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SCIENTIFIC PAPER

# NOTE ON SIMPLE AND LOGARITHMIC RETURN

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**Abstract:** *In this paper we describe and clarify the definitions and the usage of the simple and logarithmic returns for financial assets like stocks or portfolios. It can be proven that the distributions of the simple and logarithmic returns are really close to each other. Because of this fact we investigate the question whether the calculated financial risk depends on the use of simple or log returns. To show the effect of the return-type on the calculations, we consider and compare the riskiness order of stocks and portfolios. For our purposes, in the empirical study we use seven Hungarian daily stock prices and for the risk calculation we focus on the following risk measures: standard deviation, semivariance, Value at Risk and Expected Shortfall. The results clearly show that the riskiness order can depend on the use of the return type (i.e. log or simple return). Generally, often – due to missing data or the nature of the analysis – one has to use approximations. We also examine the effect of these approximations on the riskiness order of stocks and of portfolios. We found differences in the riskiness order using exact or approximated values. Therefore, we believe, if this is possible, exact values instead of approximated ones should be used for calculations. Additionally, it is important that one uses the same type of return within one study and one has to be aware of the possible instabilities when comparing return results.*

**Keywords:** *simple return, logarithmic return, riskiness order, stock, portfolio (JEL. Code:* C18)

## INTRODUCTION

In the financial area of today an important question is: how one defines and measures the risk of financial assets such as stocks and portfolios. Furthermore, it is not enough only to measure risks they also need to be compared to help us to take decisions on different financial questions. Because of these comparability reasons one uses instead of the prices the returns of an asset.

In this paper we will be dealing with the simple and the logarithmic returns. It is self-evident and natural that these two returns are different from each other. For example Hudson considers this relationship by comparing means and concludes that the mean of the logarithmic returns is less than the mean of the simple return (computed on the same set of returns) (Hudson, 2010). We will discuss the possible correlations and differences between the two returns from an other point of view. For our purposes it is important to understand how the choice of the return-type effects the riskiness order of the considered set of assets. For example, do we consider a stock respectively a portfolio equally risky (compare to the others) using simple or logarithmic returns to calculate the risk. To answer this question we will do an empirical study.

The objective of this paper is to describe and to clarify the definitions and the usage of the simple and logarithmic returns. In the first part of the theoretical background we will state the definitions of the one- and multi-period simple and log return and we will describe the relationship between them. These definitions will be extended to portfolios in the second part of the theoretical background. The second part of the study is the empirical part. First, we would like to confirm in practice – via using Hungarian stock data – mathematical formulas, equations and results presented in the theoretical part. Second, we will answer to our main question, i.e. whether using simple or logarithmic return could have an effect on our decision.

## RETURNS, THE THEORETICAL BACKGROUND

In this theoretical part of the paper we summarize the important definitions, expressions connected with simple and logarithmic returns and we clarify and establish relations between the two notions. Definitions of the following chapter are based on Tsay (Tsay, 2005) and Calafiore (Calafiore, 2014).

### *Asset Return*

First, we will define the simple and logarithmic return of an asset. In addition we will show the most important equations and expressions connected with the topic.

### *a. Simple Return*

The Oxford dictionary defines the return as a profit on an investment over a period of time, expressed as a proportion of the original investment. In the next paragraphs we express returns in a more mathematical framework.
In the case of asset returns let us consider a time horizon $[0,T]$ . Furthermore $P_0$ be the price of an asset at time $0$ and $P_T$ the price of an asset at time $T$. If there is no cash flow in this $[0,T]$ time interval, we speak of the *one-period simple net return* and we introduce the notation $R^S_{[0,T]}[1]$. So the one-period simple net return of an asset can be defined by

$$R^S_{[0,T]}[1] := \frac{P_T - P_0}{P_0} = \frac{P_T}{P_0} - 1. \qquad (1)$$

The corresponding *one-period simple gross return* of an asset is given in terms of the simple return:

$$GrR^S_{[0,T]}[1] := 1 + R^S_{[0,T]}[1] = \frac{P_T}{P_0}. \qquad (2)$$

Later on if we speak of the simple return we think of the one-period simple net return.
For the multi-period case, let us divide the interval $[0,T]$ into $n$ pair wise disjoint subintervals: let $t_0 := 0$ be the $0^{th}$ time point, $t_n := T$ be the last time point and let $t_i$ be the time points in-between, such that $t_{i-1} < t_i$, $i = 1,\ldots,n$. According to our definition we can calculate on these subintervals the one-period simple gross return:

$$GrR^S_{[t_{i-1},t_i]}[1] = 1 + R^S_{[t_{i-1},t_i]}[1] = \frac{P_{t_i}}{P_{t_{i-1}}}$$ and thus the "return" on the whole $[0,T]$ interval must be the product of the gross returns of the subintervals. This return is called the *n-period simple gross return*:

$$\prod_{i=1}^{n} GrR^S_{[t_{i-1},t_i]}[1] = \prod_{i=1}^{n}\left(1 + R^S_{[t_{i-1},t_i]}[1]\right) = \frac{P_{t_1}}{P_{t_0}}\frac{P_{t_2}}{P_{t_1}},\ldots,\frac{P_{t_n}}{P_{t_{n-1}}} = \frac{P_{t_n}}{P_{t_0}} =: GrR^S_{[0,T]}[n]. \qquad (3)$$

We would like to add that since $t_0 = 0$ and $t_n = T$ the n-period simple gross return equals the one-period simple gross return:

$$GrR^S_{[0,T]}[n] = \frac{P_{t_n}}{P_{t_0}} = \frac{P_T}{P_0} = GrR^S_{[0,T]}[1]. \qquad (4)$$

Analogously to the one-period case, we define the *n-period simple net return* by using the n-period gross return and subtracting one:

$$\mathrm{R}^{\mathrm{S}}_{[0,\mathrm{T}]}[\mathrm{n}] := \frac{\mathrm{P}_{t_n}}{\mathrm{P}_{t_0}} - 1. \qquad (5)$$

With Equation ( 3 ) and ( 5 ) at hand one can rewrite the n-period simple gross returns by:

$$GrR^S_{[0,T]}[n] = \frac{P_{t_n}}{P_{t_0}} = 1 + R^S_{[0,T]}[n] = \prod_{i=1}^{n}\frac{P_{t_i}}{P_{t_{i-1}}}. \qquad (6)$$

### *b. Logarithmic Return/Continuously Compounded Return*

To understand the logarithmic return, simply the log return, let us divide the interval $[0,T]$ into $n$ equidistant intervals. In this paragraph we use the same notation as it was introduced for the multi-period simple return. Assume now, that on every $[t_{i-1},t_i]$ subinterval the return $R$ is the same, moreover that it is the $n$ th part of some one-period return on $[0,T]$, denoted by $R^*_{[0,T]}[1]$, i.e. $R := \frac{R^*_{[0,T]}[1]}{n}$. In this case, Equation ( 3 ) can be written as follows:

$$GrR^S_{[0,T]}[n] = \prod_{i=1}^{n}\left(1 + R^S_{[t_{i-1},t_i]}[1]\right) = (1+R)^n = \left(1 + \frac{R^*_{[0,T]}[1]}{n}\right)^n \qquad (7)$$

Since $t_0$ and $t_n$ are the 0th and the last time points respectively, Equation ( 7 ) can be written as follows:

$$GrR^S_{[0,T]}[n] = \frac{1}{\mathrm{P}_0}\prod_{i=1}^{n-1}\frac{P_{t_i}}{P_{t_i}}\mathrm{P}_{\mathrm{T}} = \frac{\mathrm{P}_{\mathrm{T}}}{\mathrm{P}_0}, \qquad (8)$$

and therefore using Equation ( 7 ) and Equation ( 8 ) it holds that

$$\frac{P_T}{P_0} = \left(1 + \frac{R^*_{[0,T]}[1]}{n}\right)^n. \qquad (9)$$

Let us make the length of the $[t_{i-1},t_i]$ subintervals smaller and smaller. This means that the number of equidistant subintervals of $[0,T]$ must grow, $n \to \infty$ . Hence we have to compute limits:

$$\lim_{n\to\infty}\frac{P_T}{P_0} = \lim_{n\to\infty}\left(1 + \frac{R^*_{[0,T]}[1]}{n}\right)^n, \qquad (10)$$

and therefore it follows by the definition of the exponential function that

$$\frac{P_T}{P_0} = e^{R^*_{[0,T]}[1]}. \qquad (11)$$

Since we are interested in returns, we apply the logarithm:

$$\ln\left(\frac{P_T}{P_0}\right) = R^*_{[0,T]}[1]. \quad (12)$$

The return in Equation ( 12 ) is called the one-period logarithmic return of an asset. So, we define the *one-period log return* as the logarithm of the one-period simple gross return and we use the notation $R^L_{[0,T]}[1]$:

$$R^L_{[0,T]}[1] := \ln\left(\frac{P_T}{P_0}\right) = \ln\left(1 + R^S_{[0,T]}[1]\right). \quad (13)$$

Similarly to the simple return's case, one defines the *n-period logarithmic return*:

$$R^L_{[0,T]}[n] := \ln\left(\frac{P_{t_1}}{P_{t_0}}\frac{P_{t_2}}{P_{t_1}},\ldots,\frac{P_{t_n}}{P_{t_{n-1}}}\right) = \sum_{i=1}^{n}\ln\left(\frac{P_{t_i}}{P_{i_{t-1}}}\right) = \sum_{i=1}^{n}. \quad (14)$$

We can see that in this case the n-period log return is the sum of the $n$ one-period log returns. And this is one of the reasons why one uses the log return rather than the simple return: adding numbers close to zero is not a problem, but multiplying numbers close to zero can cause arithmetic overflow. In addition it is easier to derive the time series properties of sums than of products (Daníelsson, 2011). Analogously to the simple return's case since $t_0$ and $t_n$ are the 0th and the last time points respectively:

$$R^L_{[0,T]}[1] = R^L_{[0,T]}[n]. \quad (15)$$

We would like to add, that more generally on every interval one can calculate the return. In this study we will use daily asset prices and thus daily returns. So, the considered time interval will always be one day. Therefore, the one-period simple and logarithmic return can be written as follows:

$$R^S_t := R^S_{[t-1,t]}[1] = \frac{P_t}{P_{t-1}} - 1 \quad (16)$$

and

$$R^L_t := R^L_{[t-1,t]}[1] := \ln\left(\frac{P_t}{P_{t-1}}\right). \quad (17)$$

Later in this study we will use Equation ( 16 ) and Equation ( 17 ) for the calculations and we will speak of the return at time point $t$. Note that one can easily see the relation between the simple and log return:

$$R^S_t = e^{R^L_t} - 1 \quad (18)$$

and

$$R^L_t = \ln\left(1 + R^S_t\right). \quad (19)$$

It can be deduced – using an approximation of the logarithm that $\ln(1+x) \approx x$, if $x$ is near to zero – that if the simple return is near to zero it is in addition very comparable to the log return (proof follows just by substitution of $x$ with the simple return):

$$R^L_{[t_{i-1},t_i]}[1] = \ln\left(\frac{P_{t_i}}{P_{t_{i-1}}}\right) \approx \frac{P_{t_i}}{P_{t_{i-1}}} - 1 = R^S_{[t_{i-1},t_i]}[1], \text{ if } R^S_{[t_{i-1},t_i]}[1] \approx 0. \quad (20)$$

## *Portfolio Return*

In this section we will focus on how to calculate the simple and the logarithmic return of a portfolio. We use the following notation:

$n$ : the number of assets in the portfolio
$i$ : refers to the assets in the portfolio, $i = 1,\ldots,n$
$S_t$: the amount of money invested in the portfolio at time $t$
$S_{t,i}$: the amount of money invested in asset $i$ at time $t$
$P_{t,i}$: the price of asset $i$ at time $t$
$w_{t,i}$: relative weights of the asset $i$ in portfolio at time $t$
$k_i$: number of asset $i$ in portfolio

Let us consider a portfolio which consists of n assets. Using the notation above it is natural that the amount of money invested in asset $i$ at time $t$ can be expressed by

$$S_{t,i} = k_i P_{t,i} = w_{t,i} S_t, \quad (21)$$

and the amount of money invested in the portfolio at time $t$ is given by

$$S_t = \sum_{i=1}^{n} S_{t,i} = \sum_{i=1}^{n} k_i P_{t,i}. \quad (22)$$

With equation ( 21 ) and ( 22 ) in hand we can express the relative weights at time $t$ :

$$w_{t,i} = \frac{S_{t,i}}{S_t} = \frac{k_i P_{t,i}}{\sum_{i=1}^{n} k_i P_{t,i}}. \quad (23)$$

These relative weights change in time according to the asset prices. In this study later on, if we speak of weights we always think of these relative weights. Note, that the relative weights sum up to one:

$$\sum_{i=1}^{n} w_{t,i} = 1. \quad (24)$$

## *a. Simple Return of a Portfolio*

In this section we will show how to calculate the simple return of a portfolio (denoted by $R^S_t$). Similarly to the simple return of an asset we can define the *simple return of a portfolio* at time $t$ the gain (or loss) in value of the portfolio relative to the starting value, mathematically (Bacon, 2011):

$$R^S_t := \frac{S_t}{S_{t-1}} - 1. \quad (25)$$

Using the fact that the weights sum up to one and the equation $R_{t,i}^S = \frac{P_{t,i}}{P_{t-1,i}} - 1$, where $R_{t,i}^S$ is the simple return of asset $i$ at time $t$, Equation ( 22 ) can be rewritten as

$$S_t = \sum_{i=1}^{n} k_i P_{t,i} = \sum_{i=1}^{n} k_i P_{t-1,i}\left(1+R_{t,i}^S\right) = \sum_{i=1}^{n} S_{t-1,i}\left(1+R_{t,i}^S\right) = \sum_{i=1}^{n} S_{t-1} w_{t-1,i}\left(1+R_{t,i}^S\right) \quad (26)$$

and thus we can express the simple return of a portfolio at time $t$ by

$$R_t^S = \frac{S_t}{S_{t-1}} - 1 = \sum_{i=1}^{n} w_{t-1,i} R_{t,i}^S. \quad (27)$$

We can see that the portfolio simple return is the sum of the weighted simple returns of the constituents of the considered portfolio.

### *b. Logarithmic Return/Continuously Compounded Return of a Portfolio*

The *logarithmic return of a portfolio* (denoted by $R_t^L$) at time $t$ can be defined analogously to the logarithmic return of an asset:

$$R_t^L := \ln\left(\frac{S_t}{S_{t-1}}\right). \quad (28)$$

Moreover using the relation between logarithmic and simple return (see Equation ( 13 ) and Equation ( 27 ) the logarithmic return of a portfolio can be calculated in the following way:

$$R_t^L = \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln\left(1+\sum_{i=1}^{n} w_{t-1,i} R_{t,i}^S\right) = \ln\left(\sum_{i=1}^{n} w_{t-1,i} e^{R_{t,i}^L}\right) \quad (29)$$

where $R_{t,i}^L$ is the log return of asset $i$ at time $t$.
Unfortunately the log return of a portfolio does not have a similar convenient property as it was developed in Equation ( 27 ) for the case of the simple return, so it cannot be written as the sum of the weighted log returns of the constituents of the considered portfolio. Similarly to the return of an asset – by using the $\ln(1+x) \approx x$ approximation – one can show, that if the simple returns of a portfolio are close to zero then the simple returns and the log returns of a portfolio are similar to each other:

$$R_t^L = \ln\left(1+R_t^S\right) \approx R_t^S. \quad (30)$$

Using the assumptions that the simple returns are close to zero, and the definition of the exponential function one can nevertheless deduce the following linear approximation:

$$R_t^L \approx R_t^S = \sum_{i=1}^{n}\left(\sum_{j=0}^{\infty}\frac{\left(R_{t,i}^L\right)^j}{j!} - 1\right) w_{t-1,i} \approx \sum_{i=1}^{n}\left(\left(1+R_{t,i}^L\right)-1\right) w_{t-1,i} = \sum_{i=1}^{n} R_{t,i}^L w_{t-1,i}. \quad (31)$$

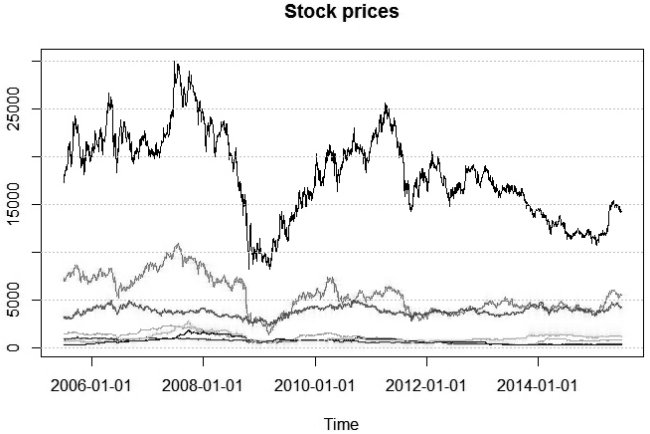
So in this case $$R_t^L \approx \sum_{i=1}^{n} R_{t,i}^L w_{t-1,i}. \quad (32)$$

## EMPIRICAL STUDIES

### *The data*

For the empirical calculation we will work with Hungarian daily stock prices between 01.07.2005 and 29.06.2015. The data was downloaded from the Budapest Stock Exchange homepage (www.bet.hu). We focus on seven stocks, namely FHB, MOL, MTELEKOM, OTP, Pannergy, Raba and Richter and analyze them in the mentioned time interval. The missing values were filled by the previous day data. To perform the analysis we use the mathematical software R.
We plot the stock prices in (Figure1), which shows that prices cannot be used for comparisons.

***Figure1: Stock prices (gray: FHB, black: MOL, red: MTELEKOM, green: OTP, purple: Pannergy, light blue: Raba, magenta: Richter)***




### *Comparing simple and logarithmic returns*

We could see in the asset and in the portfolio case that if the simple returns are close to zero then the simple and log returns are close to each other. In the first part of this empirical study we will check this theoretical fact in practice. The first price data is from 01. 07. 2005 and we consider them as price data at time $t = 1$. The last ones are from 29. 06. 2015 and we consider them as price data at time 2607. Note, that the "first" returns can be calculated on the interval $[t = 1, t = 2]$ and they are denoted by $R_{2,i}^S$ and $R_{2,i}^L$ respectively, for all the seven stocks ($i = 1, \ldots, n$).

***Table 1: Basic statistics of simple returns: minimum, first quartile, median, mean, third quartile, maximum***

| | FHB | MOL | MTELEKOM | OTP | Pannergy | Raba | Richter |
|---|---|---|---|---|---|---|---|
| **Min.** | -0,178975 | -0,149750 | -0,118151 | -0,149854 | -0,802770 | -0,149809 | -0,900179 |
| **1st qu.** | -0,011340 | -0,011768 | -0,009009 | -0,013054 | -0,007828 | -0,008961 | -0,009508 |
| **Median** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Mean** | 0,000040 | 0,000183 | -0,000170 | 0,000272 | -0,000118 | 0,000435 | -0,000062 |
| **3rd qu.** | 0,009742 | 0,011833 | 0,008635 | 0,013605 | 0,006466 | 0,008554 | 0,009654 |
| **Max.** | 0,232339 | 0,150583 | 0,123894 | 0,232639 | 0,149826 | 0,280193 | 0,094983 |

***Source: own calculation***

## *a. Stock returns*

First we calculated the daily simple and logarithmic returns of all the individual stocks using Equation ( 16 ) and Equation ( 17 ). In order to show the results more clear we introduced two outliers in the case of the Richter and Pannergy stocks (check the minimum values). The basic statistics are summarized in Table 1 and Table 2. From these summaries we can clearly see that in both cases the returns are close to zero: the medians are zero and the interquartile ranges are relatively small. Later on in this study we will use this modified data. Comparing Table 1 and Table 2 one can say that the distributions of the simple and logarithmic returns are really close to each other.

## *b. Portfolio returns*

Let us consider a portfolio: We assume that we own a portfolio consisting of one from all the seven stocks, i.e. $k_i = 1,\ i = 1,\ldots,7$ (see notation in the theoretical part). First we calculate the simple and the log returns of the portfolio using Equation ( 27 ) and Equation ( 28 ) respectively. The values are summarized in box plots, see (Figure 2). As we mentioned in the theoretical part, the log return and the simple return should be similar if the simple returns are close to zero (see Equation ( 30 )). In (Figure 2) we can clearly see that in the case of our data the simple and the log returns are close to zero except one outlier in both cases. This means that the simple and a log return values are very close to each other. This conclusion could be confirmed by taking a look at (Figure 3), where the simple return of the portfolio was plotted against the log return of the same portfolio. Except one outlier all the values are lying on the 45° line.

***Figure 2. Box plots of portfolio simple (left) and portfolio logarithmic return (right) values***

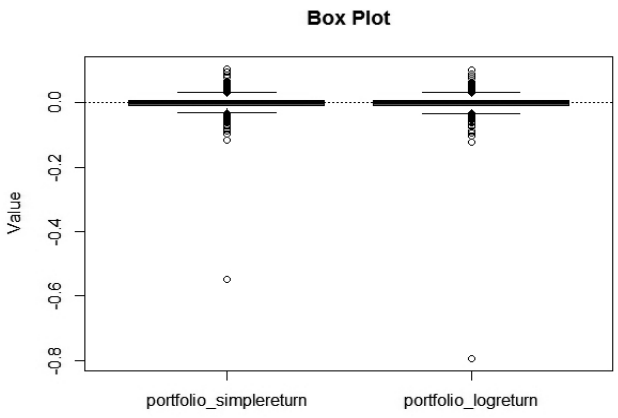
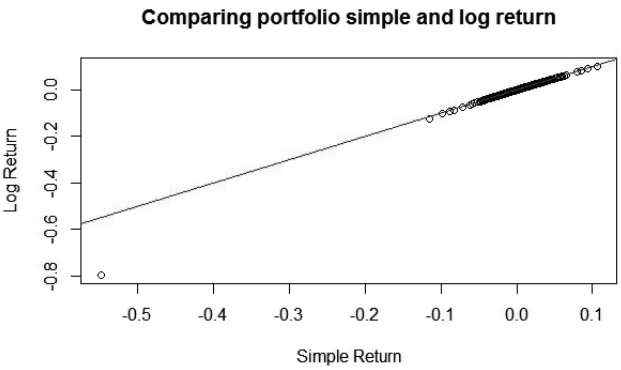



***Figure 3. Comparing portfolio simple and logarithmic returns***




## *Comparing riskiness order*

From the fact, that the distributions of the stock simple and logarithmic returns are really close to each other (see section '*Comparing simple and logarithmic returns*') we may conclude that it does not depend on whether we use simple or log returns for the financial calculations. We will check this assumption using different risk measures and using the ordering method described in the introduction.

We calculate four from the most often and widely used risk measures: the standard deviation, the semivariance, the Value at Risk and the Expected Shortfall. Detailed descriptions of this risk measures one can find for example Bugár (Bugár, 2006) and Embrechts (Embrechts, 2005).

In the next step we state how to calculate these risk measures in the case of a realization of a random variable.

Let $r = (r_1,\ldots,r_n)$, where $r_i$ is the i th return $(i = 1,\ldots,n)$ and $\overline{r}$ the average of these returns $(\overline{r} = \frac{1}{n}\sum_{i=1}^{n} r_i)$.

## *a. Standard Deviation*

$$\sigma(r) \approx \sqrt{\frac{\sum_{i=1}^{n}(r_i - \overline{r})^2}{n-1}} \qquad (33)$$

*Table 2: Basic statistics of logarithmic returns: minimum, first quartile, median, mean, third quartile, maximum*

| | FHB | MOL | MTELEKOM | OTP | Pannergy | Raba | Richter |
|---|---|---|---|---|---|---|---|
| **Min.** | -0,197201 | -0,162225 | -0,123734 | -0,162347 | -1,623385 | -0,162294 | -2,305382 |
| **1st qu.** | -0,011404 | -0,011837 | -0,009050 | -0,013140 | -0,007859 | -0,009001 | -0,009553 |
| **Median** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Mean** | -0,000245 | 0,000075 | -0,00314 | -0,000089 | -0,000620 | 0,000206 | -0,000767 |
| **3rd qu.** | 0,009695 | 0,011764 | 0,008597 | 0,013513 | 0,006445 | 0,008518 | 0,009608 |
| **Max.** | 0,208914 | 0,140269 | 0116799 | 0,209157 | 0,139610 | 0,247011 | 0,090739 |

***Source: own calculation***

### *b. Semivariance*

$$\mathrm{SV}(\mathrm{r}) \approx \frac{\sum_{i=1}^{n} \left(\min\left\{\mathrm{r_i} - \bar{\mathrm{r}}, 0\right\}\right)^2}{\mathrm{n}} \qquad (34)$$

### *c. Value at Risk - VaR (at $\alpha$ level)*

$$VaR_{\alpha}(r) \approx -\hat{F}^{\leftarrow}_{\{r_1,\ldots,r_n\}}(\alpha), \qquad (35)$$

where $\hat{\mathrm{F}}_{\{\mathrm{x}_1,\ldots,\mathrm{x}_n\}}(\mathrm{x}) := \frac{1}{\mathrm{n}}\sum_{i=1}^{n} 1_{\{\mathrm{x}_i \le \mathrm{x}\}}$ is the empirical distribution

function and $1_{\{\mathrm{x}_i \le \mathrm{x}\}}$ is the indicator function of the set $\{\mathrm{x}_i \le \mathrm{x}\}$.

### *d. Expected Shortfall – ES (at $\alpha$ level)*

$$ES_{\alpha}(r) \approx -\frac{\sum_{i=1}^{k} r_i^*}{k}, \qquad (36)$$

where $\mathrm{k} = \lceil \mathrm{n}\alpha \rceil = \max\{\mathrm{m} \mid \mathrm{m} < \mathrm{n}\alpha,\ \mathrm{m} \in \mathbb{N}\}$ and $r_i^*$ is the *i* th element in the increasing order of the returns $r_i$: $r_1^* \le r_2^* \le \ldots \le r_n^*$.

The only risk measure which satisfies the expected properties (monotonicity, subadditivity, positive homogeneity, cash invariance/translation invariance) is the Expected Shortfall. Further discussion on this topic for example (Acerbi, 2002) and (Artzner, 1999).

### *a. Stocks*

First we consider the stock returns and we calculate the standard deviation, the semivariance, the VaR and the ES values. They are shown in (Figure 4). The purple bars indicate the values calculated using simple returns and the blue bars indicate the values calculated using log returns. We can see that in the case of the semivariance and VaR (at both $\alpha = 0,05$ and $\alpha = 0,01$ levels) the order does not depend on the type of return. If we use the semivariance as a risk measure the riskiest stock is the Richter, followed by Pannergy, OTP, FHB, MOL, Raba and MTELEKOM. In the case of VaR, the riskiest stock is the OTP, followed by the stocks MOL, FHB, Richter, Raba, Pannergy and MTELEKOM. At $\alpha = 0,01$ level the order is the following: OTP, FHB, MOL, Raba, Pannergy, MTELEKOM, Richter.

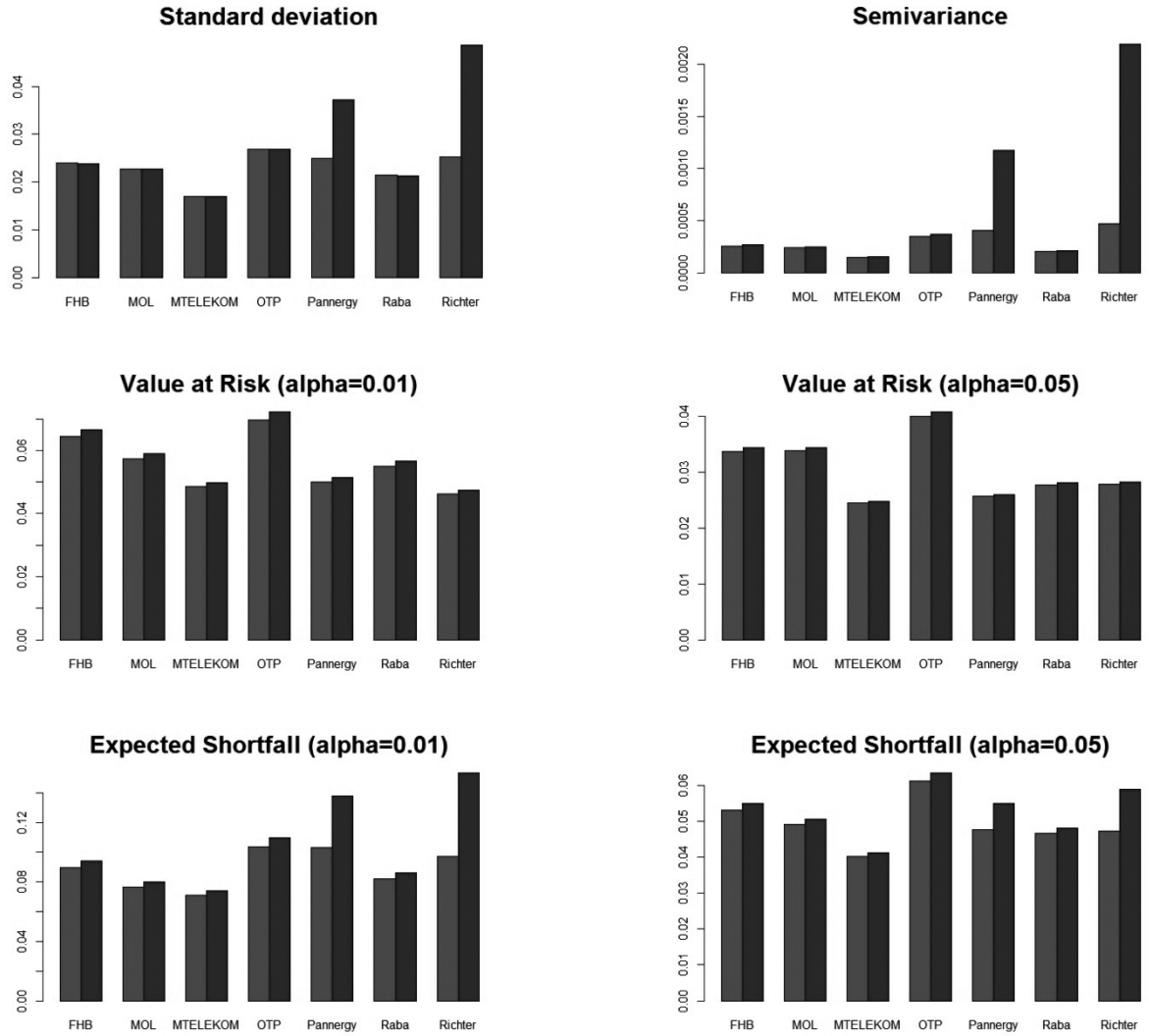
In the case of standard deviation and ES the contrary was observed: the order does depend on the type of return. Using the simple return for risk calculation the OTP stock has the highest standard deviation value. The OTP is followed by Richter, Pannergy, FHB, MOL, Raba and MTELEKOM. If we use the log return for risk calculation, then the aforementioned order changes: the riskiest is the Richter, followed by Pannergy, OTP, FHB, MOL, Raba, MTELEKOM. For example, the OTP stock what was the riskiest using simple returns, in the case of the log return is only on the 3rd place. Let us consider now the ES. At the level of 5%, using simple returns for the calculations we got the following order: OTP, FHB, MOL, Pannergy, Richter, Raba, MTELEKOM; while using log returns the order changes as follows: OTP, Richter, Pannergy, FHB, MOL, Raba, MTELEKOM. We can see, that for example the Richter stock is the second riskiest stock in the case of using log returns, but it is just on the 5th place in the case of simple returns. At the level of 1% the riskiness order also different concerning simple or log returns. In the case of simple returns the riskiest asset is the OTP, followed by Pannergy, Richter, FHB, Raba, MOL and MTELEKOM. In the case of logarithmic returns the riskiest asset is the Richter, followed by Pannergy, OTP, FHB, Raba, MOL and MTELEKOM.

This calculation shows, that despite the fact that the simple and log returns are comparable, our assumption, that the result does not depend on whether we use simple or log return seems to be not correct. We could show, that the only coherent risk measure, the Expected Shorfall, gives different riskiness orders for the same stocks depending on whether it was calculated using simple or log returns. And this can lead to different decisions.

### *b. Portfolios*

In the case of portfolios, we consider seven portfolios, each of these portfolios consist of six distinguishing stocks (we just leave away one of the seven stocks) and we calculate the risk of all these portfolios in order to generate a “riskiness order”. To calculate the risk we consider the two most often used risk measures: the Value at Risk (see Equation ( 35 ))

***Figure 4. Standard Deviation, Semivariance, VaR and ES values calculated using simple (purple bars) and logarithmic (blue bars) returns***




***Source: own calculation***

and the Expected Shortfall (see Equation ( 36 )) at $\alpha = 0,05$ and $\alpha = 0,01$ levels. The results are shown in Table 3. The numbers in the table show the riskiness order of the portfolio calculated by using VaR and ES in case of simple returns respective log returns at two different alpha levels. One can observe, that VaR is stable on both levels, meaning the order does not depend on the choice of return. In the case of the ES at $\alpha = 0,01$ level – similarly to the VaR – the two orders are the same. To the contrary for $\alpha = 0,05$ level: the first and the second portfolio switched positions. And our decision can be influenced by this different riskiness order.

We can clearly see from the results, that not only the type of the return, or the chosen risk measure but also the level of alpha (given the risk measure) has a decisive effect on the order, and hence on the decision. For example at $\alpha = 0,05$ level the ES measures Portfolio7 is one of the riskiest portfolio. But, in contrast, at $\alpha = 0,01$ level, Portfolio7 is the least risky portfolio from these seven portfolios.

***Table 3. Riskiness order of portfolios using simple and logarithmic returns***

| | VaR | | | | ES | | | |
|---|---|---|---|---|---|---|---|---|
| alpha | 0,05 | | 0,01 | | 0,05 | | 0,01 | |
| return | simple | log | simple | log | simple | log | simple | log |
| Portfolio1 | 6 | 6 | 4 | 4 | 6 | 6 | 5 | 5 |
| Portfolio2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| Portfolio3 | 5 | 5 | 3 | 3 | 5 | 5 | 4 | 4 |
| Portfolio4 | 7 | 7 | 7 | 7 | 3 | 3 | 2 | 2 |
| Portfolio5 | 4 | 4 | 6 | 6 | 7 | 7 | 6 | 6 |
| Portfolio6 | 3 | 3 | 5 | 5 | 4 | 4 | 3 | 3 |
| Portfolio7 | 1 | 1 | 1 | 1 | 1 | 2 | 7 | 7 |

### *Using approximations*

In literature one can regularly see that the relative weights are substituted by $1/n$, where $n$ is the number of assets in the portfolio. One reason for this could be that one wants that the weights are constant in time, because the relative weights are changing in time since they are calculated from the prices (see Equation ( 23 )). Another argument for using approximation is that in practice sometimes one does not know the asset prices (for example, a simulation result gives only returns). In the absence of the prices one cannot calculate the relative weights and in the absence of the relative weights it is not possible to calculate the portfolio return.
In this last part of our study we would like to show – using our ordering method – the effect of an approximation on the riskiness order. We will consider again the portfolios which were constructed in section '*Comparing riskiness order, Portfolios'*. To calculate the simple return of a portfolio we will use equation ( 27 ), and we will approximate the weights with $1/n$: $w_i \approx \frac{1}{n}$, $i = 1,\ldots,n$ and such that we consider an equally weighted portfolio. This approximation turns to exact equation if we consider a portfolio which consists of same number of all the assets ($k_i = k_j$, $i \neq j$, $i, j = 1,\ldots,n$) and the asset prices are the same. So, we will use the following approximation for the portfolio simple return:

$$\hat{R}_t^S = \frac{1}{n}\sum_{i=1}^{n} R_{t,i}^S. \qquad (37)$$

In the case of log returns we will consider two different approximations. For the first one we use Equation ( 28 ) and the same assumption as before: we assume that the weights are $1/n$ ($w_i = \frac{1}{n}, i = 1,\ldots,n$). Therefore:

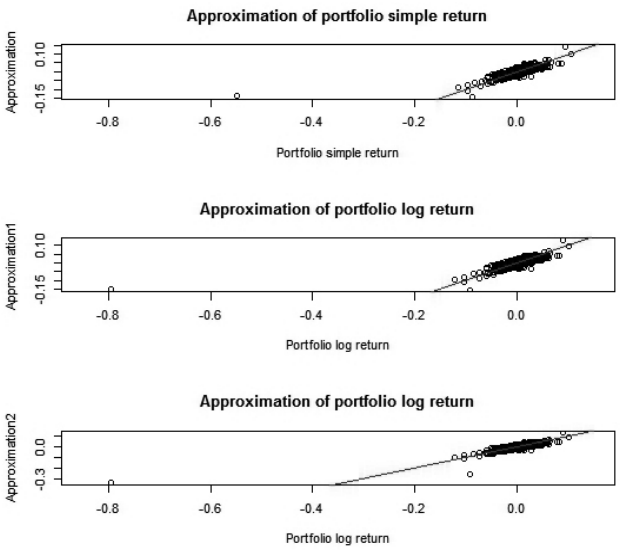
$$\hat{\mathrm{R}}_{\mathrm{t}}^{\mathrm{L}} = \ln\left(\frac{1}{n}\sum_{i=1}^{n} e^{R_{t,i}^L}\right). \qquad (38)$$

For the second approximation we will use Equation ( 31 ), which is already an approximation and as a further assumption we consider the weights equal to $1/n$ ($w_i = \frac{1}{n}, i = 1,\ldots,n$ ), similarly to the previous ones.
So, the second approximation for the portfolio log return can be expressed in the following way:

$$\hat{R}_t^L = \frac{1}{n}\sum_{i=1}^{n} R_{t,i}^L. \qquad (39)$$

In (Figure 5) we can see how far away the approximated values are from the exact values. The first plot shows the approximated values calculated using Equation ( 37 ), the middle one shows the approximated values evaluated using Equations ( 38 ) and the third plot shows the approximated values calculated using Equation ( 39 ). In all the three cases there is one outlier. Up to this the point clouds are still distributed along the 45° line. We may conclude from this, that using these approximations we can get similar result than using not approximated, exact return values. We would like to answer the following questions. First we will check whether the riskiness order changes if we use approximated simple or approximated logarithmic returns. Second, we will compare these orders in the case of approximated and exact simple and logarithmic returns. To calculate the risk we will use again the VaR and the ES risk measures at two different $\alpha = 0,05$ and $\alpha = 0,01$ levels.

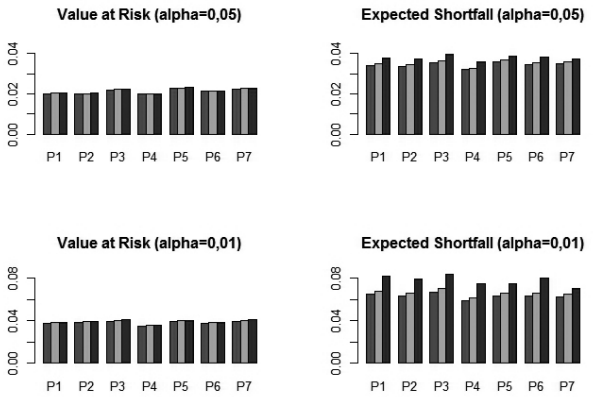
***Figure 5. Approximations of portfolio simple and logarithmic returns***




The results are shown in (Figure 6). P1, P2,..., P7 indicate the seven different portfolios. The purple bars stand for the risk calculated using the approximated simple return data (see Equation ( 37 )), the orange and blue ones for the risk calculated using the approximated log return data (see Equation ( 38 ) and Equation ( 39 ) respectively). In the left column of the figure we can see the Value at Risk values and in the right column we can see the ES values. Similarly to the previous cases the VaR seems to be more stable, since the order does not depend on whether we use simple or log return. At $\alpha = 0,05$ level the riskiest portfolio is P5 followed by P7, P3, P6, P1, P2 and P4. And at $\alpha = 0,01$ level we calculated the following order: P7, P3, P5, P2, P6, P1, and P4. These orders are the same using approximated simple or one of the log return data. If we take a look at the ES values we can see that here the order of the portfolios changes depending on the type of used approximation method. At $\alpha = 0,05$ level we got the same order as in the case of the approximated simple return and the first approximation of the log return (see Equation ( 38 )), namely: P5, P3, P7,P6, P1, P2, P4. But this is different from the order which we get if we use the second approximation of the log return (see Equation ( 39 )), that is: P3, P5, P6, P1, P2, P7, P4. These results are also shown in the last three columns of Table 4

and Table 5, where $R^L$ and $R^S$ denote the simple and logarithmic returns respectively, while $\hat{R}^S$ denotes the approximated simple returns, and $\hat{R}_1^L$ (see Equation ( 38 )) and $\hat{R}_2^L$ (see Equation ( 39 )) the approximated log return.

***Figure 6. VaR and ES values for seven portfolios using approximated data***




Finally let us examine whether the type of data used has an effect on the order, i.e. whether approximated or exact data. In Table 4 we can see the results using risk measure VaR and in Table 5 we can see the results using risk measure ES. The result clearly shows a totally different riskiness order on all the cases. For example at 5% level the VaR ranked Porfolio2 on the second place using not approximated data and it is on the sixth place when measured with approximated data. It is similar in the case of ES: Portfolio2 is in the second or first place (depending on the type of the return) using not approximated data but in contrast the portfolio is on the fifth or sixth place using approximated data. Or Portfolio5 is on one hand the less riskiest portfolio if we calculate ES at alpha=0,05 level from the exact simple or logarithmic return, but on the other hand it is the riskiest portfolio if we calculate the ES at alpha=0,05 level from approximated data. Similar results have been found on the level of alpha=0,01. For example in the case of VaR Portfolio5 is on the sixth place if the value calculated from not approximated data and on the third place if the ES is calculated from approximated returns. The ES is less stable. Depending on the type of return or whether we use approximation the order can vary strongly, see for example Portfolio4 or Portfolio5.

***Table 4. Order of the portfolios using the risk measure VaR on the level of alpha=5% and alpha= 1%.***

| | VaR | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| alpha | 0,05 | | | | | 0,01 | | | | |
| return | $R^S$ | $R^L$ | $\hat{R}^S$ | $\hat{R}_1^L$ | $\hat{R}_2^L$ | $R^S$ | $R^L$ | $\hat{R}^S$ | $\hat{R}_1^L$ | $\hat{R}_2^L$ |
| Portfolio1 | 6 | 6 | 5 | 5 | 5 | 4 | 4 | 6 | 6 | 6 |
| Portfolio2 | 2 | 2 | 6 | 6 | 6 | 2 | 2 | 4 | 4 | 4 |
| Portfolio3 | 5 | 5 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 |
| Portfolio4 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| Portfolio5 | 4 | 4 | 1 | 1 | 1 | 6 | 6 | 3 | 3 | 3 |
| Portfolio6 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | 5 | 5 |
| Portfolio7 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |

***Source: own calculation***

***Table 5. Order of the portfolios using the risk measure ES on the level of alpha=5% and alpha= 1%.***

| | ES | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| alpha | 0,05 | | | | | 0,01 | | | | |
| return | $R^S$ | $R^L$ | $\hat{R}^S$ | $\hat{R}_1^L$ | $\hat{R}_2^L$ | $R^S$ | $R^L$ | $\hat{R}^S$ | $\hat{R}_1^L$ | $\hat{R}_2^L$ |
| Portfolio1 | 6 | 6 | 5 | 5 | 4 | 5 | 5 | 2 | 2 | 2 |
| Portfolio2 | 2 | 1 | 6 | 6 | 5 | 1 | 1 | 5 | 4 | 4 |
| Portfolio3 | 5 | 5 | 2 | 2 | 1 | 4 | 4 | 1 | 1 | 1 |
| Portfolio4 | 3 | 3 | 7 | 7 | 7 | 2 | 2 | 7 | 7 | 5 |
| Portfolio5 | 7 | 7 | 1 | 1 | 2 | 6 | 6 | 4 | 3 | 6 |
| Portfolio6 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | 5 | 3 |
| Portfolio7 | 1 | 2 | 3 | 3 | 6 | 7 | 7 | 6 | 6 | 7 |

***Source: own calculation***

## SUMMARY AND CONCLUSION

In this study our goal was to clarify the notion of simple and logarithmic return and to show the differences and the connections between them. In the theoretical part we stated the definitions of the one- and the multi-period simple and logarithmic returns.

Equations - presented in the stock case – show, that the logarithmic return has an advantage against the simple return, namely that the multi-period logarithmic return can be calculated as a sum of the one-period logarithmic returns, while the multi-period simple return is the product of the one-period simple returns, which can lead to computational problems for values close to zero.

In the case of a portfolio it is important to highlight, that the portfolio weights depend on the price of stocks in the portfolio. So they change in time. In the case of an equally weighted portfolio one has to balance regularly the portfolio. It is also important to note, that the simple return of a portfolio is the sum of the weighted simple returns of the constituents of the considered portfolio. In contrast, the logarithmic

return of a portfolio can only be approximated by the sum of the weighted logarithmic returns of the constituents of the considered portfolio.

In addition we could see, that if the simple return values are close to zero, then the distribution of the simple and logarithmic returns are very near to each other. This raises the question whether the used return-type (i.e. simple or log return) has an effect on the calculations and thus on the results. In the empirical part of our study we wanted to answer this question. We were interested in whether the used return-type in the calculations results in a different riskiness order. First we compared the order in the case of the stocks. We found that while in the case of semivariance and VaR the order does not depend on the type of return, in the case of standard deviation and ES it does. After the stocks we considered portfolios. Six different portfolios were compared and ordered according to their risks. The result of our calculation shows, that the VaR does not depend on the use return-type, but in the case of the ES we got different orders in the two cases.

Furthermore, we investigated what is the effect on this order if we use approximated return values – for example we considered equal weights, which is common in practice - instead of the exact values. We have found in every case different riskiness orders, sometimes even serious differences. Therefore, we believe, if this is possible, exact values instead of approximated ones should be used for calculations.

In summary, even though the two return-type values are very similar, it is not necessary that the riskiness orders are the same. It is important that one uses the same type of return within one study and one has to be aware of the possible instabilities when comparing return results.

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