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A multidimensional network analysis of rivals' cooperation

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ABSTRACT

The paper empirically elaborates and tests a novel measurement model called a complex model of network structure exploration (CMNSE). This new measurement model gauges simultaneously, on one hand, structural features of nodes with degree distribution, on the other hand, the spatial characteristics of edges of a real network are investigated with quadrat analysis. The topology of the newest form of an economic network called coopetitive network is tested by CMNSE. Empirical findings reveal that this coopetitive network does not have scale-free property since its degree exponent belongs to the anomalous regime. Moreover, the spatial position of a hub and nodes can be pinpointed with CMNSE highlighting that nodes are clustered significantly in the periphery of the networked space, but the hub is localized in the center of the network.

Kulcsszavak
hálózati struktúra, skálafüggetlenség, kvadrát analízis, valódi hálózatok

Ebben a tanulmányban kidolgozunk és empirikusan tesztelünk egy újszerű hálózati mérési modellt, amit a hálózati struktúra komplex mérési modelljének (CMNSE) neveztünk el. Ez egyrészt méri a csúcspontok strukturális jellemzőit fokszámeloszlással, másrészt vizsgálja az élek térbeli vonásait kvadrátanalízissel. A gazdasági hálózatok legújabb típusát, egy magyar koopetitív hálózat topológiáját kvantifikáljuk CMNSE modellel. Az eredmények szerint ennek az üzleti hálózatnak nincs skálafüggetlen tulajdonsága, mivel fokszámkitevője rendkívül alacsony. Továbbá, a hálózat térbeli elemzésével megállapítható, hogy a gócpont a hálózat által lefedett tér centrumában található, míg a csúcspontok a hálózat peremén sűrűsödnek.

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Introduction

The paper develops and empirically tests a measurement model, referred to as a complex model of network structure exploration (CMNSE), analyzing the structure of a real network in a multifaceted way. Obviously, the network topology must be scrutinized to understand functions, interior features, robustness, vulnerability, accomplishment, effects, and externalities of complex systems (Barabási 2016, Chagnon et al 2016, Newman 2000). Only the architecture of *real* networks is examined by CMNSE. The notion of real network refers to a type of network wherein vertices are not randomly linked to each other and that can be found in the real world as well; the real networks cannot be described with Erdős-Rényi random graph theory (Bollás 2001). In this respect, real networks might be regarded as non-random empirical networks. Furthermore, the notion of network structure here indicates that nodes and edges are organized and arranged in the networked space.

The paper is constructed as follows. Section 2 reviews the theoretical underpinnings and the most relevant empirical results of network structure investigations. After every detail of CMNSE is demonstrated in the Method section, the model is tested empirically network of wine shops (NWS) in Budapest (capital of Hungary) in the Results section. Finally, the Conclusion section summarizes the main points of CMNSE, empirical findings, and future research directions as well.

Theoretical background and empirical overviews

Network structure has been studied regularly by degree distribution (i.e., how many linkages belong to vertices in a certain network). For example, the topology of biological (Barabási 2016, Barabási-Oltvai 2005, Kveler et al. 2018, Santolini-Barabási 2018, Shen et al 2015), medical (Barabási 2016, Barabási-Oltvai 2005, Csermely – Korcsmáros – Kiss 2013b), ecological (Melián et al. 2009, Palla et al. 2005, Poisot-Gravel 2014), societal (Barabási et al. 2002, Glowacki et al. 2016, Jackson 2016, Kossinets-Watts 2006, Palla et al. 2005), economic (Anand-Craig-Goetz 2014, Goyal 2007, Jackson 2016, Knieps 2015, Zhang-Du 2017) and informational (Jeong-Albert-Barabási 1999, Gleeson et al 2016, Stopczynski-Pentland-Lehmann 2018) networks have so far been characterized with degree distribution. Csermely et al. (2013a, 2013b) classify accurately types of structural properties of linkages. Besides, some multidimensional studies, however, combine several network indices to measure network schemes. Such as, Uchida-Shirayama (2008) inspects a network system by employing the same time degree distribution, clustering coefficients, short average path length, and degree correlation. Hagberg-Swart-Schult (2008) also integrates network diameter, betweenness centrality, shortest path, degree distribution, and clustering coefficient to describe the properties of a network structure. Chagnon et al. (2016) explore ecological network topology by synthesizing six network metrics: C-score index, nestedness, betweenness centrality, modularity, power-law fit to degree distribution, and interaction strength asymmetry. Russel et al. (2017) use simultaneously closeness centrality, betweenness centrality, and clustering coefficient to design the network structure of children's societal environment engaged in post-traumatic stress. Apparently, empirical network measurements have so far usually applied degree distribution and hardly ever multilayer approaches.

Notwithstanding, a few empirical inquiries have so far analyzed structural features of node structure applying heuristic measure-based methods or probabilistic inference-based model (Chai et al 2013, Chen et al 2016, Daqing et al 2011, Duan et al. 2013). Obviously, these do not explore the structural traits of edges. Structural characteristics of vertices and edges have so far been scrutinized separately and not together.

Of course, the above-mentioned methods are adequate and useful, but both were used separately in studies thus entirely network topology could not be described. Simply put, since a real network subsumes vertices *and* edges, structural and spatial traits of both must therefore be examined simultaneously to map network architecture. This complex analytical approach cannot be omitted in scientific research.

Simply put,

the structure of a real network =
structural features of *edges*
+
structural features of *nodes*.

In this paper, CMNSE is developed to perceive and quantify simultaneously spatial and structural properties of vertices *and* links. The next section describes and explains the main features of the CMNSE.

The network model

Two components of CMNSE can be distinguished. The first one inspects structural features of links by degree distribution. The second one dissects the structural characteristics of nodes by quadrat analysis. Apparently, a real network emerges as a complex system thus it must be studied in a complex way. Firstly, the paper concentrates on how structural properties of linkages are measured.

Measuring structural features of edges

A real network (*N*) embraces a finite, nonempty set $V = \{v_i\}$ of vertices (*V*) and a finite, nonempty set $E = \{e_j\}$ of edges (*E*). The number of vertices in *V* is defined as i, and the number of edges in *E* is referred to as j. Since CMNSE is built on previous scientific inquiries, the structural feature of edges is gauged consequently with degree distribution. Degree distribution p_k can be estimated well with the maximum likelihood technique:

$$p_k \sim Cb^{-\gamma} \tag{1}$$

where *C* is a constant, b means a variable and $-\gamma$ expresses degree exponent (Mocnik 2018). According to the discrete formalism of degree distribution, constant *C* is determined by the normalization condition

$$\sum_{i=0}^{\infty} p_k = 1$$

(2)

Applying equation (1) we obtain

$$C \sum_{i=0}^{\infty} k^{-\gamma} = 1$$

(3)

hence

$$C = \frac{1}{\sum_{i=1}^{\infty} k^{-\gamma}} = \frac{1}{\xi(\gamma)}$$

(4)

where $\xi(\gamma)$ appears as the Riemann-zeta function. For k>0 the discrete power-law degree distribution possesses the form

$$p_k = \frac{k^{-\gamma}}{\xi(\gamma)}$$

(5)

The value of $-\gamma$ shows structural characteristics of network connections, the CMNSE, as a result, concentrates on the value of the degree exponent (Barabási 2016, Goh-Kahng-Kim 2001).

Measuring structural features of nodes

Gauging of structural patterns of nodes might be more difficult than degree distribution. The starting point of CMNSE is that every real network possesses spatial extension, size, form, and dimension. For example, the geographical distances among vertices are relatively long in a motorway network (Adamatzky et al. 2017), networks for commodities delivery (Barthélemy 2017), river networks (Rodrigue-Ronaldo 1997), power grid networks (Kim et al. 2018), and street networks (Gil 2016) as well. Shorter physical distances can be found regularly in social networks (Barthélemy 2011, Latour 2011) or networks of small-and-medium-sized enterprises (Balister et al. 2018, Törnroos et al. 2017). Nevertheless, the physical distances among vertices can be measured in centimeters or millimeters in underground hyphal networks

(Friese-Allen 1991), three-dimensional integrated circuits (Wong 2007), neurons' networks in the brain (Dehmamy-Milanlouei-Barabási 2018), circulatory network systems (West-Brown 2003) or other cell networks (Gartner-Prescher-Lavis 2017) as well. Obviously, all real networks have spatial extension irrespective of their sizes, ages, or types.

Spatial characteristics of real networks permit the description and capture of structural traits of vertices, the spatial position of hubs and nodes, and the physical interplay among them (Mocnik-Frank 2015). By considering the spatial distribution of vertices, we can answer the question of where network agents are clustering or thinning out in the networked space. Nodes of spatial distribution are measured with quadrat analysis (i.e., it focuses on spatial patterns and allocation of nodes by comparing the number of vertices among the cells; the sizes of a grid have no mathematical rules or theorems, it is defined always by the researcher) (Brinkhoff – Kresse 2012, Jinghu-Junfeng-Yibo 2015, Reginald 1977). The spatial distribution of nodes is analyzed empirically by quadrat analysis because it is regarded as a useful, simple, elegant, and reliable method (Robinson et al 2016).

At this point, the networked space must be defined and delimited geographically. Firstly, the network map has to be drawn in which a vertex depicts the geographical location of a business entity and a bond between nodes presents partnership (Figure 3). The outermost vertices (i.e., those nodes that can be found topologically the outermost of the network) are connected to each other (see green dash line in Figure 3) obtaining the physical boundary of the networked space.

Subsequently, grids are superimposed over the spatial layout of the networked space and the number of events falling in each grid area is counted. The results of quadrat analysis are characterized by the variance-to-mean ratio (VMR) test (O'Sullivan-David 2010, Robinson et al 2016). To implement VMR, the mean grid count (μ) must be calculated:

$$\mu = \frac{V}{x} \tag{6}$$

where x expresses the number of quadrats.

After this, $x(a-\mu)^2$ is computed where a means the number of events. The variance (s^2) is obtained:

$$s^2 = \frac{1}{V}\sum_{i=1}^{v}(a-\mu)^2 \tag{7}$$

and

$$VMR = \frac{s^2}{\mu} \tag{8}$$

If VMR<1, the variance is low, regularly is zero, and the distribution of nodes is uniform. When the spatial allocation of points is random/stochastic (i.e., follows the Poisson distribution pattern) then VMR=1, namely the mean and variance are equal. If VMR>1 (variance is greater than mean), the distribution is clustered. In a nutshell, point distribution could be clustered (attracting), stochastic/random (Poisson), and uniform (repelling) (Robinson et al 2016).

To summarize, the structural features of edges are gauged with degree distribution, and the structural features of nodes are measured by quadrat analysis. Firstly, in the CMNSE, degree distribution should be calculated to obtain structural traits of links. After this, the structural attributes of edges must be defined. To implement it, the spatial boundaries of a real network must be delimited. Later, quadrats are superimposed over the map of networked space and spatial patterns of points thus are analyzed with VMR. CMNSE synthesizes results of degree distribution and quadrat analysis to study network structure in a complex way.

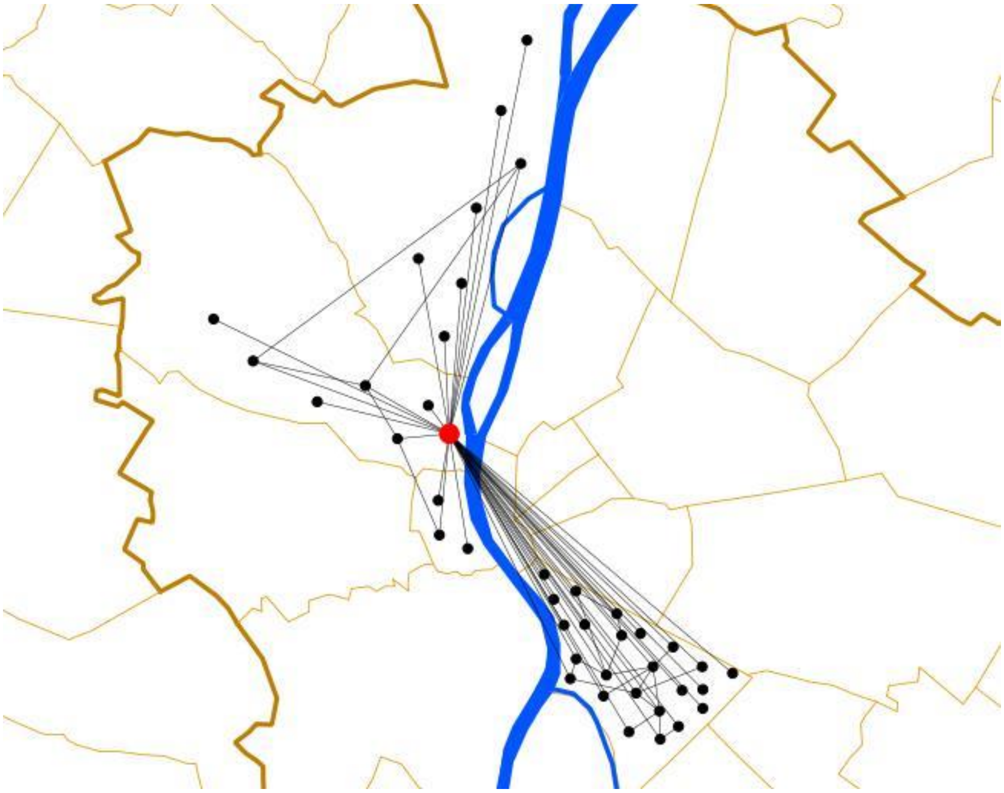
In the next section, a structure of a new type of economic network will be examined empirically by testing CMNSE.

Results and discussion

General features of a new form of economic network

In this section, the topology of a totally new type of economic network, called coopetitive network (i.e., dynamic inter-firm relationship in which rivals compete and collaborate with each other simultaneously in some business fields to realize higher profit rate), is studied with CMNSE. More specifically, in 2011 some owners of wine shops in Budapest constructed informally a coopetitive network in Budapest to reduce purchase prices together and increase their complex accomplishments; the spatial layout of a network of wine shops (NWS) is depicted in Figure 1.

Figure 1. Spatial layout of coopetitive network of wine shops in Budapest



NWS solely includes micro-enterprises and small and medium-sized entrepreneurs. Coopeting partners cooperate in only two business activities, namely the mutual purchase and transport of bottles of wines. Network agents purchase in a bulk from wineries to receive discounts and transport products together to decrease expenditures and increase, as a result, profit. However, rivals compete in several business fields such as for more consumers, well-qualified employers, innovations, relational capital, recipes for special foods, and reliable accountants, to name just a few. Similar coopetitive networks may be found in Eastern European regions as well (Jóna-Tóth 2017, Jóna 2018). Nevertheless, NWS has been operating approximately for 5 years, these days it went out of the local business life.

In NWS a node demonstrates the physical location of a firm, and an edge means undirected and unweighted coopetitive interactions (emails, phone communications, face-to-face conversations, etc.) among rivals. Coopetitive activities (mutual transportation and purchasing) are planned, managed, and coordinated by emails hence the length of the linkage between rivals is defined as the Euclidean distance (i.e., it refers to the length of the line segment among actors). The Euclidian distance based on the Pythagorean theorem is obtained in a simple way in two dimensions:

$$ed(p,q) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} \quad (9)$$

where *ed(p,q)* means Euclidian distance between nodes, *p* and *q* pinpoint the positions of nodes in polar coordinate (Barthélemy 2011, O'Sullivan 2014). In the practice, every length of linkage is measured separately, expressed in kilometer and the longest is the spatial diameter of the network.

In brief, NWS is referred to as a bottom-up real network wherein loops and isolated nodes ($k_i = 0$) cannot be found (k_i expresses the node degree). NWS functionalizes as an informal network without any formal contracts among actors.

The sociological snowball method was employed to map the whole network and to muster the raw network dataset (Heckathorn – Cameron 2017).

Testing complex model of network structure exploration (CMNSE)

Firstly, the degree distribution of NWS is measured. After this, the results of quadrat analysis are characterized by VMR. Finally, both findings are interpreted together to describe the multilevel way of the topology of a real network.

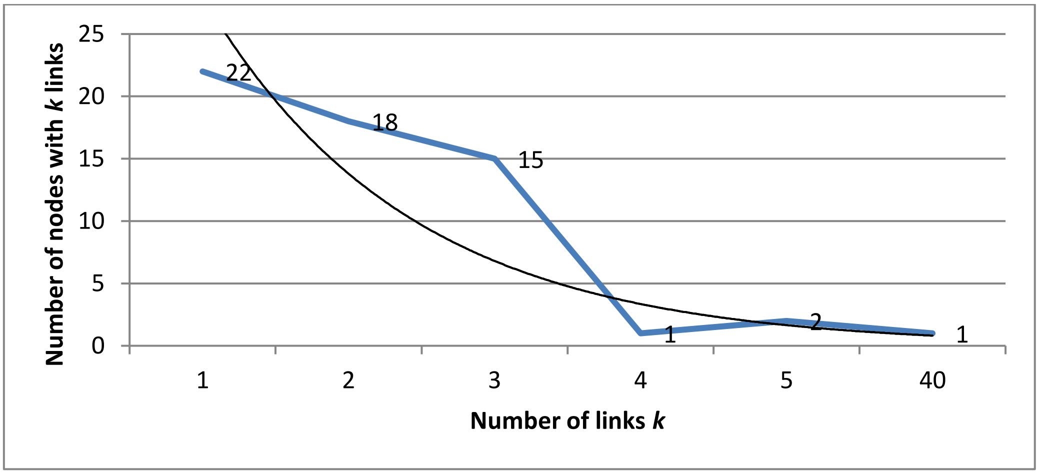
Degree distribution of NWS

NWS encompasses 41 vertices (*V*=41) and 55 undirected, unweighted edges (E=55):

$$E = \frac{1}{2}\sum_{i=1}^{V} k_i \tag{10}$$

The average degree of NWS $\left(\langle k \rangle = \frac{2E}{V}\right)$ is 2.68 meaning that a network player interacts with more than two enterprises. Notwithstanding, if results of degree distribution are analyzed (Figure 2), a hub, the most connected vertex, can be recognized with 40 links. More precisely, a hub (focal firm) ties to every node, but more than 53% of vertices possess only one connection ($k \ll \langle k \rangle \ll k_{hub}$); NWS is regarded as a sparse network. Such a centralized network has been formed because rivals loathe each other because of their harmful earlier business experiences. Coopeting partners hardly ever interact with each other, they do not trust each other but they believe in the hub that mediates among rivals and fills structural holes (i.e., it is a gap in the network among disconnected nodes. The hole is bridged by the hub to integrate the whole network) in NWS as well (Jóna-Tóth 2017, Jóna 2018).

Figure 2. Degree distribution of network of wine shops



Resource: my calculation.

Barabási-Ravasz-Vicsek (2001) and Barabási (2016) suggest that regularly real networks have power-law degree distribution. Notwithstanding, according to Figure 2, NWS does not have scale-free network property since degree exponent $\gamma = 0{,}707$ meaning belonging to the anomalous regime (anomalous regime: $\gamma \leq 2$; scale-free regime: $2 < \gamma < 3$; random network regime: $\gamma > 3$) (Luitz 2015, Newman 2005). Of course, the estimated value of γ must be handled with caution because NWS<50 (Barabási 2016: 157). NWS, however, is not a scale-free network but operates effectively with anomalous topological edge modes. It implies that the robustness of NWS is relatively high against random targeting and attacking, but it is low against consciously attacking. The special type of interconnectivity of NWS causes high vulnerability in the network. Furthermore, this paper emphasizes still that NWS has not power-law degree distribution because the number of ties of neighboring vertices is often limited if real networks are embedded in the local or global spatial layout. It is proved with some tangible examples by Mocnik–Frank (2015) and Mocnik (2018).

In addition, the anomalous regime indicates that a giant component develops quickly in NWS; it links to every node and has a special, well-qualified ability to acquire connections. This super component appears as the main actor bridging among competitors and managing coopetitive activities in NWS.

4.2.2. Quadrat analysis of NWS

Firstly, grids embracing 13x13 matrixes are superimposed over the spatial layout of NWS and the number of events falling in each grid area is computed (Figure 3 and Table 1). The longer side of the rectangle shaped cell is 1626 metres, and the shorter side is 1237 metres.

Figure 3. Quadrat analysis of NWS

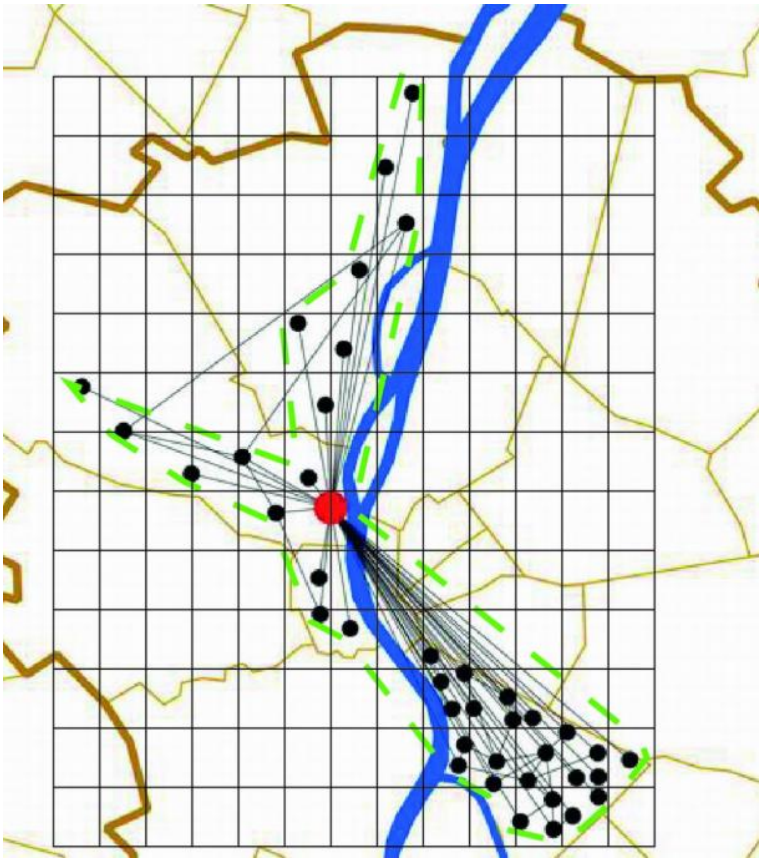


Table 1 demonstrates how nodes are clustered extremely in the networked space (see the values in the lower right corner in Table 1), the degree of the cluster is computed with equations (2), (3), and (4).

Table 1. The spatial pattern of NWS is visualized by quadrat analysis.

0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0	0
0	1	1	0	1	1	0	0	0	0	0	0	0
0	0	0	0	1	**1**	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	3	3	1	0	0
0	0	0	0	0	0	0	0	2	2	2	4	1
0	0	0	0	0	0	0	0	0	0	3	2	0

Resource: my own calculation. The bold number (almost in the center of the matrix) illustrates the geographical location of the hub in the networked space.

Table 2. Quadrat counts and calculation of the variance for the NWS

Number of events (*a*)	Number of quadrats (x)	$a-\mu$	$(a-\mu)^2$	$x(a-\mu)^2$
0	141	-0.2426	0.0588547	8.2985127
1	20	0.7574	0.57365476	11.4730952
2	4	1.7574	3.08845476	12.35381904
3	3	2.7574	7.60325476	22.80976428
4	1	3.7574	14.11805476	14.11805476
Total	**169**			**69.05324598**

Source: my calculation. Mean cell count (μ): $\mu = \frac{V}{x}$; $\mu = \frac{41}{169} = 0.2426$. Variance: $\frac{69.05324598}{41} = 1.6842255$. Variance/mean cell count (VMR): $\frac{1.6842255}{0.2426} = 6.942396$

Table 2 reports that $\mu = 0.2426$, s^2=1.6842255, and VMR=6.942396. Since VMR is greater than 1 ($VMR \gg 1$), spatial patterns of NWS are regarded as a clustered, centralized, and complex system but it is not a star graph.

Besides, the quadrat analysis permits the description of the physical distance between nodes. Empirical findings show that the average distance of linkages is 8.206 meters, the longest distance is 21.140 meters, and the shortest distance is 364 meters in NWS.

Moreover, by applying quadrat analysis, the spatial position of the hub can be identified. Firstly, those grids must be found in which giant components exist. It can be obtained simply if we pinpoint and mark the cells in which the most connected vertex/vertices appear. By using this method, the physical position of a hub could be stated in NWS located in almost the center of a networked space (Table 1). Interestingly, the position of the hub can be discovered relatively far from the groups of clustered network players, the average distance between the super component and southwestern clustered nodes is almost 4.5 km.

The paper presupposed that most vertices would allocate very close to the geographical position of the hub. Notwithstanding, this empirical measurement does not verify it because the hub is pinpointed relatively far from clustered

vertices. Nevertheless, the coopeting connections between the focal firm and network agents overcome physical distance; the physical distance can be defeated and shortcut by utilizing social proximity in NWS (Boschma 2005).
To summarize, the degree distribution points out that the hub ties to every competitor while rivals rarely connect to each other; the NWS possesses anomalous topological edge modes following a dispersion pattern. The quadrat analysis, however, accentuates that the super-components of the NWS geographically and physically exist relatively far from clusters of vertices; the hub emerges in almost the center of the networked space (Table 1) meanwhile clustered network agents are located on the periphery. Both edges and nodes are significantly clustered in NWS.

Summary and conclusion

This paper demonstrates a new CMNSE measurement model with which the complex structure of a real network can be analyzed and quantified. The CMNSE synthesizes two well-known techniques of degree distribution and quadrat analysis as well. This model perceives the structural and spatial properties of edges and vertices at the same time to present the nature, function, and robustness of a real network. Why is it important fundamentally? In the 21st century, useful networks (societal, market-based economic, motorway, street, infrastructural, etc.) and harmful networks (terrorist, hackers, gossip, drug, mafia, hoax, virus, etc.) can be compartmentalized. The first one should be improved and the second one must be destroyed. By applying the CMNSE, both can be conducted. First, the key is to find the hub and its spatial position in the networked space that manages and controls the whole network (Barabási 2016). If a hub location is pinpointed, it may be supported or targeted depending on whether they are useful or harmful. Diverse impacts spread through the hub in the network, it hence is enough to support or impede the hub to manage or control the whole network.
More broadly, scientific papers have so far scrutinized the hub by degree distribution although these focus on merely answering the question of whether the hub exists or not in a real network. Nevertheless, by applying the CMNSE that synthesizes well-known methods of degree distribution and quadrat analysis, we could define the numbers and spatial, physical locations of hubs to understand network structure and, subsequently, impact operational mechanisms of complex systems.

The main limit of the CMNSE is that only structural traits of two-dimensional networks can be described. Topologies of three-dimensional networks cannot be characterized by this model. The next step is to develop CMNSE to scrutinize the structure of the three-dimensional network in the future.

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