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Guided Discovery in Hungarian Education Using Problem Threads: The Pósa Method in Secondary Mathematics Classrooms

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Abstract. In Hungary, 'guided discovery' refers to instruction in which students learn mathematical concepts through task sequences that foster mathematical thinking. A prominent figure of guided discovery is Lajos Pósa, who developed his method to teach gifted students. Rather than teaching mathematics through thematic blocks, the Pósa Method employs webs of interconnected problem threads in which problems are built on each other, and different threads are presented simultaneously, so that students work on problems from multiple threads at the same time. It was found that this method has been successful as extracurricular training for gifted students since the 1980s; however since 2017, as part of an ongoing research, the method has been applied to mainstream curriculum in two public secondary school classrooms. The present paper examines the design and implementation processes of problem threads in this public secondary school context.

Key words and phrases: Lajos Pósa, Pósa Method, guided discovery, problem thread.

MSC Subject Classification: 97D40.

Introduction

Teaching mathematics should always aim to foster in students a deep understanding of the content presented. One method that strives to this end is the Pósa Method, developed by Lajos Pósa for advanced math students in Hungary since the 1980s (Győri & Juhász, 2017). This method utilizes problem threads, which are series of problems that students complete, discovering mathematics from different topics simultaneously (Katona & Szűcs, 2017). While this method has mainly been used as an extracurricular program, since 2017 the Pósa Method has been used to teach content in two public secondary school classrooms as a study supported by the Content Pedagogy Research Program of the Hungarian Academy of Sciences. Little research regarding the use of the Pósa Method exists, so this paper aims to fill that gap by examining the design and implementation of problem threads in the Pósa Method when used in the context of public secondary school mathematics.

More specifically, when looking at how problem threads are designed, this paper investigates first the main characteristics of the Pósa Method in two Hungarian public secondary mathematics classes and the process of designing problem threads. Second, the classroom experience and the implementation of problem threads are explored in more detail. The research is qualitative in nature, utilizing observations, interviews, and questionnaires to obtain data.

This paper first examines the existing literature regarding the Pósa Method in its original context as an extracurricular program. Next, it explores the context of the present study. Following this, the paper details the methodology used for data collection before discussing the results of both the design and implementation processes of using problem threads in two public secondary classrooms.

Literature Review

Guided Discovery

When considering discovery learning, particularly guided discovery, in Hungarian education, especially of gifted students, one would be misguided if they failed to notice Lajos Pósa. However, Pósa was not the first Hungarian mathematician to see value in discovery learning; he followed in the footsteps of Tamás Varga. Varga is considered the pioneer of using guided discovery learning in Hungarian public primary schools because of his work leading the reform movements of Hungarian mathematics education in the 1960s and 70s (Gosztonyi, 2016). Pósa has taken Varga's work a step further by applying principles of guided discovery to contexts for advanced mathematics students.

Before detailing the Pósa Method, it is important to note what exactly is meant by 'discovery learning,' and more specifically, 'guided discovery.' According to Honomichl and Chen (2012), discovery learning "is a technique for helping learners create and organize knowledge. Involving mindful participation and active inquiry, it typically takes place during problem-solving situations. The learner draws upon past knowledge and experience to infer underlying strategies and gain an understanding of concepts" (p. 615). When originally introduced into educational contexts, students learning by discovery were usually expected to work independently, with minimal intervention from the teacher (Honomichl & Chen, 2012). Since then, the focus has shifted to a particular type of discovery learning called 'guided discovery.' Honomichl and Chen (2012) define this as being "similar to the Vygotskian concept of scaffolding: guidance that is dynamic and responsive to the learner's current state of experience and ability, with inexperienced learners receiving greater guidance or supervision and experienced learners receiving less intervention" (p. 615). They suggest that this can be done by strategically presenting the material, giving feedback, or by asking probing questions. Both Varga and Pósa's work using discovery learning falls into this category of guided discovery.

The Pósa Method

According to Győri and Juhász (2017), after completing his own education, Pósa founded his first weekend camp for mathematically gifted students in 1988; these camps still continue today. Meant to be independent from the regular mathematics education children receive in school, these camps focus on gaining a deeper understanding of concepts and ways of thinking studied in their math classrooms, as well as fostering creative and persistent thinking. This is achieved through what is called the Pósa Method, a type of guided discovery learning that utilizes a web of problem threads.

In addition to Győri and Juhász (2017), Katona and Szűcs (2017) decribe the Pósa Method that will be the source of the following paragraphs. The Pósa Method relies on a web of problem threads; this is how students experience the guided discovery of mathematics. A problem thread is a series of problems that are connected to each other in some way. These problems can be connected in different ways. Sometimes, problems are connected because they build on each other; that is, the solution for one problem is based on or made easier by the solution to a previous. Pósa describes this as "building a staircase to a goal" (as cited in Stockton, 2010, p. 3). Problems can also be connected because they have a common feature, or kernel, such as a method of solving. Moreover, different threads are not completely disconnected from one another. Instead, they often run simultaneously and meet at common problems that use ideas from multiple threads, creating a web-like structure; from this comes the phrase web of problem threads.

Two goals of the Pósa Method are to foster student enjoyment of mathematics and to promote learning of mathematics through discovery. Furthermore, Győri and Juhász (2017) note the following key principles must be employed when problem threads are utilized in educational settings, so that students have the best guided discovery experience. First, it is customary that students are given more than one problem to work on at a time, each from a different problem thread. Providing options gives students autonomy and allows students to choose the topics or questions that they would prefer to work on first. It also makes the experience more enjoyable for the students. Another important aspect of the Pósa Method is allowing students time to think so that they are able to come to a solution on their own. Closely related, the way a teacher dialogues with a student is crucial. Teachers should let students think on their own without letting them remain stuck; hints that help but do not give away the solution are necessary for students to have the best experience.

The next principle of the Pósa Method deals with how students should interact in the classroom. Group work is an important part of the discovery process in this method; however, individual thinking is crucial for developing critical thinking skills. Thus, working on the presented problems should involve time for individual thinking followed by working in small groups if students have run into similar difficulties. There is one rule with group work: if a student has solved a problem, they cannot help other students. Ultimately, the goal of group work in the Pósa Method is to allow students to be inspired by one another's thinking, problem solving, and brainstorming, while not taking away the opportunity for every student to discover a solution for themselves.

The final few principles involve the students' work and role in the classroom. Students must be allowed to pose questions, and teachers must help students learn to ask good, interesting questions. Often, students' questions are even integrated into the curriculum for future use. Additionally, students should be allowed the freedom to make mistakes, thereby removing the fear of being shamed for an incorrect answer, since this fear hinders the process of exploration and discovery. Finally, students should be seen as the instructors' colleagues, fostering good relationships and giving them a comfortable environment in which they can enjoy the process of discovery.

The Hungarian Education System

As described above, the Pósa Method was originally created for use in weekend math camps, not for traditional secondary school classrooms. In some ways, the Pósa Method must be adapted in this context to better fit the structure and process of Hungarian education and curriculum. A few relevant aspects of Hungarian education are discussed below, and the research in this study investigates what the Pósa Method and the design and implementation of problem threads look like in this context.

The Hungarian mathematics curriculum is guided by the set of general competencies in the National Core Curriculum, which is compulsory for all schools in the country. The requirements stated in the National Core Curriculum are then further broken down and specified by the local curriculum and then by each school's Pedagogical Programme (The Government of Hungary, 2011). Based on the requirements laid out by the school's Pedagogical Programme, teachers create a schedule determining how exactly the required curriculum is covered throughout secondary school (Stockton, 2010). Teachers in Hungary are given some flexibility in this process because they will stay with one group of students as they progress through secondary school. For students, this means that they will usually have one mathematics teacher for the entirety of their secondary school career, giving the teacher and students the opportunity to form relationships and giving the teacher all four years of secondary school to cover the required curriculum (Stockton, 2010).

Furthermore, this structure means that students are not enrolled in separate math subjects each year, as is often the case in other countries. Instead, Hungarian secondary school students are simply enrolled in 'Mathematics' (Stockton, 2010), applying a cyclic nature of learning mathematics, as topics are revisited each year, adding more content and deepening students' understanding each time (Gosztonyi, 2015). Additionally, using this structure, students can more easily see the relationships between different aspects of mathematics as they learn them in tandem, rather than as disparate subjects (Stockton, 2010). In most schools, the ultimate goal of a student's mathematical education is for them to pass the secondary school leaving examination, which serves as both a secondary school exit exam as well as a college entrance exam (Ministry of Human Capacities, 2011).

Methodology

Applying the Pósa Method in public secondary schools means that those classes must cover the material given by the National Core Curriculum while aligning with the aims of the Local Curriculum and the school's Pedagogical Programme, as well as prepare students to perform well on the leaving examination. These must be done while also fulfilling the goals of the Pósa Method to foster student enjoyment of mathematics and promote learning of mathematics through discovery. Since the Pósa Method depends primarily on a web of problem threads, the focus of the study begins with the design of these threads, followed by their implementation in classrooms.

Research Questions

This study revolves around two aspects. First, the background and the designing aspect of problem threads are investigated. Questions under this topic include: (1.1) What are the main characteristics of the Pósa Method in two Hungarian public secondary mathematics classes? (1.2) What is the process of designing problem threads for two Hungarian public secondary mathematics classes? Second, the classroom experience and the implementation of problem threads are addressed in greater detail. Research questions included here are: (2.1) What does a class look like when using the Pósa Method in two Hungarian public secondary mathematics classes? (2.2) How are problem threads implemented in two Hungarian public secondary mathematics classes? (2.3) What are the students' experiences and opinions in regards to using the Pósa Method in two Hungarian public secondary mathematics classes?

Procedures

As all the research questions are of exploratory nature, the study uses qualitative data collection which involves verbal and verbalized data. When analyzing the data, the 'insider' perspective makes the study more grounded, discovery-oriented, exploratory, expansionist, descriptive, and inductive (Larsen-Freeman & Long, 1991). The rich data is ungeneralizable in qualitative research because of the special characteristics of single case studies, but the data could possibly be transferable to similar contexts. This data-driven study perceives no preconceptions since the research questions are open, using question words as How? What? The data collection for this study comes from observations, interviews, and questionnaires, and the analysis of this data primarily looks for patterns by analyzing the content and relating the content to the context (Seliger & Shohamy, 1989). In order to ensure quality control, triangulation is applied through collecting data from interviews with teachers, observations of their classes, and questionnaires for their students (Dörnyei, 2007).

During November 2019, three visits were made to two different schools: one elite school and one vocational school, as well as a math camp, all using the Pósa Method in Budapest, Hungary. The math camp was observed by two English-speaking researchers and one Hungarian translator, one classroom was observed by four English-speaking researchers and a Hungarian translator, and the second classroom was observed by two English-speaking researchers. In this last case, a translator was unnecessary because the class was instructed in English. In addition to the observations, an interview with each of the teachers of the observed classes and a student questionnaire for each of the classrooms were completed. The interview questions focused on interviewees' perspectives and relations to the Pósa Method; their definitions, goals, and perceived values of using problem threads; the design and implementation of problem threads; the structure of their lesson instruction; advantages and disadvantages of using the Pósa Method in public education. Meanwhile, the questionnaire focused on students' perspectives of their current mathematics lessons which uses the Pósa Method.

Participants

As mentioned earlier the Content Pedagogy Research Program of the Hungarian Academy of Sciences supports the experiment of implementing the Pósa Method in three groups of two secondary public schools in Budapest. One interviewee is the mathematics teacher who adapts the problem threads and implements the Pósa Method in the elite public secondary school, who had been the student of Pósa and currently teaches in the math camps. The other interviewee had been the student of the former, whose current task is to further adapt and implement the problem threads for the vocational group. The questionnaires were filled in by the students of the two interviewees in the experiment. The observations were made to their classes. This report focuses only on the public secondary mathematics classes which use the Pósa Method instead of the Pósa math camp itself.

It should be noted that the interviews were conducted in English, which is not the native language of the interviewees; however, the interviewees possess sufficient

English skills for the interviews to be completed successfully. In addition, the questionnaire, which was created in English, was translated to the native language of the students, Hungarian, by the researchers. Students completed the questionnaires in Hungarian, and their answers were translated back to English for the purpose of analyzing the data. Although most of the questions were completed on a 5 point Likert Scale, readers should note that small language nuances might still be present during the translation stage of the questions and answers themselves. Still, researchers believe that the nuances do not strongly affect the analysis of the results.

Data analysis was informed by two processes. First, answers were compared from the two teachers who were interviewed. Second, we looked at the connections between the interview answers, the observed lessons and the students' responses in the questionnaire. More specifically, we examined the similarities and differences in the two teachers' comments about the Pósa Method and whether their expectations of utilizing the Pósa Method in their lessons were met by their students.

Results and Discussion

Due to the qualitative nature of data collection, the study relied on emerging themes. Thus, the results and discussion are presented in the following structure. First, the teachers' point of view on designing and then implementing problem threads are presented, comparing the intended implementations with the observations and the students' answers. Second, two main goals that emerged, students' motivation and success, are analyzed and compared from all the data sources: literature, interviews, observations, and questionnaires.

Designing Problem Threads

During the interviews, the two teachers were asked to describe the main characteristics of the Pósa Method and the problem threads. Their answers mirrored what was found in the literature review. The characteristics of the Pósa Method that were emphasized the most were the principles of students taking joy in thinking about math and students posing good questions. When describing the problem threads, both teachers stated that the problems within a thread have a strong connection to each other, the connection not necessarily being the content of the problem, but sometimes the strategy for solving the problem or the way of thinking that is required to solve the problem. The teachers also noted that the problem threads continue throughout many years, with as many as tens to hundreds of problems belonging to a thread.

The interviewees were then asked about the process of designing the threads. The first step in creating a thread is to determine the end goal of what students should learn or what mathematical thinking skills they should acquire. The next step is to break up the end goal into smaller steps, and then decide on specific problems that correspond to those steps. The last step in the design process is to revise the thread as needed. There exist gaps in difficulty between the problems within a thread. If the gap is too small from one problem to the next, students will not be challenged. In contrast, if the gap from one problem to the next is too big, students will find the material too challenging. Thus, if the teachers notice an issue with this, they will revise the thread accordingly. The problems of a thread should generally be challenging but solvable. As far as the order of the problems in a thread goes, the teachers stated that the problems usually increase in difficulty, however, there are times when the teacher will introduce a challenge problem in the beginning of the thread that students do not have the tools to solve just yet, and the following problems lead up to the challenge problem. Some problems within a thread have a connection to the real world, while others are more abstract. In order to make these abstract problems interesting for students, the teachers will emphasize to the students that mathematics is abstract in its true essence, and that it is important for them to learn. The teachers will also occasionally create a context or narrative for abstract problems or relate them to the history of mathematics or to a famous mathematician, though these connections should come naturally and not be forced.

According to the interviewees the main difference between the groups applying the Pósa Method is the level of difficulty of the problems. For talented students they should be more challenging and the gaps are much bigger, while for public school students the gaps are smaller, and students need more reinforcement. In the case of the vocational school, the gaps are even smaller, and the end of a problem thread (for example, the level of abstraction) varies between students greatly. These differences between students in the vocational school's group resulted in a main set of problems that all students go through, and an extra set of more difficult problems used for differentiation.

Implementing Problem Threads in the Classroom

Both contexts follow the same implementation process. A typical lesson that is taught using problem threads starts with a discussion of homework problems, which will be described further shortly. Afterwards, students work on ‘open’ problems, or problems whose solutions have not yet been discussed together as a class. The teachers shared that there are about 3 to 5 open problems during one 45-minute class period. In both lessons that were observed, which were double lessons lasting 90 minutes, there were 5 open problems. In the questionnaire, students were asked how they felt about the amount of work their teacher assigns in class. The results are displayed in Table 1 of Appendix A. As the table portrays, 21 out of the 31 surveyed students (68%) believe their teacher assigns "just enough" work during class. After students work on open problems, if time allows, there is a class discussion about the solutions of the open problems. Consistent to what was mentioned in the literature review, the teachers stated that they try to give students as much time as is needed for them to arrive at the solution of a problem. In the observed lessons, the students were given 45 to 50 minutes to think about the problems, either individually or in pairs or groups. In the questionnaire, students were asked to rate how much they agree or disagree with the statement "I have enough time to work on the problems in class." There were 24 students (77%), who expressed that they "agreed" or "strongly agreed" with the statement (Table 2, Appendix A). For the lessons that were observed, there remained roughly half the duration of the lesson for a class discussion, however, there are instances when no time remains for a class discussion, as students have not yet solved the open problems. At the conclusion of the lesson, students are assigned the problems that are still open as homework (usually about 1 to 3 problems).

As discussed in the literature review, one of the main characteristics of the Pósa Method and using problem threads to teach is the discovery aspect. In the interviews, the teachers stated that students are not given theorems or formulas for mathematical concepts. Instead, through the problems in the problem thread, the students are driven to discover the theorem or formula on their own. Students were asked in the questionnaire to what extent they agreed with the statement "I enjoy figuring out solutions by myself." Roughly 80% of the surveyed students "agreed" or "strongly agreed" with the statement (Table 3, Appendix A). This result directly connects to how much students like the discovery aspect of learning math through problem threads. Another question asked students to rate their agreement with the statement "I like being told how to solve a problem." In contrast to the results in Table 3, although the majority of students expressed that they enjoyed figuring out solutions on their own, only 5 out of the 31 students (16%) "disagreed" or "strongly disagreed" with the statement "I like being told how to solve a problem" (Table 4, Appendix A). Another aspect of discovery that the teachers noted is the fact that students are not explicitly told which problems belong to the same thread, though many are able to deduce these connections on their own. One student stated that "I can find connections between different topics, which makes me understand the material easier," and five other students explicitly stated the same idea.

Another characteristic of the Pósa Method and the problem threads, also mentioned in the literature review, is the autonomy that students have in regards to the problems. Teachers articulated that students can choose the order in which to solve the open problems, as well as the method of solving the problems. In the questionnaire, students were asked to identify their accordance with the statement "I enjoy being able to choose which problems to work on in class" for which 27 out of the 31 students (87%) "agreed" or "strongly agreed" (Table 5, Appendix A). Students were also asked about their strategies for choosing which problems to solve during the lessons. Students were given a list of strategies and asked to mark all that applied (Table 6, Appendix A). The two most popular strategies that students employ in their selection of problems are working on problems that they find interesting first and working on problems that they think will be easy first. When asked "How do you feel about working on problems from different topics at the same time?" only 2 students worded concerns that they would like to practice the same method of solving a problem more, 5 students were neutral about it, and 24 students (77%) especially liked it. Their reasons are that it makes the lessons more varied and interesting, they can always find a problem to work on, and they can realize connections between different topics better.

The final characteristic of the Pósa Method and the problem threads that will be discussed is the development of students' abilities to pose good questions. During the lessons, it was observed that a common theme in the problem threads is a problem or several problems dealing with specific parameters, followed by a problem that generalizes for *n*. In the interviews, teachers expressed that they do not design all of the problems in a problem thread to have a generalizable solution; however, they always intend for students to ask the question of whether a problem has a generalizable solution or not. Some of these questions happen to be open problems in the field of mathematics, which allows students the opportunity to think like mathematicians. In the observed lessons, some students automatically started generalizing problems while others were satisfied with their solutions, so it became a tool for differentiation.

Problem Threads and Student Motivation

Both interviewees said a major driving factor behind the decision to use the Pósa Method to teach in their classroom was to boost student motivation and interest in learning the material. They felt that successful teaching could develop students' abilities to think creatively and critically about mathematics. Ideally, students should end the course of study having a positive attitude towards math, regardless of whether they plan to pursue it beyond secondary school. Responses to the student questionnaire indicate that this aim is being achieved. Over 90% of students surveyed indicate that they "agree" or "strongly agree" with the statement "I enjoy learning math through the Pósa Method". Furthermore, 87% of survey respondents "agreed" or "strongly agreed" with the statement "Learning math through the Pósa Method improves my thinking skills". No students indicated that they "strongly disagreed" with either statement.

Teachers also reported that they wanted their students to acquire the skills necessary to think about interesting and challenging questions. Both teachers reported that their students were able to take an existing question and extend it to a more challenging or abstract version. In doing so, the students were observed building their mathematical reasoning skills. During classroom observations, students were seen extending ideas from previous problems and finding new ways to engage with the material. Additionally, teachers valued student input in solving all problems and seemed to view students as equal partners in the problem solving process. Generally, it seems students agree that the problems being posed in class are interesting. Particularly, this interest has improved over time. 25% of the students (8 respondents) reported that they "disagreed" or "strongly disagreed" with the statement "In primary school, I thought the problems and topics we would learn in secondary school would be interesting". Only one student disagreed with the statement "Now, I find the problems we learn interesting" and over 77% of respondents "agreed" or "strongly agreed". One student stated that they appreciate the Pósa Method because it "makes it possible to look at a problem from different viewpoints".

Problem Threads and Student Success

One concern that has been raised in relation to using the Pósa Method to teach in public secondary schools is ensuring that all of the required secondary school material gets covered over four years. Since the current project is only in its third year, the teachers were not entirely sure how this critique would play out, but both stated that they were not concerned about this being an issue. Additionally, both teachers felt that teaching through problem threads would better prepare students for the leaving examination than the traditional methods. First, a traditional Hungarian classroom would cover a particular topic for four to six weeks. In contrast, since problem threads extend over longer time frames, students using the Pósa Method can study a particular topic for months on end which may help to build deeper understanding of topics. Second, because topics from multiple threads are posed simultaneously, students are able to gain confidence with identifying what strategy is needed to solve a given problem. During classroom observations, these predictions appeared to be correct. Students appeared to know how to solve the posed questions even when they came from different content areas.

From the questionnaire, 80% of the students responded that they "agreed" or "strongly agreed" with the statement "Learning through the Pósa Method will prepare me well for the leaving examination." In the open-ended survey question, one student stated "I think [the Pósa Method is] good because we can understand connections better, and we will have a greater insight overall." Still 13% of respondents "disagreed" or "strongly disagreed" with the statement "Learning through the Pósa Method will prepare me well for the leaving examination." When using problem threads, there is less time to practice rote processes and memorize mathematical ideas. One student said, "The problem is that when I can solve a problem, then comes a new one, and as I didn't have the chance to practice the previous, I forget it." Another student stated "I'm a little confused because I don't know which methods I should use in which problems." The teachers stated that this may be a possible detriment when students are asked to write the leaving examination.

A further goal of the interviewed math teachers is to prepare their students to have reasoning skills to use beyond the classroom. They want their students to be able to think logically and see how math can be utilized in daily life. When possible, they integrate real-world problems into their classrooms with the hope that students make the connection between the material being learned and the types of questions they might face in the future. Both concrete and abstract problems were used during the observed lessons. Of all the stated goals of the teachers, this is one of the least recognized by the students. Only 65% of the students "agreed" or "strongly agreed" with the statement "I can use the problem solving skills I learned from math to help in my everyday life." How to reconcile this misalignment is a potential place for further research.

Conclusion and Further Research

This study investigates how problem threads in the Pósa Method are used in Hungarian public secondary mathematics classrooms. Analysis is drawn from a review of the current literature on the Pósa Method and problem threads, interviews with two teachers who are implementing the Pósa Method and problem threads in their mainstream secondary school classrooms, observations of the two teachers' lessons, and a questionnaire completed by the students of these teachers. Most of the information found in the literature review regarding the design, implementation, and goals of the Pósa Method and problem threads within the desired context was consistent with the data obtained from the interviews, observations, and responses to the questionnaire. The two teachers use the Pósa Method and problem threads in order to challenge students with interesting and enjoyable problems, help them discover mathematical concepts for themselves, pose meaningful questions, and develop their thinking and problem solving skills. These goals have been confirmed by 77-91% of their students. The novelty of the findings is that students do not mind learning through problem threads and working on problems from different topic areas at the same time; moreover, they find it more interesting and enjoy discovering connections and different viewpoints. These results provide better, more holistic understanding of the use of problem threads in Hungarian public secondary mathematics classrooms.

An area for further investigation will be the effects that the Pósa Method and problem threads have on student achievement on the leaving examination. The outcome of the exams will inform teachers whether or not the method is an effective teaching practice in regards to student achievement on standardized tests. After the examinations and detailed analysis of the four-year long experiment, further, possibly larger scale experiments with control groups can be designed.

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Appendix A

Response	Too little work	Little	Just enough	A lot	Too much
Number of Students	1	3	21	6	0
Percentage	3.2%	9.7%	67.7%	19.4%	0%

Table 1. Responses to "How do you feel about the amount of work your teacher assigns in class?"

Response	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Number of Students	1	1	5	13	11
Percentage	3.2%	3.2%	16.1%	41.9%	35.5%

Table 2. Responses to "I have enough time to work on the problems in class."

Response	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Number of Students	0	2	4	7	18
Percentage	0%	6.5%	12.9%	22.6%	58.1%

Table 3. Responses to "I enjoy figuring out solutions by myself."

Response	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Number of Students	1	4	14	8	4
Percentage	3.2%	12.9%	45.2%	25.8%	12.9%

Table 4. Responses to "I like being told how to solve a problem."

Response	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Number of Students	0	0	4	4	23
Percentage	0%	0%	12.9%	12.9%	74.2%

Table 5. Responses to "I enjoy being able to choose which problems to work on in class."

Response	*Number of Students*	*Percentage of Whole*
I work through all of the problems in order.	4	12.9%
I try to work on all of the problems, in any order.	12	38.7%
I work on the problems I find interesting first.	21	67.7%
I work on the problems I think will be easy first.	22	71.0%
I skip problems that I don't think are interesting.	8	25.8%
I skip problems that I don't know how to do.	13	41.9%
I only work on problems I find boring if I have extra time.	14	45.2%
I only work on difficult problems if I have extra time.	12	38.7%
I ask a classmate for help when I'm stuck.	14	45.2%
I ask the teacher for help when I'm stuck.	12	38.7%

Table 6. Responses to "What is your strategy for choosing which problems to solve in class?"

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