18/2 (2020), 143–156
DOI: 10.5485/TMCS.2020.0513
tmcs@science.unideb.hu
http://tmcs.math.unideb.hu

Teaching Mathematics and Computer Science

Solution of an open reality based word-problem in two secondary schools

Szilárd Svitek, Judit Szitányi and Gabriella Ambrus

Abstract. This survey through an open reality based word problem is intended to assess - in two secondary schools in Komárom (Hungary) and in Komarno (Slovakia, Hungarian name: Révkomárom) in grade 10 - the ability of students to realize openness of a task. The comparison is justified by the fact that the language of teaching is Hungarian in both secondary schools, but with different curricula. This survey is related to the Content Pedagogy Research Program by the Hungarian Academy of Sciences. It is preceded by several surveys with a word problem (Pocket Money) of the third author and led by her between 2012 and 2015, and within that project in 2017 within a large sample test, among about 1500 students and university students in Hungary (Ambrus, 2018) (Ambrus et al., 2019). In our research we wanted first to assess how openly work students in two schools of the two cities mentioned in solving the same task. The answer to this question was similar to the large sample test results, so most of the students worked in a closed way, when solving this word problem. So we went on and tried to explore how students thought about their own solution given to this task, through mixed-type interviews.

Key words and phrases: problemsolving, open task, interview, educational research.

MSC Subject Classification: 97D70, 97F90, 97D50, 97M10.

Theoretical background

Open tasks

In today's mathematics education, closed tasks dominate (Büchter & Leuders, 2005). A task can be considered closed if the initial conditions and the question (end state to be achieved) and the way of solution (solution path) are clearly given in the task. We can talk about open tasks if at least one of the above conditions is not met. Pehkonen (1995) calls a task open only if the initial or target state of the former is not precisely defined. Wiegand and Blum (1999) grouped the open tasks according to the openness/closeness of the initial state – solution path – target state as follows.

	Starting position	Solution path	Target state
Type 1	clear	clear	clear
Type 2	clear	clear	unclear
Type 3	unclear	clear	clear
Type 4	unclear	clear	unclear
Type 5	unclear	unclear	clear
Type 6	unclear	unclear	unclear

Table 1. Grouping of open tasks by Wiegand and Blum (1999)

Open-ended tasks can be entirely "within" mathematics, but can also be based on a near-realistic situation. In addition, open-ended problems can also help in both vertical and horizontal construction of the mathematical concept network (Greefrath, 2004). Open-ended tasks can provide enough motivation for students to see and understand real-life problems with a mathematical eye.

Open real-world text tasks

In the case of open tasks based on a real situation, the initial state can be influenced by several different factors. By fixing these in different ways, several different (correct) results can be obtained. A real and then a mathematical model based on the given assumptions leads to the preparation of a solution. Finally, when evaluating the solution, we again need to consider the starting factors. It is necessary to decide whether the solution we have received is acceptable under the pre-determined conditions. It is also worth looking at the correctness of our result from the perspective of the original task, as it may be necessary to change the determining factors to find a new model and thus a new solution. This is actually the process of modeling (Greer, 1997), and the tasks that can be solved in this way are also called modeling tasks (Blum & Leiss, 2006). Open tasks based on the real situation are also of particular importance in acquiring competencies related to the modeling process, as they can help students develop creativity, critical and problem-solving thinking (Chapman, 2006). They can also be motivated to understand the mathematical concept and the underlying mathematical content. With the help of reality based word problem, students can acquire knowledge that can be used in practice.

However, solving modeling tasks also causes difficulties for students, the main reason for which is usually the openness of the task. Simple open problems, such as traditional word tasks based on a real situation, where mostly less modeling is required, could in principle be closer to learners, but in practice this is not the case. On the one hand, students tend to be intimidated by word tasks in general (Jahnke, 2001). On the other hand, it is also not true that these open tasks are handled/solved by the students according to the situation (Verschaffel & De Corte, 1997). Yet properly applied (non-forced) reality based word problems could even help students become familiar with open-ended tasks.

In science and everyday life, uncertainty is a given. Uncertainty is therefore a phenomenon at the heart of the mathematical analysis of many problem situations. Test formulated in PISA study are always based on real-world situations that contain problems or tasks that must be solved. Therefore we can establish, that real-world tasks are an important part of the PISA study.

The Pocket Money problem

The situation of the open word problem used as well in the present study is easy imaginable and realistic for the students, its wording is simple, it is similar to the traditional, closed word tasks.

The Pocket Money problem

Ever since the Pisti family moved into a new flat, he receives a weekly pocket money of HUF 1000, which he has always set aside. How many days have they lived there when he has already raised HUF 10 000? Please justify your answer (Ambrus, 2016).

In the case of this task, the problem basically is that not everything is given in the initial state (students only think that they handle the task yes and closed), and only by specifying and clarifying additional conditions can a specific solution be made that takes the situation into account.

Thus, in the division of Wiegand and Blum (1999), according to the table, the task belongs mostly to type 5, possibly to type 3 (since changing the conditions does not substantially change the solution, only the result), because the goal is clear: give the number of days spent in the new apartment.

The task itself does not require deeper mathematical knowledge, and by the fifth grade of the primary school, or even earlier, students already have the competences needed to solve the problem. The topic of the assignment (pocket) money - can make students more interested in the subject.

At first glance, the solution seems simple; you don't need to use complicated math formulas to solve it. The openness of the problem stems mostly from the fact that two variables are not predefined in the problem. It is when Pisti and his family moved into their new home and when the boy gets his pocket money (which day of the week). Once we discover this, or we discover it with the students, the task immediately becomes "open" (it was open before the discovery, but since the solver of the task did not think about the real situation, can treat it as a closed task). For a detailed solution to this problem, see Ambrus (2016), now we mention only some important details from it. So, if you were treating it as an open-ended task, you can gave accordingly to the given (complementary) assumptions several correct solutions, and finally i.e. the 245 ± 6 intervals.

As can be seen from the detailed solution, students can solve the problem at different level. Obviously, solutions of different levels and complexities could be expected from different grades and from students with different performances in mathematics. But now we were only thinking of a basic solution regarding higher grades as well. This is 245 days in the case of a closed interpretation, and in the case of an open interpretation this means a solution based on some assumptions or a list of specific solutions from the already mentioned interval, for example $245 + 6$ days. After specifying the condition, only addition, multiplication and division are required to produce a numerical solution.

Background and purpose of the research

Within the Academic Project, a large sample survey has already been conducted among primary and secondary school students with the Pocket Money problem (or a given version of it in a smaller number of numbers, but with the same mathematical content, in the case of lower grades) (Ambrus et al., 2019). In this survey, on the one hand, the question of how many students managed to give an open (realistic) solution to the Pocket Money problem was examined, on the other hand, after solving the problem, they answered questions related to their learning of mathematics by filling in a questionnaire.

A solution was considered as open, - as in the previous studies - if the learner has indicated that there may be several good solutions, or that the solution depends on assumptions. That was also an open solution in which false or inaccurate results were given, but the learner pointed out in a right way that there could be several solutions.

The same task was chosen for our survey and the conditions of the survey were also the same, that the data would be comparable with the results of the mentioned previous survey.

One of the main aspects of our study was to achieve that we could compare the results of selected students in the two cities. To do this, we had to collect data that would allow the most relevant comparisons of the results of the students of the two schools, taking one-one school in each city. In solving this problem, as in previous surveys, the students' attention was specifically called to justify their answer.

Based on our experiences, and based on the large sample results already achieved, we assumed that the overwhelming majority of students think closed when dealing with this real-world problem (Ambrus et al., 2019). The other assumption was that there would be no significant difference in performance between students in the two countries. We based this assumption on similar syllabus contents and PISA surveys results (OECD, 2016).

However, since this survey, like the previous ones, was a written one, one can only guess at the mindset of the students. Therefore, we also planned an additional study - personal interviews. The purpose of the interviews was to gain a deeper insight into the students thinking, also giving them the opportunity to express themselves verbally.

Method

Our present study consisted of two main parts:

(1) In the first part, the students solved first the Pocket Money problem and then the questionnaire. The analysis of the questionnaire data is not covered in this research, the emphasis was on the interviews.

(2) The other part was the interview with students about their solution which was not a part in the case of the previous surveys.

From the possible ways of making interviews the mixed type interview was chosen. At the beginning of the interview, the interviewer asks the subject structured questions to determine the guideline of the interview and then move on to the background, allowing the interviewee to speak. As a result, the interviewer will have to respond flexibly in this part of the interview, possibly asking further questions to help. It is also important that the whole interview is not limited in time, contains no questions to decide (this way not to affect the subject of the interview). It is important for the interviewer to remain neutral during the interview, this way the interviewee is not influenced in any way. Many literature describes, analyzes and uses interviews as a method, but Trautmann said, none of the interview concepts presented in the literature was appropriate regarding the students he worked with, that is why we chose his method (Trautmann, 2010). Thus, the mixed interview proved to be an obvious solution, as it gave students the opportunity to express themselves as freely as possible, while providing comparable answers through introductory and follow-up questions.

The sample for the survey

Apart from their geographic location, we chose the two cities, Komárom (Hungary) and Révkomárom (Slovakia), - separated only after the Second World War in the 20th century -, because of the similarity of the content of the framework curricula (National core curriculum (NCC) in Hungary and The International Standard Classification of Education (ISCED3) in Slovakia-regarding secondary (this is what the number three means) schools) and the similar results in PISA 2015 survey in these countries (OECD, 2016). It is clear from the curricula of the two countries that, while Hungary provides precise hours for the elaboration of certain subjects and sections, Slovakia gives the schools a little more space to determine for themselves the amount of lessons they will devote to mathematics.

Introduced in Slovakia in 2015, the innovative public education program distinguishes grammar schools from vocational secondary schools and secondary vocational schools. The 2015 innovation in the Slovak State Educational Programm does not specify in which grade what should be taught, in what order and to what extent, but the performance and content standards of the individual main and sub-topics already define the curriculum quite precisely, which is similar to the Hungarian NCC 2012, which is still fully valid at the time of the survey.

It is prescribed total of 12 hours of mathematics per week for Hungarian minority grammar schools in Slovakia. Separate curricula have been defined for each specialization in vocational secondary schools and secondary vocational schools, with different number of classes. The number of classes per week, broken down by grade, can be determined by the schools themselves. In the examined secondary school, the number of hours in mathematics per week, broken down by grade, was as follows: 1st Grade: 5; 2nd Grade: 2; 3rd Grade: 2; 4th Grade: 0.

Number of classes devoted to mathematics in Hungarian secondary schools, divided into four grades 1st Grade: 3; 2nd Grade: 3 + 1; 3rd Grade: 3 + 1; 4th

Grade: 3 + 1. (+1 is the number of math lessons in the number of classes that can be planned.)

The striking difference in the number of classes of mathematics in the two countries may also be due to the fact that while in Hungary the degree in mathematics is compulsory, in Slovakia it is currently only available as an optional subject in the baccalaureate. Both ISCED3 and NCC describe the goals that students should reach, what skills, knowledge and competences they should acquire. While NCC also lists related areas with other subjects, this is not the case at ISCED3. Both countries are members of the Eastern European bloc, characterized by decades of direct economic, political or military ties to the former Soviet Union (Csíkos et al., 2019). Both countries achieved similar results in the 2015 PISA survey, but overall, Hungary was performing better in terms of final results. Only by examining mathematical competence can we say that Hungary was ahead of Slovakia in the 2015 survey. It is important to note that the difference between the two countries is not significant. In the years prior to the 2015 PISA survey, however, Slovakia performed better in mathematical competence. (OECD, 2016)

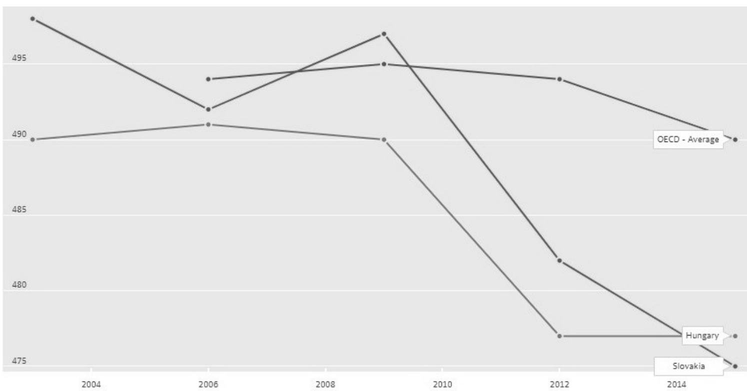

Figure 1. The mean score in PISA study of Hungary and Slovakia in the part mathematics until 2015 (OECD, 2016)

In selecting the schools, we took into account the average grades of the students there, which was average (mark three). In Slovakia, as in Hungary, the grading system consists of five levels. This was also to make the most relevant comparison possible. We had a problem with the Slovak part, where the primary school consists of 9 grades. Thus, in order to meet the 10th grade criterion (that is, the given age group), the first grade students of the selected secondary school in Slovakia participated in the survey. So the entire 10th grade of the two schools, one-one in each city, participated in the survey, a total of 120 students (52 students in Hungary and 68 students in Slovakia, n = 120).

The survey

In the first part of the survey students were asked to work on the Pocket Money problem. The students worked alone, had not met the task before, had not been involved in any preparation or assistance. They had 10 minutes to solve the task, but it was possible to work on it additional 5 minutes, but this was not used by the students. The survey was conducted in the respective classes in both schools at the same time with the help of colleagues, who received beforehand the necessary information to conduct the investigation.

On the second part, students received a questionnaire. The questions were intended to determine the student's relationship with the mathematics and its learning, as well as questions, reflecting on the task, they had 10 minutes for the answers. As mentioned, the questionnaire and the results obtained are not addressed in the present study.

The vast majority of students gave the "expected" solution/answer, the 245 (days), which indicates closed way of thinking in this very case. Only one student from the Slovak school answered on a different (open) way. There were four other students who did not provide the expected answer: 245. One of these four students, was a high school student in Komárom who presumably read the question superficially (the answer was 35 days), which meant of course the number of the weeks. This error was also reported in the case of a student at the school in Slovakia, where 35 was also the answer. Another two Slovak students made a calculation error, one giving 275 and the other 265.

The answers given to the Pocket Money problem suggest that the students - at least in this case - seem to be thinking in a closed way. In the large sample survey with the same method (Ambrus et al., 2019), 1346 students were interviewed, of which 279 students (20.73%) were in grade 10. Of these students, 25 (8.96%) were identified as "realistic" (open answering) respondents.

In our present survey only one student gave a realistic answer which means 0,83%. This number is clearly different from the results of the large sample survey, which also provided a reason to supplement the research with interviews.

	10th grade students	realistic answer	percentage
large sample survey	279	25	8.96%
present survey	120	1	0,83%

Table 2. The results of 10th graders

The interview

A total of 15 interviews were conducted in the two schools (with 6 students from secondary schools in Komárom and with 9 students from Komarno). The selection for the interview was made randomly, except one person. We certainly wanted to be able to interview the student who gave the realistic (open) answer. Interviews were always conducted in the school of the student, in a separate room. The students were interviewed individually by the same interviewer, the first author. We started the conversation with all the students with the same questions, and set the guideline for the discussion. These were:

- Read through your solution, it can be out loud!
- What has come to your mind now about the task and your solution / solution?
- Did you think about the situation / aspect?
- Have you ever seen a task like this?
- Did you like the task? Why?

In addition to the warm-up questions, we asked later control questions, which were in other forms, but in terms of content, the same questions were asked. We tried to verify in this way the information we received during the query. Interviews with students lasted 4-5 minutes on average. For the coding the students' answers, we introduced KM (for Hungarians, Komárom) and KN (for Slovaks, Komárno), which were assigned a number, based on the number of interviews that took place at that school. (The code for the student, who gave the 245 ± 6 is KN3, and will be marked with * for easier identification.)

During the interview, the first thing was, that the students received the sheet with the task and their solution again. They could not write in it anymore, they had a separate (blank) sheet for taking notes. Students did not use this opportunity, but rather expressed themselves verbally. We asked them to re-read the task and the solution they provided. Regarding the question of what came to mind first after reading the assignment we can distinguish two main groups based on their answers. At one, the students emphasized the ease of the task

(e. g. according to them all the data was provided, elementary knowledge was also enough for the solution, we use the ET (easy task) notation regarding their answers). The other group included those who cited it as a real life task because of pocket money. Their group is denoted by RL (real life). Presumably, money, as a central element, brought this opinion. It is an interesting contradiction that some students referred to it as a real life task, but they did not ask for the unspecified variables of the task, to know which day they moved in and the day the pocket money was given.

Q: What came to your mind the first time you read the assignment?

- *KM1*: I thought a lot about this task because I found it too easy. I looked for the knack in it but couldn't find it. (ET)
- *KM2*: Simple logic can solve the problem. (ET)
- *KM5*: Very simple, clear task. (ET)
- *KN1*: I tried to assume that a month consists of 31 days, but then I realized that not every month is made up of the same number of days. (RL) -/ *here it is noticeable that the student immediately begins to explain the course of his/her solution, his/her strategy, and misses the step of understanding the problem.*
- *KN3**: This is a real life example because we work with money in it. (RL)
- *KN5*: This is a simple task. (ET)
- *KN6*: It seemed simple enough and it wasn't difficult to solve the problem. (ET) – *by simple task the student understood the closed task based on the interview, where the triple of the initial state - solution path - target state was given*

We were wondering if they saw a task like this before, or that with their current knowledge "what kind" of task it is considered by them. It is clear from their answers to the question that this problem was treated as a routine word problem, where the data was extracted from the text and the solution calculated.

Q: Have you ever seen a task like this?

- *KM1*: Yes, I have encountered a text assignment in elementary school. (ET)
- *KN3**: I have not encountered such a task yet. (RL)
- *KN6*: Yes, I did a lot of math competitions. - here the student notes on another question, reflecting on his answer, that the aforementioned mathematical competition was Pythagoras. (ET)
- *KM6*: ... because all the data was provided. (ET)

Based on their answers to the question, they mainly compared the textual tasks with the Pocket Money problem, and classify this task among those where the answer has to be formulated in text.

From the interviews, we also collected the most thoughtful expressions, relevant to our research (which represent students mindsets) that came up during the interview, but were not answers to any of the questions directly. These can help answer the question of what the students thought and how they thought about the problem. All such manifestations that were noticeable were summarized below.

- *KM4*: I think this worked out well.
- *KN2*: I think I did the job well.
- *KN5*: I think I described what was needed.
- *KN6*: I went all the way I had to.
- *KN6*: The example was not complex, there was nothing to misunderstand, there was nothing to mislead. I didn't have to think too much about it.

It is also clear from these manifestations that students, rather than characterizing their own thinking, try to imitate the reflection that they think their teacher would say about their work.

Based on their responses to the interview, we identified three category of interviewees. In this case, the group is not the ideal formulation, since only one student is placed in one of these categories. The first and largest category is the closed thinking category. This is not surprising, considering previous surveys. The student KN3 *is the only one of the category of open-thinking (the group expression here would be a strong exaggeration). We could also distinguish a third category who are trying to think openly (at least not as expected). Meeting expectations and having little experience with these types of tasks will eventually lead to closed thinking. This (classroom culture) is referred to by Greer (1997), who refers to R. B. Davis's, 1989 research with the following quote:

"Was the boy really thinking about solving the actual problem, or was he trying to accommodate himself to the peculiar tribal culture of the American classroom?"

Interviews may place someone in the third group, that is, those who try to think openly, if they have indicated during the interview that they have tried to meet expectations (be they teacher-or curriculum related), or made a statement that he/she had solved the task well, but was unsure during the interview. Students with the following code can be listed here: KM4, KN2, KN5, KN6.

Of the students, this phenomenon was most noticeable at KN6 and KN5. After the interview, KN5 said: This is what we are expected to do, this is what we will need for graduation.

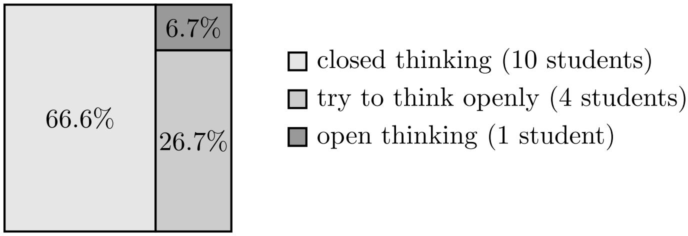

Figure 2. Percentage distribution of students into the category's – based on interviews

It may be noted, that at the end of the interviews, there was no one who clearly stated or just would have suggested that the task was open. Although this was not stated as an expectation during the interview planning, the reference to openness could possibly have been formulated. This could have added additional questions to the interview, but this did not happen.

Summary

A word problem play an important role in mathematics education, helping students to apply formal mathematical knowledge and theoretical skills acquired in the school (Verschaffel & De Corte, 1997). The survey was based on a concrete close-to-reality task, after which students had to complete a questionnaire. We did this in 1-1 secondary schools in two countries. (N = 120). Because the answers were unanimous, we conducted interviews (k = 15). In our interviews, we were mindful of giving our subjects as much freedom as possible, so that they were able to tell and convey their own way of thinking (Küsters, 2009). From our original assumptions, it has been proved, based on preliminary research, that the majority of students think in a closed way and that there will be no significant difference in the performance of students in the two countries. What we found most often during the interviews was, that they did not think about the real situation and had no contact with these types of examples in their studies. Because they needed little mathematical knowledge to solve the problem, they called it easy, which could also lead to thinking "mechanically", with the usual (this leads to a result of 245 days in the case of a closed interpretation) solution schemes. It also turned out that most of them wanted to meet expectations, they wanted to solve the task the way they learned beforehand by word problems. From the students solutions, and the interviews, it can be seen (both individually and as a whole) that students made hardly any connection between the mathematics and the real life content of the task.

Acknowledgements

This research has received support from the Content Pedagogy Research Program of the Hungarian Academy of Sciences (MTA-ELTE Complex Mathematics Education Research Group).

References

Ambrus, G. (2016). The Pocket Money Problem. In A. Kuzle, B. Rott, & T. H. Cadez (Eds.), *Problem Solving in the Mathematics Classroom - Perspectives and Practices from Different Countries* (p. 49-59). Münster: WTM-Verlag.

Ambrus, G. (2018). Egy nyitott, valós szituáción alapuló feladat variációi az oktatásban és a didaktikai kutatásban. *Gyermeknevelés*, *6*, 55-65. doi: 10.31074/gyn201815565

Ambrus, G., Herendiné Kónya, E., Kovács, Z., Szitányi, J., & Csíkos, C. (2019). A cross-sectional analysis of students answers to a realistic word problem from grade 2 to 10. *International Electronic Journal of Mathematics Education*, *14*(3), 513-521. doi: 10.29333/iejme/5753

Blum, W., & Leiss, D. (2006). "Filling up"–the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In M. E. Bosch (Ed.), *Cerme-4–proceedings of the fourth conference of the european society for research in mathematics education.* Guixol.

Büchter, A., & Leuders, T. (2005). *Mathematikaufgaben selbst entwickeln. Lernen fördern – Leistung prüfen.* Berlin: Cornelsen Scriptor.

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics*, *62*, 211-230.

Csíkos, C., Andras, S., Rausch, A., & Shvarts, A. (2019). Mathematical learning and its difficulties in eastern european countries. In A. F. et al. (Ed.), *International handbook of mathematical learning difficulties* (p. 145-163). Cham: Springer. doi: 10.1007/978-3-319-97148-3

Greefrath, G. (2004). Offene Aufgaben mit Realitätsbezug. Eine Übersicht mit Beispielen und erste Ergebnisse aus Fallstudien. *mathematica didactica*, *27*(2), 16-38.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and instruction*, *7*, 293-307.

Küsters, I. (2009). *Narrative Interviews: Grundlagen und Anwendungen* (2nd ed.). VS Verlag für Sozialwissenschaften.

OECD. (2016). *PISA 2015 Results: Excellence and Equity in Education (Volume I).* Organization For Economic Co-Operation and Development. doi: 10 .1787/9789.2642.66490

Pehkonen, E. (1995). Introduction: Use of open-ended problems. *International Reviews on Mathematical Education*, *27*(2), 55-57.

Trautmann, T. (2010). *Interviews mit Kindern: Grundlagen, Techniken, Besonderheiten, Beispiele* (1st ed.). VS Verlag für Sozialwissenschaften.

Verschaffel, L., & De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school? In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (p. 69-97). Psychology Press/Erlbaum (UK): Taylor and Francis.

Wiegand, B., & Blum, W. (1999). Offene Probleme für den Mathematikunterricht - Kann man Schulbücher dafür nutzten? *Beiträge zum Mathematikunterricht*, 590-593.

SZILÁRD SVITEK
EÖTVÖS LORÁND UNIVERSITY, FACULTY OF SCIENCE

E-mail: svitekszilard@gmail.com

JUDIT SZITÁNYI
EÖTVÖS LORÁND UNIVERSITY, FACULTY OF PRIMARY AND PREE-SCHOOL EDUCATION

E-mail: szitanyi.judit@tok.elte.hu

GABRIELLA AMBRUS
EÖTVÖS LORÁND UNIVERSITY, FACULTY OF SCIENCE

E-mail: ambrus.gabriella@ttk.elte.hu

(Received July, 2020)