International Journal of Horticultural Science 2009, 15 (1–2): 123–126
Agroinform Publishing House, Budapest, Printed in Hungary
ISSN 1585-0404

Evaluation of a simple fruit tree structural model

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Summary: A simple three element tree structure model of Láng, 2006 was tested in plum orchard using two different inertia fruit tree shakers. The first was a slider crank type one; the second had rotating eccentric weights. The parameters of both were chosen to give similar frequency and amplitude output in average orchard conditions. Orchard experiments were carried out shaking the trees with both machines at several frequencies and shaking heights. The measured acceleration and amplitude values were plotted on diagrams together with the calculated acceleration and amplitude curves of the fruit tree-shaker machine model. Choosing the right fruit tree parameters, such as apparent spring constant, damping coefficient, reduced trunk mass and coefficient of elasticity of the trunk the measured and calculated values coincided well. This proves the ability of the fruit tree model for optimising the shaker parameters to any given orchard.

Notation:

b	horizontal distance of the root ends from the centre line of the trunk, m
C	the virtual turning centre of the tree model,
a	acceleration, ms^{-2}
c	apparent spring constant of the limb at the spot of shaking, mN^{-1}
h	the vertical depth of the nodes A and B underneath the soil surface, m
k	viscous damping coefficient of the limb at the spot of shaking, Nsm^{-1}
K	constant
M_f	mass of the shaker frame, kg
M	reduced mass of the tree limb at the clamping point of the shaker, kg
M_{red}	reduced mass of the mass of M to the root ends, kg
M_t	total mass of the limb-shaker system, including M_f, M and m, kg
m	total unbalanced masses, kg
O	trunk position on the soil surface,
r	eccentricity of unbalanced masses, m
t	time, s
X	amplitude of displacement in horizontal direction, m
y	vertical distance from ground level, m
ρ	vertical distance of the virtual turning centre C from O, m
ω	shaking frequency, rad s^{-1}

Key words: fruit tree modeling, shaker, harvesting, frequency and amplitude

Introduction

At shaker harvest fruit detachment is mostly influenced by the frequency and amplitude of the upper end of their stem. Modelling the fruit tree may supply reliable data for the shaker design, concerning shaking frequency, amplitude and the size of masses, taking part in the shaking process.

In the differential equation of the fruit tree-shaker system of Fridley and Adrian (1966) the tree was replaced by a three-element model, which was vibrated by a sinusoidal changing force, generated by unbalanced masses (*Fig. 1.*):

$$M_t\ddot{x}_M + k\dot{x}_M + \frac{1}{c}x_M = mr\omega^2 \sin\omega t \qquad (1)$$

where: M_t is the total mass of the limb-shaker system in kg;
$\ddot{x}_M$ is the trunk acceleration in ms^{-2};
k is the viscous damping coefficient of the limb in Nsm^{-1};
$\dot{x}_M$ is the limb velocity in ms^{-1};
c is the apparent spring constant of the limb in mN^{-1};
x_M is the limb displacements in horizontal direction in m;
m is the total unbalanced masse of the shaker in kg;
r is the eccentricity of the unbalanced masses in m;
ω is the shaking angular frequency in rad s^{-1};
t is the time in s.

For the calculation of the trunk displacement amplitude X, the following well known equation can be used:

$$X = \frac{mr\omega^2}{\sqrt{\left(\frac{1}{c} - M_t\omega^2\right)^2 + (k\omega)^2}} \tag{2}$$

The peak acceleration of the vibrating trunk is:

$$a_x = X \cdot \omega^2 \tag{3}$$

Horváth & Sitkei (2001) presumed that during shaking the input energy is mostly absorbed in the soil through the rooting system therefore, the trunk cannot be regarded as a vertical cantilever. It translates and turns during shaking and vibrates a certain amount of soil around the tree. They measured the translations of the tree at shaking the trunk in different heights, than calculated the virtual centre of turning of it. It was found that the location of this centre changes with the height of shaking, and so does the reduced mass measured at the clamping points. Their conclusion was that the increase of the reduced mass means increasing mass of soil vibrating with the trunk. Evaluating run-out acceleration curves of a trunk shaker the logarithmic decrements for different trunk cross-sections was defined and compared with data obtained by a presumption, based on the relation of reduced masses of soil and canopy (*Horváth & Sitkei*, 2002).

Láng (2003, 2006) presented a three element tree structure model built of elastic trunk and main roots. The model enabled the calculation of the virtual centre of turning in function of the height of force applied. It made possible also the transfer of a defined reduced mass, apparent spring constant and viscous damping coefficient value of one trunk cross section to any other. Using Eqn. 1 the shaker machine –tree interaction could be described for any shaking height on the trunk.

In this paper the model of *Lang*, 2006 was tested in a plum orchard. Two different inertia type shaker were built an used in the tests by shaking the trees at several frequencies and shaking heights. Orchard tests and calculated model results are compared and conclusions are drawn for the design of shaker machines.

Materials and methods

The three element tree structure model

The simple fruit tree model of *Láng*, 2006 is shown on *Figure 1*.

The function, describing the position of virtual turning centre C is

$$\rho = \frac{b^2}{y+h} \tag{4}$$

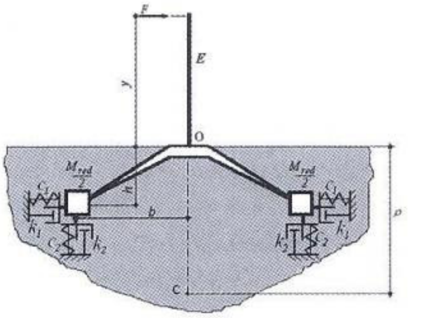

Figure 1. The three element tree structure model

where b is the distance between the centre line of the model and the end of the main root,
h is the depth of the end of the main root,
y is the shaking height on the trunk

The function of the reduced tree mass in function of shaking height y is

$$M(y) = \frac{(y_{def} + \rho_{def})^2}{(y+\rho)^2} \frac{(\rho - h)^2 + b^2}{(\rho_{def} - h)^2 + b^2} M_{def} \tag{5}$$

where M_{def} is a defined reduced mass value at y_{def} shaking height and at ρ_{def} virtual turning centre position

The damping coefficient at any y shaking height is

$$k(y) = \frac{(y_{def} + \rho_{def})^2}{(y+\rho)^2} \frac{(\rho - h)^2 + b^2}{(\rho_{def} - h)^2 + b^2} k_{def} \tag{6}$$

where k_{def} is the viscous damping coefficient value at y_{def} shaking height and at ρ_c virtual turning centre position

For any y height of the trunk the resultant spring constant can be calculated as follows

$$c(y) = c'(y) + c''(y) \tag{7}$$

The first part of the sum is due to the elastic turning of the main roots

$$c'(y) = \frac{(y+\rho)^2}{(\rho - h)^2 + b^2} \frac{(\rho_{def} - h)^2 + b^2}{(y_{def} + \rho_{def})^2} \frac{c'_{def}}{2} \tag{8}$$

where c'_{def} is the defined spring constant of the main roots measured at y_{def} shaking height and at ρ_{def} virtual turning centre position

The second part of Eqn. 7 comes from the elastic bending of the trunk at a known y height

$$c''(y) = \frac{y^3}{3 \cdot I \cdot E} \tag{9}$$

Now the trunk amplitude at y ρ_{def} shaking height can be calculated, using Eqn. 2

$$X(y) = \frac{m \cdot r \cdot \omega^2}{\sqrt{\left(\frac{1}{c''(y)} - (M(y) + m + M_f) \cdot \omega^2\right)^2 + (k(y) \cdot \omega)^2}} \qquad (10)$$

where M_f is the mass of the shaker machine frame

The equation of trunk peak acceleration in function of any y shaking height gives the transformed Eqn. 3 as follows

$$a_x(y) = X(y) \cdot \omega^2 \qquad (11)$$

Eqns. 10 and 11 describe the amplitude and acceleration of the trunk not only in function of shaking height but also in function of the position of virtual turning centre ρ, and of the constants K_1 K_2 and K_3 as well as of trunk diameter and modulus of elasticity.

The inertia shaker machines

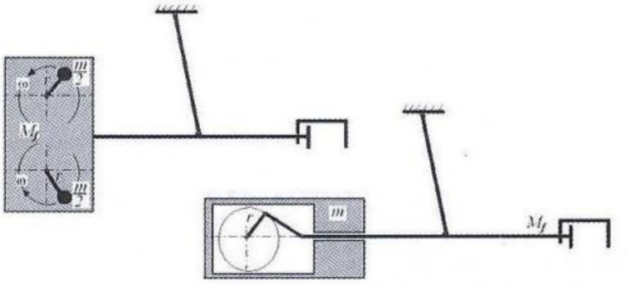

Figure 2. The two inertia type shakers in test

In the orchard test two inertia type shakers were used. Both were rod type machines, one of them supplied with slider crank type shaker unit, the other one with two rotating eccentric masses *(Fig. 2)*.

The main parameters of the two units are summarised in *Table 1*. Those were chosen to give similar amplitudes and acceleration when shaking average size trees at the same frequencies.

Table 1. The main parameters of the two shaking units

Shaker type	m kg	M_f kg	r cm
Slider crank	111	75,2	3,5
Rotating eccentric masses	43,5	180,5	9,4

Field tests

Plum trees of a 6 year old orchard were shaken by the two machines at different frequencies and shaking heights. Trunk diameters of the trees in test ranged between 11,8 to 13,8 cm. Accelerometer was fixed on the shaker boom and acceleration versus time curves were registered in each test.

Shaking frequencies were set up by changing the PTO speed and ranged between 6 and 14 Hz.

During evaluation of the acceleration versus time curves the average acceleration peaks were read and the amplitudes were calculated as the second integral of the peak acceleration – time function.

Definition of the model tree parameters

The model parameters were calculated using multi-variant iteration. From earlier tests and experiments the initial values for the root's end position were taken for *b*=600 mm and *h*=185mm. For the trunk's modulus of elasticity $E=10^{10}$ Pa was chosen.

The parameters M_{def}, K_{def}, and c'_{def} were estimated by drawing peak acceleration-time and amplitude-time curves and comparing them with measured data. Parameters were changed until the measured and calculated values coincided well in all test arrangements.

Results and discussion

Figure 3 shows the calculated (Eqns. 2–11) peak acceleration versus shaking height curves of a tested plum tree shaken by the slider crank type machine at 10,5 Hz and by the shaker with rotating eccentric masses at 7,1 Hz. The measured peak accelerations for the two machines at 3 different shaking heights are also plotted on the diagram. Shaking was carried out at 690, 520 and 280 mm trunk heights.

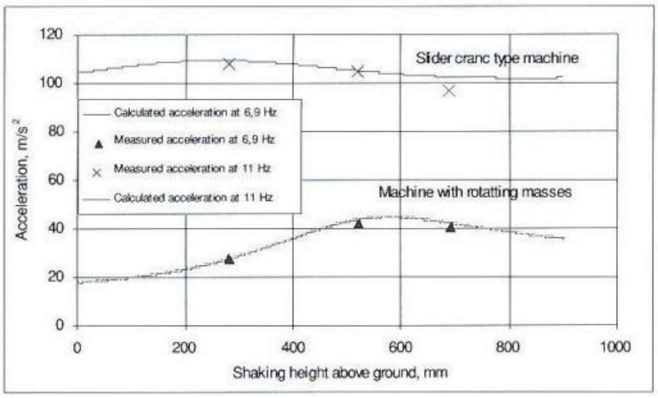

Figure 3. Calculated and measured peak accelerations on the trunk of a plum tree shaken by the two type inertia shakers at 3 different heights

Figure 4, 5 and 6 show the calculated peak acceleration vs. frequency and amplitude vs. frequency curves for the tree trunk in *Figure 3* at the shaking heights 690, 520 and 280 mm respectively. Measured peak acceleration values at different shaking frequencies as well as the trunk amplitudes, calculated from them are also plotted on the diagrams.

For the tree introduced here *h*=800 mm, *b*=190 mm and $E=10^{10}$ Pa proved to be the best value. In the equations 5, 6 and 8 the reduced mass M, damping coefficient k and spring constant of the roots c measured at 80 cm trunk height was replaced. In the case of the tree presented here *M(80)*= 120 kg, *k(80)*=5000 Ns/m and *c(80)*=0,0016 mm/N gave the best fitting.

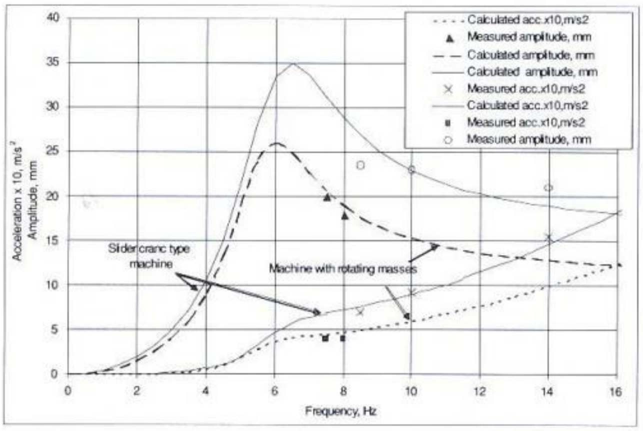

Figure 4. Calculated peak acceleration vs. frequency and, amplitude vs. frequency curves, measured accelerations and amplitudes of the tree trunk at the height of 690 mm when shaken by the two types of machine

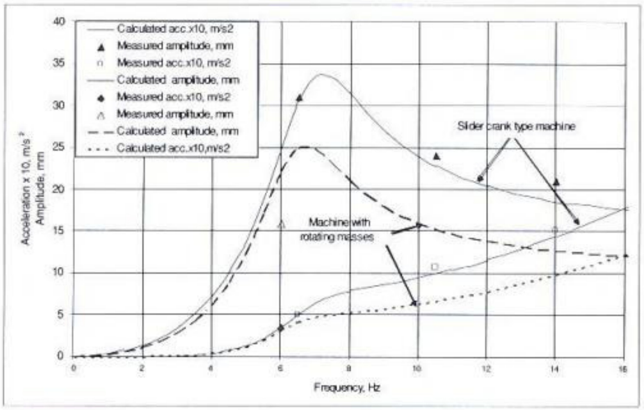

Figure 5. Calculated peak acceleration vs. frequency and, amplitude vs. frequency curves, measured accelerations and amplitudes of the tree trunk at the height of 520 mm when shaken by the two types of machines

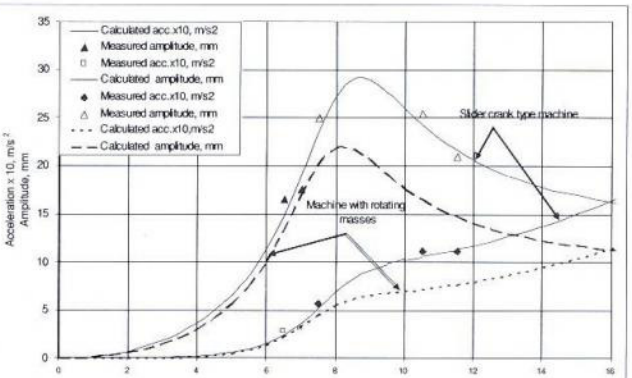

Figure 6. Calculated peak acceleration vs. frequency and, amplitude vs. frequency curves, measured accelerations and amplitudes of the tree trunk at the height of 280 mm when shaken by the two types of machines

Similar results were achieved at two other trees involved in the experiments.

Taking in account the geometric asymmetry of real fruit trees the calculated and measured values coincide acceptably. This means that the fruit tree model introduced above is an appropriate tool for fitting the shaker machine to the tree parameters more accurately than before.

Conclusion

According practical experiences the fruit detachment efficiency is influenced by the frequency of shaking and the amplitude and acceleration of the fruit bearing branches.

As *Figs. 4–6* show increasing frequency results not necessarily in increasing acceleration and amplitudes. The above Figures show also the natural frequency of the tree-machine system, which changes with the shaking height, due to the changes of all parameters involved.

Following from Eqns. 10 and 11 there are four parameters by which the amplitude and acceleration can be influenced: the frequency of shaking, the inertia mass, the mass of the machine frame and the height of shaking. Using the tree parameters M, k and c, the desired amplitude and acceleration at a shaken trunk height can be set by choosing appropriately the four parameters free to change.

As an example the resulting amplitude and acceleration of the shaker with rotating masses in *Figure 4* can be improved by reducing the mass of machine frame from 180,5 to 110 kg (*Figure 7*). This seems to be technically the simplest method for changing the output of the shaker machine.

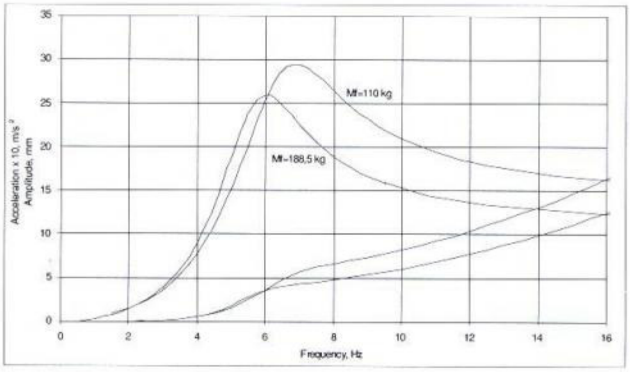

Figure 7. The effect of machine frame mass reduction to the trunk amplitude and acceleration

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