THEORETICAL AND COMPARATIVE STUDY REGARDING THE GALERKIN-VLASOV METHOD USE FOR STUDY THE FREE VIBRATIONS OF WOOD FLAT PLATE.

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Abstract. As for the historic record of the problem related to the study of flat plates, the first results were out for publishing at the end of the 18th century, the beginning of the 19th century, having Chladni E, Strehlke, Konig, R, Tanaka S, Rayleigh L, Ritz W and later on Gontkevich V, Timoshenko S, Leissa as pioneers. Each of the above mentioned authors have had significant contributions regarding the development of methods in order to solve the plates and establish some rigurous solutions of their differential equations of equilibrium.
The making of constructions, machines and different high-perfomance appliances, whose functioning should take place in safety conditions, have required theoretical studies of rich complexity, as well as practical experiments, within which the problem of their free and forced vibrations represent an important category in the respective theme of research.

Keywords: plate, Galerkin-Vlasov, clamped, , free, edge variational method.

INTRODUCTION

The rectangular flat plates, as well as the the angular ones in general, often intervene as strength elements in the structures of civil and industrial constructions, their actual shape and support mode being imposed by different conditions in the exploatation of the buildings, such as the lay-out of some technological appliances. The results of the scientific research regarding the flat plates, as well as the practical importance related to the knowledge of their way of behaviour in different loading and support situations within some structures (machines, buildings, equipments) are emphasized in numerous treaties, books, scientific papers published throughout the centuries.

The object of the paper consists of the study of the free vibration of rectangular wood flat plate having clamped edges. The analysed flat plates are thin, elastic and isotropic with stiffness at bending and meet the availability conditions of Kirchhoff hypotheses [8], [10]. The suggested calculus method is an adaptation of the variational method Galerkin-Vlasov from the static calculus of dynamic plates and has been elaborated in such a way that the calculi necessary for determining the dynamic characteristics of the plates are made on the basis of some programmes designed by the author himself.

MATERIALS AND METHODS

Using differents variational methods Bors [1], Fetea [2] , Iguchi [5] and Szillard [7], determinated the solution in the case of freely vibrating plates reduces to a homogeneous differential equations

$$\nabla^4 w(x,y,t) + \frac{\rho h}{D}\ddot{w}(x,y,t) = 0,$$

where x and y are Cartesian coordinates in the plane of the middle surface, ρ density, w displacements and t time. For the case of freely vibrating plate, the external forces is zero and the effects of the rotational inertia forces are neglected. We assume the solutions of equation in the form

$$w(x,y,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\Phi_{ij}(x,y)\cdot\eta_{ij}(t) = \sum\sum\Phi_{ij}(x,y)\cdot A_{ij}\cdot\sin(\omega_{ij}t-\varphi_{ij}),$$

where

$$\Phi_{ij}(x,y) = X_i(x)\cdot Y_j(y)$$

Represents the shape function of the vibration, while the time-dependency of the displecements, $\eta(t)$, are assumed to be harmonic. The solution, $w(x,y,t)$, must satisfy the boundary conditions of the plate and the initial conditions of the motion at t=0; these conditions are $(w)_{t=0} = (\dot{w})_{t=0} = 0$. $X_i(x)$, represents the *i*th mode of a freely vibrating uniform beam, with length *a* and $Y_j(y)$ is the *j*th mode of a beam of length b. The equation of the normal free vibration modes

$$\nabla^4\Phi(x,y) = \lambda\Phi(x,y),$$

in which ∇^4 is the double Laplacean operator, together with the boundary conditions, represents a Sturm-Liouville problem, whose solving with the suggested method leads to the charateristics of pulsations and vibration shapes. By applying the variational Galerkin-Vlasov method, the problem of free flexural vibration of plates can be reduced to the solution of the following variational equations

$$\nabla^4\Phi_{ij}(x,y) = \lambda_{ij}\cdot\Phi_{ij}(x,y),$$

$$\iint_A\left[\nabla^4\Phi_{ij}(x,y) - \frac{\rho\cdot\omega_{ij}^{\ 2}}{D}\Phi_{ij}(x,y)\right]\cdot\Phi_{ij}(x,y)dxdy = 0.$$

By introducing the notations

$$I_1 = \iint_A [\Phi_{ij}(x,y) \cdot \nabla^4 \Phi_{ij}(x,y)] dxdy,$$

$$I_2 = \iint_A \Phi_{ij}^2(x,y) dxdy,$$

an approximate analytical expression for the pulsations of the free flexural vibration of plates with uniform thickness is

$$\omega_{ij}^2 = \frac{I_1}{I_2} \frac{D}{\rho} = \lambda_{ij} \frac{D}{\rho}.$$

where *i, j*, can takes the values 1,2,3,.... Introducing the ∇^4 the double Laplacean operator, in equation and separate the independent variabiles x and y, we obtained for the expression of the parameter of specific pulsations

$$X_i^{''''}(x) \cdot Y_j(y) + 2 \cdot X_i^{''}(x) \cdot Y_j^{''}(y) + X_i(x) \cdot Y_j^{''''}(y) = \lambda_{ij} \cdot X_i(x) \cdot Y_j(y)$$

$$\lambda_{ij} = \frac{\int_0^a X_i^{''''}(x) \cdot X_i(x)dx \cdot \int_0^b Y_j^2(y)dy + 2 \cdot \int_0^a X_i^{''}(x) X_i(x)dx \cdot \int_0^b Y_j^{''}(y) \cdot Y_j(y)dy + \int_0^a X_i^2 dx \cdot \int_0^b Y_j^{''''} \cdot Y_j(y)dy}{\int_0^a X_i^2(x)dx \cdot \int_0^b Y_j^2(y)dy}$$

The pulsations of plate can be calculated using the expression

$$\omega_{ij} = \frac{1}{a^2} \sqrt{\lambda_{ij} \frac{D}{\rho}}$$

RESULTS AND DISCUSSION

Through a careful analysis of field literature regarding the results obtained for the static and dynamic calculus of flat plates with different boundary conditions, there are to be noted different authors' concerns to elaborate a more exact calculus method that ensures an economical projection in safety conditions.

There has also been noted that most of the complications coming up in solving the flat plates are bound to the existence of free edges, these difficulties being reflected by the impossibility of finding some functions to describe the state of in-plate stress, with the rigurous consideration of the static conditions on a free edge.

The object of the paper consists of the study of the free vibration of rectangular wood flat plates having clamped edges. The analysed flat plates are thin, elastic and isotropic with stiffness at bending and meet the availability conditions of Kirchhoff hypotheses[8],[10]. The suggested calculus method is an adaptation of the variational method Galerkin-Vlasov from the static calculus of dynamic plates and has been elaborated in such a way that the calculi necessary for determining the dynamic characteristics of the plates are made on the basis of some programmes designed by the author himself. The normal vibration modes for the combinations of conditions at the ends of the beam are determined. The following cases are studied:

- doubly-clamped beam, having the shape function $y_i = G_i(x)$,

Considering the variational method Galerkin-Vlasov, we determining the vibration modes,self-pulsations ω_{ij} (represented by pulsation parameter $\sqrt{\lambda_{ij}}$)and the vibration functions $\Phi_{ij}(x,y)$

$$G_i(x) = \left(\cosh \beta_i \frac{x}{a} - \cos \beta_i \frac{x}{a}\right) - k_i \cdot \left(\sinh \beta_i \frac{x}{a} - \sin \beta_i \frac{x}{a}\right)$$

$$G_j(y) = \left(\cosh \beta_j \frac{y}{b} - \cos \beta_j \frac{y}{b}\right) - k_j \cdot \left(\sinh \beta_j \frac{y}{b} - \sin \beta_j \frac{y}{b}\right)$$

$$\Phi_{ij}(x,y) = G_i(x) \cdot G_j(y).$$

Bors [1] and Fetea [2], determinate the parameters for $\beta_i = \alpha_i \cdot l$ and k_i, for the 3 mode shape of a beam double clamped.

i	$\beta_i = \alpha_i \cdot l$	k_i
1	4,730041	0,982502
2	7,853205	1,000777
3	10,995608	0,999966

$$[(G_i''''(x)\cdot G_j(y)+2\cdot G_i''(x)\cdot G_j''(y)+G_i(x)\cdot G_j''''(y)]\cdot G_i(x)\cdot G_j(y)=\lambda_{ij}\cdot G_i^2(x)\cdot G_j^2(y)$$

$$\lambda_{ij}=\frac{\left(\frac{\beta_i}{a}\right)^4\int_0^a G_i''''\cdot G_i dx\cdot\int_0^b G_j^2 dy+2\cdot\int_0^a\left(\frac{\beta_i}{a}\right)^2 G_i'' G_i dx\cdot\int_0^b\left(\frac{\beta_j}{b}\right)^2 G_j''\cdot G_j dy+\int_0^a G_i^2 dx\cdot\int_0^b\left(\frac{\beta_j}{b}\right)^4 G_j''''\cdot G_j dy}{\int_0^a G_i^2 dx\cdot\int_0^b G_j^2 dy}$$

By introducing the notations $\alpha=\frac{b}{a}$, $u=\frac{x}{a}$, $v=\frac{y}{b}$, we obtain the parameter pulsation expression

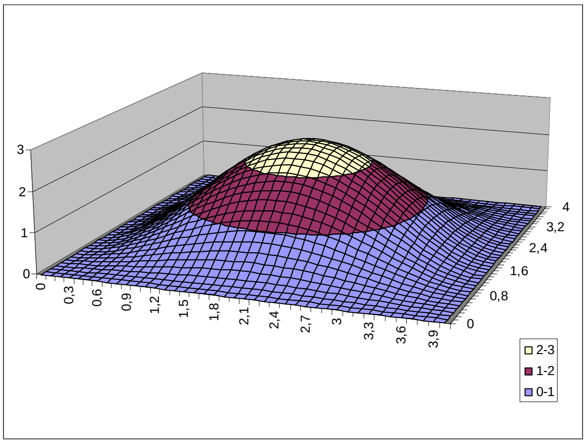
$$\lambda_{ij}=\frac{\frac{1}{a^4}[\beta_i^4\int_0^1 G_i''''\cdot G_i du\cdot\int_0^1 G_j^2 dv+2\cdot\alpha^2\cdot\beta_i^2\cdot\beta_j^2\int_0^1 G_i'' G_i du\cdot\int_0^1 G_j''\cdot G_j dv+\alpha^4\cdot\beta_j^4\int_0^1 G_i^2 du\cdot\int_0^1 G_j''''\cdot G_j dv}{\int_0^1 G_i^2 du\cdot\int_0^1 G_j^2 dv}$$

For the flat plate clamped on the boundary we determine the values of the specific pulsations adequate to the first 7 normal vibration modes considered. The values of the parameters of specific pulsations are presented in the table

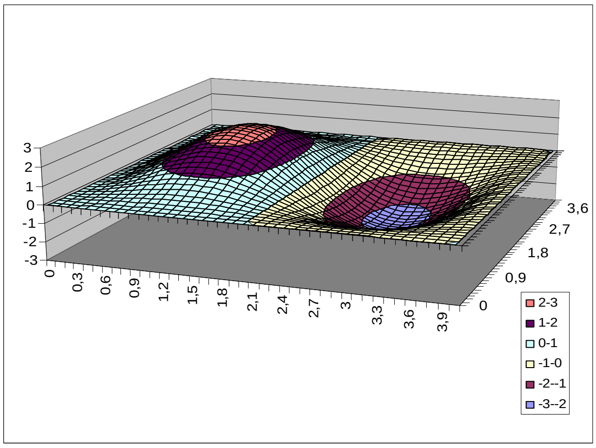
Vibration mode	Mode (1,1)	Mode (1,2)	Mode (1,3)	Mode (2,1)	Mode (2,2)	Mode (2,3)	Mode (3,1)
$\sqrt{\lambda_{ij}}$, $\alpha=1$	36,0036	74,52	129,21	74,52	108,97	165,2	129,21

For the rectangular plate having the edge ratio $\alpha = a/b = 2$, the value of fundamental pulsation parameter obtained by applying the suggested method is $\sqrt{\lambda_{11}} = 24{,}05$.

The modes shapes vibration and the functions $\Phi_{ij}(x,y)$ obtaining using the variational method are



Mode Shape 11

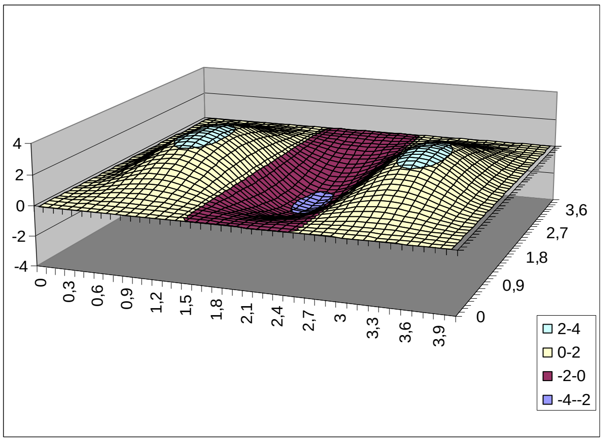


Mode Shape 12

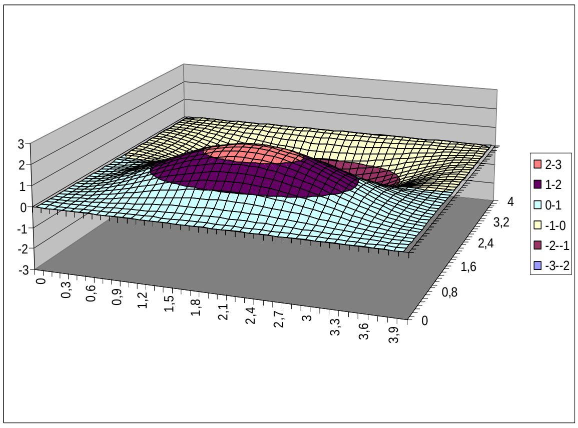
					Vibration functions mode (1,1)						
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,035	0,117	0,207	0,275	0,300	0,275	0,207	0,117	0,035	2,34E-06
0,8	0	0,117	0,3836	0,678	0,901	0,983	0,901	0,678	0,383	0,117	7,66E-06
1,2	0	0,207	0,678	1,201	1,595	1,740	1,595	1,201	0,678	0,207	1,36E-05
1,6	0	0,275	0,901	1,595	2,118	2,311	2,118	1,595	0,901	0,275	1,8E-05
2	0	0,300	0,983	1,740	2,311	2,522	2,311	1,740	0,983	0,300	1,96E-05
2,4	0	0,275	0,901	1,595	2,118	2,311	2,118	1,595	0,901	0,275	1,8E-05
2,8	0	0,207	0,678	1,201	1,595	1,740	1,595	1,201	0,678	0,207	1,36E-05
3,2	0	0,117	0,383	0,678	0,901	0,983	0,9015	0,678	0,383	0,117	7,66E-06
3,6	0	0,035	0,117	0,207	0,275	0,300	0,275	0,207	0,117	0,035	2,34E-06
4	0	2,3E-06	7,7E-06	1,36E-05	1,8E-05	1,96E-05	1,8E-05	1,36E-05	7,66E-06	2,34E-06	1,53E-10
4	0	2,3E-06	7,7E-06	1,36E-05	1,8E-05	1,96E-05	1,8E-05	1,36E-05	7,66E-06	2,34E-06	1,53E-10

					Vibration functions mode (1,2)						
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,086	0,228	0,284	0,195	1,49E-06	-0,195	-0,284	-0,228	-0,086	7,6E-05

0,8	0	0,282	0,747	0,932	0,640	4,87E-06	-0,640	-0,932	-0,747	-0,282	0,0002
1,2	0	0,499	1,322	1,650	1,133	8,61E-06	-1,133	-1,649	-1,322	-0,499	0,0004
1,6	0	0,663	1,756	2,191	1,505	1,14E-05	-1,505	-2,191	-1,756	-0,663	0,0005
2	0	0,723	1,916	2,390	1,643	1,25E-05	-1,643	-2,390	-1,916	-0,723	0,0006
2,4	0	0,663	1,756	2,191	1,505	1,14E-05	-1,505	-2,191	-1,756	-0,663	0,0005
2,8	0	0,499	1,322	1,650	1,133	8,61E-06	-1,133	-1,649	-1,322	-0,499	0,0004
3,2	0	0,282	0,747	0,932	0,640	4,87E-06	-0,640	-0,932	-0,747	-0,282	0,00024
3,6	0	0,086	0,228	0,284	0,195	1,49E-06	-0,195	-0,284	-0,228	-0,086	7,6E-05
4	0	5,64E-06	1,5E-05	1,8E-05	1,3E-05	9,7E-11	-1,3E-05	-1,9E-05	-1,5E-05	-5,6E-06	4,9E-09
4	0	5,64E-06	1,45E-05	1,8E-05	1,3E-05	9,7E-11	-1,3E-05	-1,9E-05	-1,5E-05	-5,6E-06	4,9E-09



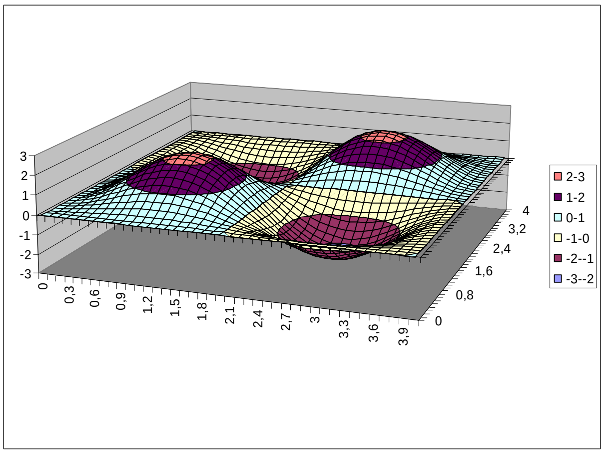
Mode Shape 13



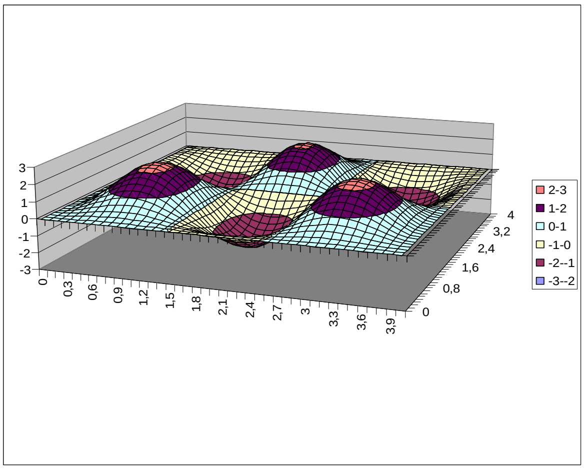
Mode Shape 21

		Vibration functions mode (1,3)									
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,145	0,285	0,164	-0,118	-0,265	-0,118	0,164	0,285	0,146	0,002
0,8	0	0,476	0,933	0,538	-0,389	-0,870	-0,389	0,538	0,934	0,479	0,008
1,2	0	0,843	1,652	0,952	-0,688	-1,540	-0,688	0,9525	1,654	0,848	0,014
1,6	0	1,120	2,194	1,264	-0,914	-2,046	-0,914	1,264	2,1967	1,127	0,019
2	0	1,222	2,394	1,379	-0,997	-2,232	-0,997	1,380	2,397	1,230	0,021
2,4	0	1,120	2,194	1,264	-0,914	-2,046	-0,914	1,264	2,196	1,127	0,019
2,8	0	0,843	1,652	0,952	-0,688	-1,540	-0,688	0,952	1,654	0,848	0,014
3,2	0	0,476	0,933	0,538	-0,389	-0,870	-0,389	0,538	0,934	0,4797	0,008
3,6	0	0,145	0,285	0,164	-0,118	-0,265	-0,118	0,164	0,285	0,146	0,002
4	0	9,5E-06	1,8E-05	1,1E-05	-7,8E-06	-1,7E-05	-7,8E-06	1,08E-05	1,9E-05	9,6E-06	1,7E-07

		Vibration functions mode (2,1)									
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,086	0,282	0,499	0,663	0,723	0,663	0,499	0,282	0,086	5,6E-06
0,8	0	0,228	0,747	1,322	1,756	1,916	1,75636	1,322	0,747	0,228	1,5E-05
1,2	0	0,284	0,932	1,650	2,191	2,390	2,191189	1,650	0,932	0,284	1,8E-05
1,6	0	0,195	0,640	1,133	1,505	1,643	1,505763	1,133	0,640	0,195	1,23E-05
2	0	1,5E-06	4,9E-06	8,6E-06	1,1E-05	1,25E-05	1,14E-05	8,6E-06	4,9E-06	1,5E-06	9,7E-11
2,4	0	-0,195	-0,640	-1,133	-1,505	-1,643	-1,50573	-1,133	-0,640	-0,195	-1,3E-05
2,8	0	-0,284	-0,932	-1,649	-2,191	-2,390	-2,19113	-1,649	-0,932	-0,284	-1,9E-05
3,2	0	-0,228	-0,747	-1,322	-1,756	-1,916	-1,75624	-1,322	-0,747	-0,228	-1,5E-05
3,6	0	-0,086	-0,282	-0,499	-0,663	-0,723	-0,66304	-0,499	-0,282	-0,086	-5,6E-06
4	0	7,6E-05	0,0002	0,0004	0,0005	0,0006	0,000584	0,0004	0,0002	7,6E-05	4,9E-09



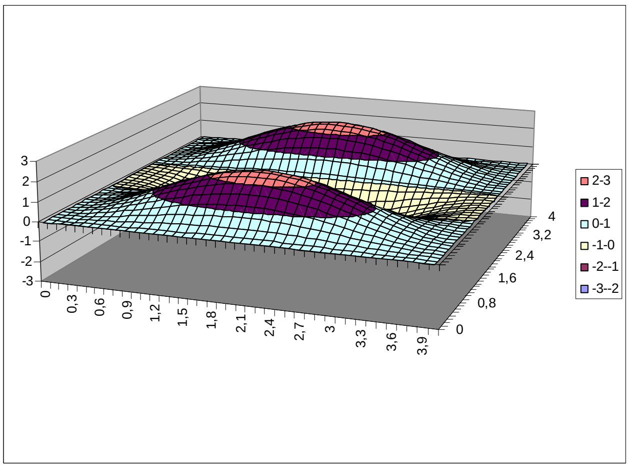
Mode Shape 22



Mode Shape 23

		Vibration functions mode (2,2)								
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,207	0,549	0,686	0,471	3,6E-06	-0,471	-0,686	-0,549	-0,207	0,0001
0,8	0	0,549	1,456	1,816	1,248	9,5E-06	-1,248	-1,816	-1,456	-0,549	0,0004
1,2	0	0,686	1,816	2,266	1,557	1,2E-05	-1,557	-2,266	-1,816	-0,685	0,0006
1,6	0	0,471	1,248	1,557	1,070	8,1E-06	-1,070	-1,557	-1,248	-0,471	0,0004
2	0	3,6E-06	9,5E-06	1,2E-05	8,1E-06	6,1E-11	-8,1E-06	-1,2E-05	-9,5E-06	-3,6E-06	3,2E-09
2,4	0	-0,471	-1,248	-1,557	-1,070	-8,1E-06	1,070	1,557	1,248	0,471	-0,0004
2,8	0	-0,686	-1,816	-2,266	-1,557	-1,2E-05	1,557	2,266	1,816	0,685	-0,0006
3,2	0	-0,549	-1,456	-1,816	-1,248	-9,5E-06	1,248	1,816	1,456	0,549	-0,0004
3,6	0	-0,207	-0,549	-0,685	-0,471	-3,6E-06	0,471	0,685	0,549	0,207	-0,0001
4	0	0,0001	0,0004	0,0006	0,0004	3,1E-09	-0,0004	-0,0006	-0,0004	-0,0001	1,6E-07

		Vibration functions mode (2,3)								
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,3509	0,6871	0,3958	-0,2863	-0,6404	-0,2862	0,3960	0,6878	0,3529	0,0061
0,8	0	0,9292	1,8195	1,0482	-0,7582	-1,6966	-0,7580	1,0488	1,8213	0,9346	0,0161
1,2	0	1,1593	2,2700	1,3077	-0,9459	-2,1166	-0,9457	1,3084	2,2722	1,1660	0,0201
1,6	0	0,7966	1,5599	0,8986	-0,6500	-1,4545	-0,6499	0,8991	1,5614	0,8012	0,0138
2	0	6,0E-06	1,2E-05	6,8E-06	-4,9E-06	-1,1E-05	-4,9E-06	6,8E-06	1,2E-05	6,1E-06	1,0E-07
2,4	0	-0,7966	-1,5599	-0,8986	0,6500	1,4545	0,6498	-0,8991	-1,5614	-0,8012	-0,0138
2,8	0	-1,1592	-2,2699	-1,3077	0,94594	2,1165	0,9457	-1,3084	-2,2722	-1,1660	-0,0202
3,2	0	-0,9291	-1,8194	-1,0481	0,75819	1,6964	0,7580	-1,0487	-1,8212	-0,9345	-0,0161
3,6	0	-0,3508	-0,6869	-0,3957	0,28624	0,6404	0,2861	-0,3954	-0,6875	-0,3528	-0,0061
4	0	0,0003	0,0006	0,0003	-0,0002	-0,0005	-0,0002	0,0003	0,0006	0,0003	5,4E-06



Mode Shape 31

		Vibration functions mode (3,1)									
y/x	**0**	**0,4**	**0,8**	**1,2**	**1,6**	**2**	**2,4**	**2,8**	**3,2**	**3,6**	**4**
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,1456	0,4769	0,8439	1,1207	1,2225	1,1207	0,8439	0,4769	0,1456	9,53E-06
0,8	0	0,2851	0,9339	1,6525	2,1945	2,3946	2,1945	1,6525	0,9339	0,2851	1,87E-05
1,2	0	0,1663	0,5380	0,9520	1,2642	1,3795	1,2642	0,9520	0,5380	0,1642	1,07E-05
1,6	0	-0,1188	-0,3891	-0,6886	-0,914	-0,9979	-0,9145	-0,688	-0,3891	-0,1188	-7,8E-06
2	0	-0,2658	-0,8708	-1,5409	-2,046	-2,2328	-2,0462	-1,5409	-0,8708	-0,2658	-1,7E-05
2,4	0	-0,1187	-0,3891	-0,6885	-0,914	-0,9976	-0,9143	-0,6885	-0,3891	-0,1188	-7,8E-06
2,8	0	0,1643	0,5383	0,9525	1,2649	1,3803	1,26497	0,95256	0,5383	0,1643	1,08E-05
3,2	0	0,2854	0,9348	1,6542	2,1967	2,3970	2,19673	1,6542	0,9348	0,2854	1,87E-05
3,6	0	0,1464	0,4797	0,8488	1,1272	1,2300	1,1272	0,8488	0,4797	0,1464	9,58E-06
4	0	0,0025	0,0083	0,0147	0,0195	0,0213	0,0195	0,0147	0,0083	0,0025	1,66E-07

CONCLUSIONS

From the study of field literature it has been noted that there is no data concerning the results obtained by other authors regarding the values of the shape functions, respectively the parameters of the rectangular plate pulsations and the pulsations proper by means of applying the Galerkin-Vlasov variational method, a reason why the results presented in the thesis represent a novelty element brought by the author.

By means of the dynamic analysis of wood plates there has been an emphasis on the complexity of the notion of dynamic calculus, which has the following as main working stages:

- establishing the dynamic model considered for the first time;
- determining the normal vibration modes (self pulsations and vectors, respectively the functions of vibration modes);
- determining the dynamic response in displacements and sectional stresses;
- checking stability and strength conditions.

There is a presentation of the percentage deviations of the vibration parameters determined by applying the suggested method in comparison to the ones determined by other authors through the use of the analytical, variational and numerical methods.

From the analysis of present data, we note that for the rectangular flat plates considered in the thesis, the percentage deviations are in the limits of high precision. To validate the suggested method, we compare the achieved theoretical results with the ones determined by other authors when applying differents methods. Comparing the values of parameters of specific pulsations, we note that the percentage deviations of the pulsation parameters determined through the suggested method are the lightest in the case of the normal vibration modes (1,1), (2,1), (2,3), (3,1), and the biggest for the modes (1,2), (1,3), (2,2), in comparison to the ones obtained by other authors

For the rectangular plate with the edge ratio $\alpha = 2$ of the plate, the authors have calculated the fundamental pulsation parameter using type Ritz solutions, its value being $\sqrt{\lambda_{11}} = 23{,}994$. For the same type of wood plate having the same edge ratio, Iguchi [5], Timoshenko [9], obtained the value the of fundamental pulsation parameter $\sqrt{\lambda_{11}} = 24{,}56$. The perecentage deviation in the value of the fundamental pulsation parameter determined through the suggested method in comparison to the value determined by Iguchi [5] and Leissa[6] using the Rayleigh-Ritz method is of 2,24 %, respectively 0,23 %, in comparison to the value determined by appplying type Ritz solutions.

The making of constructions, machines and different high-perfomance appliances, whose functioning should take place in safety conditions, have required theoretical studies of rich complexity, as well as practical experiments, within which the problem of their free and forced vibrations represent an important category in the respective theme of research. The importance of studying the vibrations of different deformable material systems (elastic systems in constructions, technological equipments, mobile or stationary machines and equipments), whose structures take in types of plates different in terms of shape, loading mode and boundary conditions characterised by forced or free vibration motions and carried on to the structure itself, has been made obvious by the system's degradation in time.

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