciples' current practices.

The name of Varga's reform project, "Complex" mathematics education can be interpreted in several ways. The complexity appears first of all in the reform project itself, touching the content and the structure of the curriculum, the teaching practices, the teaching materials, the resources for students and teachers and also teacher education. The reform is complex also in a mathematical sense: not only new mathematical domains appear, but the coherence of different domains, the rich intersections between them are particularly in focus. Finally, the complexity can be understood in a pedagogical sense: the reform emphasises the importance of rich teaching practices, using a variety of tools and teaching methods.

Varga's conception takes into account the constructivist approach (particularly influential in the 1960s), but distances itself from radical constructivism[8]. It is inspired at the same time by a dialogic approach to mathematics and its teaching, characteristic of the above mentioned Hungarian "heuristic" epistemology.

In the following, I will discuss different characteristics of the reform and show how these influences and ambitions manifest themselves in Varga's work.

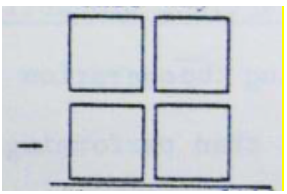
[8] This criticism towards radical constructivism is shared with other emerging approaches of the 1970s, like the *Theory of Didactical Situations* of Brousseau, but their solutions to avoid radical constructivism are different (Gosztonyi, 2017).

Tasks in Varga's approach

The tasks proposed by Varga are "rich tasks" in several senses. First of all, and similarly to other emerging didactical theories of the period[9], they are problem situations allowing students to make sense of mathematical notions and requiring their active contribution to the construction of mathematical knowledge. Varga's tasks are also rich in the sense that many of them connect several mathematical domains. Finally, they are rich by being open to multiple solution strategies.

The richness manifests itself in the collections of tasks too: the diversity of the problems' contexts; the diversity of manipulative tools, from ordinary objects of the classroom to specially designed tools like the Cuisenaire rods, the logical or the Dienes set; and the diversity of representational tools. One of the aims of this diversity is the support of a progressive abstraction process based on a variety of experiences, and the other is to present mathematics as a playful and pleasant activity.

The following example, "Subtraction by rolling a dice" represents several of the above mentioned characteristics.



The goal is to make the difference as great as possible. They can fill the boxes in any order, but only with random numbers produced by rolling dice. After a number is given by the dice, children write it into one of the boxes which is still empty and cannot change it. After filling the third box they have no choice for the fourth random number. Those with the largest difference get three points, those with the second largest get two, those with the next get one and the others none. Then they start again.

Figure 1. New topics for the elementary school math curriculum (T. Varga, 1982, p. 28)

[9] Like that of Brousseau or Freudenthal.

This game is one of the activities allowing students to discover the notion of probability, while they exercise themselves in making subtractions and they acquire a deeper understanding of the operation's properties. The competition, while a pleasant activity for pupils, motivates them to develop strategies and offers an experimental basis to compare different strategies.

The curriculum

The mathematical content and the complex, sophisticated structure of the curriculum are key elements of Varga's reform conception. Similarly to other reforms of the New Math period, Varga aims to integrate new topics in mathematics education, and to present mathematics as a coherent science, organizing the curriculum in accordance with modern mathematics. It involves basing notions on sets and relations, or the strengthened role of algebra, as in a number of other reforms of the same period – but, for Varga, it also means introducing logic, combinatorics, probability or algorithmic thinking in primary and lower secondary school education. Varga was internationally recognized for his work on teaching of logic, combinatorics and probability (T. Varga, 1972) – the specific domains studied by the Hungarian mathematicians supporting his movement.

The internal coherence of the curriculum is ensured by the parallel, spiral presentation of 5 big domains, all being present throughout the whole curriculum, with frequent and various internal connections amongst them: 1) sets and logic 2) arithmetic and algebra 3) relations, functions and series 4) geometry and measure and 5) combinatorics, probability and statistics.

Another significant characteristic of Varga's curriculum is its flexible structure: "suggested" and "compulsory" topics are distinguished from "requirements". As he explains: "many concepts and skills not appearing as requirements in the school year where they are first mentioned in the syllabus, get enrolled to them in subsequent years when they are supposed to become ripened" (Halmos & Varga, 1978, p. 231) .

This organization gives important liberty to teachers, allows differentiation amongst students, provides a rich and varied experimental basis to the progressive generalization and abstraction of mathematical notions, and supports a learning process based on mathematical discovery while elements of mathematical knowledge can emerge as tools during problem solving situations.

Resources

In the period under investigation in Hungary, only one collection of textbooks and teacher's handbooks was available, prepared by the same team as the curriculum. For the primary school, similarly to other countries in the New Math period, worksheets were available, meant to be used only as partial resources beside various activities. Official teacher's guides served as main resources for teachers. These teacher's guides follow a special structure: their main part contains quasi-continuous text mixing examples of tasks with mathematical, didactical and pedagogical commentaries. They are organized in thematic chapters, following the above mentioned five big domains of the curriculum. The tasks, small problems are described with several possible variations, suggestions for inventing new tasks and ideas for their realization: the guide often describes possible student reactions (based on the experimentations) and advises teachers how to deal with them.

After this main part, the books present a *possible* syllabus for the year, emphasizing that it is only an example and encouraging teachers to elaborate their own teaching progression for the year. In fact, following the offered syllabus is a quite complex work: because the main domains are treated in parallel, most of the lessons might contain activities from several domains which are located in different thematic chapters of the book. And since the thematic chapters are not very structured and contain many internal and implicit references, teachers have to know them quite well to use them.

For middle-school, there are textbooks and worksheets provided, with (much less detailed) teacher's guides. One unusual characteristic of these textbooks is the way they introduce new knowledge: they present fictive dialogues of students, discovering new knowledge while they discuss some mathematical problems. The teacher's guide encourages teachers to provoke similar discussions in classrooms.

Expected teaching practices: a collective re-discovery of mathematics

The analysis of these resources, their explicit content and their organisation contributes to the understanding of Varga's conception on teaching practices. In his approach, the implementation of the curriculum is based not only on the above described diverse collections of tasks but also on well structured, often long and complex teaching trajectories, organised around *series of problems*. In Varga's conception (in accordance with other authors representing the Hungarian "heuristic" approach), mathematical

problems shouldn't appear in isolated problem-situations but in connection with each other, as parts of a complex teaching process. Organizational principles of the series can be various, for example the diversity of contexts and manipulative tools and the progressively growing level of abstraction. Abstraction is a slow, progressive process in this approach; with those series of problems, it often takes several years to formulate explicit mathematical knowledge after the first experiences (for an example in combinatorics, see Gosztonyi, 2017). Heuristic strategies can also link the problems: results or solution methods of earlier problems can be reinvested – or on the contrary, challenged – by new problems. The structure of these series is not necessarily linear: a complex network can hide behind a collection of problems, where the order is strict in some aspects and flexible in others. In many cases, series also support the connection of mathematical domains with nodes of the problem-network linking several problem-threads[10].

The textbooks and (especially the primary school) teacher's handbooks present examples of those series. But these are only examples, and often not complete ones: although the handbooks give many suggestions about how to invent new tasks and how to structure series, it is mainly the teachers' responsibility to conceive their own teaching trajectories, according to their classes' specificities and needs. The handbooks explicitly request from teachers a planning work for the middle and long term, and not only for individual lessons.

Concerning the treatment of problem situations, it can take various forms including individual and group work, but collective classroom dialogue is also a very typical form, while the teacher acts as an experienced guide in the collective research process. Handbooks offer numerous advices to teacher's questions and interventions in order to react efficiently to students' contributions: to help the advancement of the collective research process while leaving an important responsibility to students in problem solving and in the construction of mathematical knowledge. The fictitious classroom dialogues presented in the middle-school textbooks also serve to support the implementation of this dialogical practice in classrooms.

In summary, we can say that in Varga's approach, the teachers' work is focused on the planning, organization and continuous support of collective re-discovery processes in

[10] The notions of „problem-threads" and „nodes" are used particularly by Lajos Pósa and his disciples in their program for gifted students, which was partly inspired by Varga's work and where this kind of structuration is particularly important. For details, see (Artigue et al., 2020; Bóra, 2020; Katona, 2020). About problem graphs, see (E. Varga, 2020).

mathematics. This work is defined on two main levels: the construction of long-term teaching processes in form of series of problems, and the management of problem-solving situations by classroom dialogues. The work expected from teachers is quite complex, covering long term planning, lesson plans, adaptation and invention of tasks, while the classroom management requires a lot of attention and thoughtful improvisation in order to scaffold conveniently the individual and collective research processes.

Varga considers teachers as creative and autonomous intellectuals with significant mathematical and pedagogical skills. This is certainly liberating for teachers, and supports the creation of a democratic milieu in classrooms, valorising also students' autonomy and creative contributions. At the same time, it creates a challenge for dissemination of the approach on a large scale.

Varga's "Guided Discovery" approach: a summary

In summary, we can say that the different elements of the reform, the curriculum, the task design, the resources, the indications about expected teaching practices are conceived following a coherent conception. This conception is partly related to some international trends of the New Math period ("modern mathematics" in the curricula, students' participation in the construction of mathematical knowledge, usage of manipulative tools etc.). At the same time, it corresponds to the above mentioned "heuristic" epistemology of mathematics, represented by Hungarian mathematicians. Problem solving and mathematical discovery is in the focus of the conception: it is supported by the flexible structure of the curriculum, the parallel, dialectic presentation of different mathematical domains, the use of various material tools and representations, the construction of long-term teaching processes in form of series of problems, and a dialogic guiding of the class. These elements allow students to advance in their own pace, to have enough time and occasion to gain various experiences and to follow a slow abstraction process through progressive generalization. Matching these characteristics, we can call Varga's conception *Guided Discovery* approach.

Many of the characteristics of Varga's *Guided Discovery* approach connect it to other contemporary approaches related to the notion of Inquiry Based Mathematics Education[11]. Varga's conception shows however some specificity: for example in his sophisticated

[11] Several of these approaches are presented in (Artigue et al., 2020).

construction of the curriculum and the idea of constructing teaching trajectories in form of series of problems. In this sense, Varga's work is susceptible to contribute to and enrich current international discussion on teaching mathematics by problem solving and inquiry.

Varga's legacy today

In spite of the partial failure of the Complex Mathematics Education reform described above, the difficulties and limits of dissemination, we have to underline that Varga's work remains influential in Hungarian mathematics education in several ways. Pálfalvi (2000) shows continuity in the curricula's conception: the main structure of the Hungarian curricula and several of its organizing principles are inspired by Varga's conception. Despite of numerous modifications, the main structure and the content of the curriculum remained quite stable until today[12]. Some of the textbook authors from Varga's team were active until the 2010s and their textbooks show also important continuity with the original versions of the 1970s – although other textbooks are also available now. Most of the teacher trainers consider Varga's "Guided Discovery" conception still relevant and find inspiration in it, especially for primary level in-service teacher training – even if only a small number of teachers implement the approach in their effective teaching practice.

The on-going MTA-ELTE Complex Mathematics Education research project aims to revisit Varga's reform, contribute to the theoretical description of his approach, make it more available on a national and an international level and connect it to other current didactical approaches; to adapt Varga's oeuvre to the current context and challenges of education; to develop extensions of this work for kindergarten and for high-school; and to contribute to its dissemination by developing new ways to support teachers' work. Several sub-projects are developed in this frame. I only mention one example here: the sub-group focusing on the teachers' work with series of problems analyses series conceived by expert teachers of the Guided Discovery approach, and develops new representational tools to make the series' structure visible. We prepare a commented collection of series of problems, and experiment the use of this collection by non-expert teachers in the frame of pre- and in-service teacher education.

[12] In the preparation of the upcoming official curriculum, to be introduced in 2020, Varga's principles were expicitly taken into account.

The Varga 100 conference was an important step towards the aims listed above: many of the presentations revisited Varga's work, and the exchange between senior and young researchers from various countries, former colleagues of Varga and PhD students certainly contributed to make Varga's oeuvre a significant part of the collective reflections in the mathematics education research community.

Acknowledgement

This article was written in the frame of the "MTA-ELTE Complex Mathematics Education in the 21th Century" project supported by the Hungarian Academy of Sciences (ID number 471028).

References

Artigue, M., Bosch, M., Doorman, M., Juhász, P., Kvasz, L., & Maass, K. (2020). Inquiry based mathematics education and the development of learning trajectories. *Teaching Mathematics and Computer Science, 18*(3), 63-89.

Balogh, M. (2014). Varga Domokos, az 'abszolút pedagógus'. *Iskolakultúra, 24*(3), 107–118.

Báthory, Z. (2001). *Maratoni reform. A magyar közoktatás reformjának története, 1972-2000*. Budapest: Ökonet.

Bóra, E. (2020). A computational thinking problem-thread for grade 7 students and above from the Pósa method. *Teaching Mathematics and Computer Science, 18*(3), 101-110.

d'Enfert, R., & Kahn, P. (Eds.). (2011). *Le temps des réformes. Disciplines scolaires et politiques éducatives sous la Cinquième République (années 1960)*. Grenoble: Presses Universitaires de Grenoble.

Dancs, G. (2016). A matematikus abszolút pedagógusról, Varga Tamásról. In Á. Hudra, E. Kiss, & L. Trencsényi (Eds.), *Abszolút pedagógusaink nyomában* (pp. 169–188). Budapest: Magyar Pedagógiai Társaság.

De Bock, D. (2020). Willy Servais and Tamás Varga – A Belgian-Hungarian perspective on teaching school mathematics. *Teaching Mathematics and Computer Science, 18*(3), 29-38.

Gosztonyi, K. (2016). Mathematical Culture and Mathematics Education in Hungary in the XXst century. In B. Larvor (Ed.), *Mathematical Cultures. The London Meetings 2012-2014* (pp. 71–89). Basel: Springer. http://link.springer.com/10.1007/978-3-319-28582-5

Gosztonyi, K. (2017, February 4). *Understanding didactical conceptions through their history: a comparison of Brousseau's and Varga's experimentations*. CERME 10, Dublin. https://keynote.conference-services.net/resources/444/5118/pdf/CERME10_0594.pdf

Gosztonyi, K., Kosztolányi, J., Pintér, K., Vancsó, Ö., & Varga, E. (2018). Varga's Complex Mathematics Education reform: at the crossroad of the New Math and Hungarian mathematical traditions. In Y. Shimizu & R. Vithal (Eds.), *School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities. Conference proceedings* (pp. 133–140). https://drive.google.com/file/d/1xR0QOjI8X_mJb5mmhH-G_OqoMaFE54Gy/view

Gurka, D. (2004). Kalmár László szerepe Lakatos Imre matematikafilozófiájának alakulásában. In G. Palló & Békés (Eds.), *A kreativitás mintázatai* (pp. 258–280). Budapest: Áron Kiadó.

Halmos, M., & Varga, T. (1978). Change in mathematics education since the late 1950's – ideas and realisation. Hungary. *Educational Studies in Mathematics*, *9*(2), 225–244.

Hungarian National Commission for UNESCO. (1963). *Report of the work of the International Symposium on School Mathematics Teaching*. Budapest: Akadémiai Kiadó.

Karp, A., & Schubring, G. (2014). *Handbook on the history of mathematics education*. New York: Springer.

Katona, D. (2020). Categorising question-question relationships in the Pósa method. *Teaching Mathematics and Computer Science*, *18*(3), 91-100.

Klein, S. (1980). *A komplex matematikatanítási módszer pszichológiai hatásvizsgálata*. Budapest: Akadémiai Kiadó.

Máté, A. (2006). Árpád Szabó and Imre Lakatos, or the relation between history and philosophy of mathematics. *Perspectives on Science*, *14*(3), 282–301.

Mérei F., & Binét Á. (1970). *Gyermeklélektan*. Budapest: Gondolat. https://www.libri.hu/konyv/merei_ferenc.gyermeklelektan.html

Pálfalvi, J. (2000). *Matematika didaktikusan*. Budapest: Typotex.

Pálfalvi J. (2019). *Varga Tamás élete*. Budapest: Typotex. https://www.typotex.hu/book/10695/palfalvai_jozsefne_varga_tamas_elete

Pléh C. (2010). *A lélektan története*. Budapest: Osiris.

https://regi.tankonyvtar.hu/hu/tartalom/tamop425/2011_0001_520_a_lelektan_tortenete/adatok.html

Romsics, I. (1999). *Hungary in the Twentieth Century*. Budapest: Corvina/Osiris.

Servais, W., & Varga, T. (1971). *Teaching school mathematics. A Unesco source book.* Middlesex: Penguin Books.

Szabó, M. (2013). Karácsony Sándor nyelvfelfogásának hatása Kalmár László korai matematikafilozófiájára. In Z. Zvolenszky (Ed.), *Nehogy érvgyülölök legyünk* (pp. 164–173). Budapest: L'Harmattan.

Szendrei, J. (2017). *In memory of Tamás Varga*. CIEAEM. http://www.cieaem.org/?q=system/files/varga.pdf

Varga, E. (2020). 'How to be well-connected?' Towards the development of Problem Graphs as design tools for teachers. *Teaching Mathematics and Computer Science*, *18*(3), 145-155.

Varga, T. (1972). Logic and probability in the lower grades. *Educational Studies in Mathematics*, *4*(3), 346–357. https://doi.org/10.1007/BF00302583

Varga, T. (1975). *Komplex matematikatanítás. Kandidátusi alkotás ismertetése* [Thesis]. MTA.

Varga, T. (1982). New topics for the elementary school math curriculum. In T. C. O'Brien (Ed.), *Toward the 21st Century in Mathematics Education* (pp. 12–34). Teachers' Center Project, Southern Illinois University at Edvardsville.

KATALIN GOSZTONYI
ELTE, INSTITUTE OF MATHEMATICS, MATHEMATICS TEACHING AND EDUCATION CENTRE
PÁZMÁNY PÉTER SÉTÁNY 1/C, BUDAPEST, HUNGARY, H-1117

E-mail: katalin.gosztonyi@gmail.com