

# A simple model for fruit tree shaking harvest

Láng, Z.

Technical Department, Corvinus University, Budapest, Villányi u. 31, 1118 Budapest, Hungary;  
e-mail: zoltan.lang@uni-corvinus.hu

**Summary:** A tree structure model was composed of trunk and main roots. It included a mass, spring and damping element, all of them reduced to the external end of the main roots. The model parameters, such as virtual turning centre, reduced mass, spring constant and damping coefficient were measured on a real cherry tree. The model was then virtually shaken at 80 cm trunk height and acceleration and displacement amplitudes versus shaking frequency were calculated. The real cherry tree was shaken also at 80 cm trunk height by an inertia type shaker machine and the same data were recorded. The acceleration amplitude vs. frequency and displacement amplitude vs. frequency functions were similar for the virtual and real tree which proves the ability of the model. Power demand and specific power demand was then calculated in function of shaking frequency. The diagrams show that the shaking frequency of 12–14 Hz of the practice is not the most efficient concerning amplitude, but is probably necessary from the point of view of acceleration needed to detachment of fruits.

**Key words:** fruit tree modeling, shaker, harvesting, optimal frequency

## Introduction

The most widespread harvesting technology for stone fruits is mechanical shaking of the tree. Both limb and trunk shaking are practised. In the first case more main branches must be shaken. This method is slower, but the removal is more perfect. It also causes less damage to the tree.

Some machines shake the tree almost at soil level to be able to place the catching surface above the shaker head. Others make it opposite. The shaker is attached at 80–90 cm high on the trunk and the catching surface is placed on the soil surface. It is however not clear, which is the better technique from the point of view of efficiency.

At shaker harvest fruit detachment is mostly influenced by the frequency and amplitude of the upper end of their stem. Modelling the fruit tree may supply reliable data for the shaker design, concerning shaking frequency, amplitude and the size of masses, taking part in the shaking process.

In the differential equation of the fruit tree-shaker system of *Fridley & Adrian* (1966) the tree was replaced by a three-element model, which was vibrated by a sinusoidal changing force, generated by unbalanced masses (*Figure 1*):

$$M_l \ddot{x}_M + k \dot{x}_M + \frac{1}{c} x_M = mr \omega^2 \sin \omega t \quad (1)$$

where:  $M_l$  is the total mass of the limb-shaker system in kg;  $\ddot{x}_M$  is the limb acceleration in  $\text{ms}^{-2}$ ;  $k$  is the viscous damping coefficient of the limb in  $\text{Ns m}^{-1}$ ;  $\dot{x}_M$  is the limb velocity in  $\text{ms}^{-1}$ ;  $c$  is the apparent spring constant of the limb in  $\text{mN}^{-1}$ ;  $x_M$  is the limb displacements in horizontal direction in m;  $m$  is the total unbalanced masse of the shaker in kg;  $r$  is the eccentricity of the unbalanced masses in m;  $\omega$  is the shaking frequency in  $\text{rad s}^{-1}$ ;  $t$  is the time in s.

For the calculation of the trunk displacement amplitude  $X$ , the following well known equation can be used:

$$X = \frac{mr \omega^2}{\sqrt{\left(\frac{1}{c} - M_l \omega^2\right)^2 + (k \omega)^2}} \quad (2)$$

The maximal acceleration of the vibrating trunk is:

$$a_x = X \cdot \omega^2 \quad (3)$$

As no data of damping coefficient and spring constant in function of shaking location was available, *Fridley & Adrian* (1966) suggested a simple function for the calculation of the limb peak-to-peak stroke  $S$  in m:

$$S = 2 \cdot X \cong \frac{2mr}{M_l} \quad (4)$$

It assumed that the shaking frequency is much higher, than the fundamental mode frequency of the limb. In lack of calculation method for the total mass  $M_l$ , it was estimated.

An attempt to include the viscous damping and elasticity parameters in the model was made by *Whitney et al.* (1990). The reduced mass, apparent spring constant and the viscous damping coefficient were measured individually on a wooden post fixed to the ground as a vertical cantilever. The data achieved were controlled in shaking experiences: an inertia type trunk shaker was clamped to the post and displacement and acting force were measured and calculated.

Comparing measured and calculated data, *Whitney et al.* (1990) found that the post acted nearly as a pure spring at the frequencies employed. They found that the trunk itself is rather elastic; a great part of the input energy during shaking harvest must be absorbed elsewhere.

*Láng* (2003, 2006) presented a three element rigid tree model built of trunk and main roots. As an average of many observations, the roots declined at  $17^\circ$  to the horizon. The

external ends of them were fixed to the soil through visco-elastic joints. Its mass was regarded constant, its elasticity and damping property resulted from the visco-elastic joints. The model enabled the calculation of the virtual centre of turning in function of the height of force applied. It made possible also the transfer of a defined reduced mass  $M_{def}$ , apparent spring constant  $c'_{def}$  and viscous damping coefficient  $k_{def}$  value of one trunk cross section to any other. Than using Eqn. 1 the shaker machine – tree interaction could be described for any shaking height on the trunk. For the calculation of trunk displacement amplitude  $X$  Eqn. 2 was used. For the average effective power demand of the shaker Eqn. 5 was applied:

$$P_{s,av} = \frac{1}{2} X^2 k \omega^2 \tag{5}$$

For the characterisation of the shaking process the specific power, as the ratio of power demand and trunk displacement amplitude was introduced (Láng 2006).

In this paper, a more sophisticated version of the above three element fruit tree model is presented and its behaviour is compared with that of a real tree. Amplitude, acceleration, power demand and specific power demand values are calculated and measured for different shaking frequencies.

## Materials and methods

### The improved fruit tree model

The model suggested by Láng (2006) was completed so that the elasticity of the trunk was also taken into account (Figure 1).

The basic equations for the new model are:

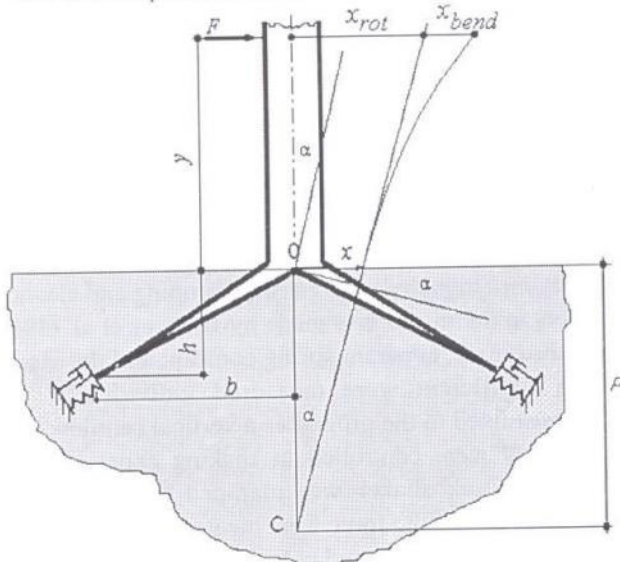


Figure 1. The improved model of trunk and main roots

1. The vertical distance  $\rho$  in m of the virtual turning centre C from O is:

$$\rho \cong \frac{b^2}{y+h} \tag{6}$$

where  $b$  and  $h$  are constants: the coordinates of the root's external ends.

2. The reduced mass  $M(y)$  for any  $y$  cross-section of the trunk can be calculated from a defined  $M_{def}$  value at  $y_{def}$  (see Figure 2):

$$M(y) = \frac{(y_{def} + \rho_{def})^2}{(y + \rho)^2} \frac{(\rho - h)^2 + b^2}{(\rho_{def} - h)^2 + b^2} M_{def} \tag{7}$$

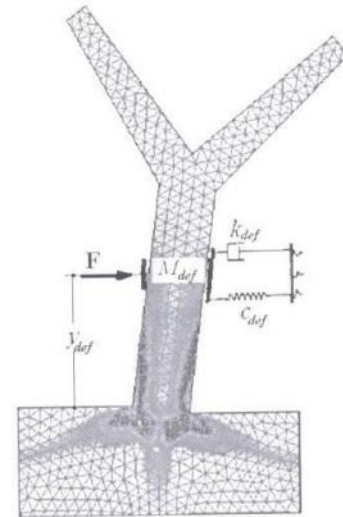


Figure 2. The explanation of defined trunk parameters

3. The horizontal displacement of the trunk during shaking may be composed of two parts: the one is due to rotation:  $x_{rot}$ , the other to the bending of the trunk:  $x_{bend}$  (Figure 1).

For the rotation:

$$x_{rot} = (\rho + y) \cdot \alpha = F \cdot c'(y) \tag{8}$$

where

$$c'(y) = \frac{(y + \rho)^2}{(\rho - h)^2 + b^2} \frac{(\rho_{def} - h)^2 + b^2}{(y_{def} + \rho_{def})^2} \frac{c'_{def}}{2} \tag{9}$$

It means that a measured spring constant  $c'_{def}$  at  $y_{def}$  can be transferred to any  $y$  trunk height.

The horizontal displacement of the trunk due to its elastic bending at  $y$  is:

$$x_{bend} = \frac{y^3}{3 \cdot I \cdot E} \cdot F \tag{10}$$

and the spring constant:

$$c''(y) = \frac{y^3}{3 \cdot I \cdot E} \tag{11}$$

The resulting spring constant for any trunk cross section is the sum of the two:

$$c(y) = c'(y) + c''(y) \tag{12}$$

4. Similarly to the mass reduction, a defined  $k_{def}$  value at  $y_{def}$  can be transferred to any other  $y$  height:



$$k(y) = \frac{(y_{def} + \rho_{def})^2}{(y + \rho)^2} \frac{(\rho - h)^2 + b^2}{(\rho_{def} - h)^2 + b^2} k_{def} \quad (13)$$

**Field tests**

To be able to test the model its parameters had to be measured. Tests were carried out in a ten years old cherry orchard on a vase form trees.

The method for calculating  $c'_{def}$  is shown on the Figure 3. A rigid finger was fixed on the bottom of the trunk. It turned together with the rooting system when horizontal force was applied. The horizontal displacement  $x_{rot}$  divided by the force  $F$  resulted  $c'_{def}$ . The average of 3 tests with different  $F$  values gave  $c'_{def} = 7.6 \cdot 10^{-4}$  mm/N.

To the calculation of the modulus of elasticity  $E$  it was expressed from Eqn. 10. For the examined tree with 135 mm trunk diameter  $E$  equals  $0.98 \cdot 10^{10}$  Pa.

The virtual turning centre of the tree could be calculated for the given circumstances, whereby the following equation applies (Figure 3):

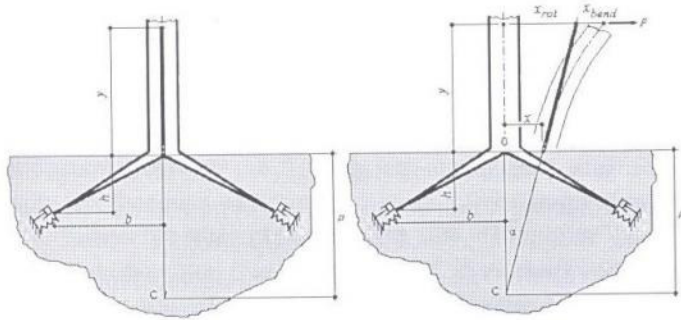


Figure 3. The method for calculating  $c'_{def}$

$$k(y) = \frac{(y_{def} + \rho_{def})^2}{(y + \rho)^2} \frac{(\rho - h)^2 + b^2}{(\rho_{def} - h)^2 + b^2} k_{def} \quad (14)$$

To define the constants  $b$  and  $h$  in Eqn. 6,  $\rho$  was calculated using Eqn. 14. As an average of 5 tests on the same tree  $\rho$  was equal to 366 mm when the force was applied at  $y = 800$  mm. Taking into consideration the  $17^\circ$  decline of the main roots,  $b$  can be expressed from Eqn. 6 ( $y = 800$  mm,  $h = b \cdot \text{tg}17^\circ$ ):  $b = 600$  mm. Using the constants  $b$  and  $h$   $\rho$  can be calculated for any  $y$  value.

The reduced mass of the tree at 80 cm trunk height was defined using Rayleigh's method (Lång, 2006). For the given tree of 135 mm trunk diameter  $M_{def}$  was equal to 230 kg.

To get the  $k_{def}$  value mechanical harvesting tests were carried out. On the trunk of the examined tree an accelerometer was fixed in the direction of shaking. Seven different shaking frequencies were applied ranging from 4.9 to 15 Hz. In both shaking and run out period accelerations versus time diagrams were recorded.

Presumed that the shaker machine and tree behave as a damped one degree-of freedom system the evaluation of the acceleration versus time diagrams followed the well known method.

The linear damping coefficient:

$$k_{def} = \frac{2 \cdot M_{y_{total}} \cdot A}{t_c} \quad (15)$$

where  $M_{y_{total}}$  is the sum of reduced masses at  $y$  participating in the vibration

$$A = \frac{\Delta y}{y_n} \quad \text{is the logarithmic decrement (see Figure 4)}$$

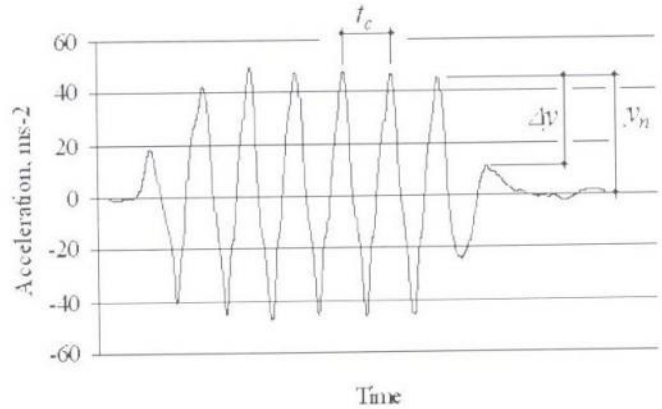


Figure 4. Method for the calculation of linear damping coefficient

The average of measured  $k_{def}$  values at seven different frequencies gave 6400 Ns/m.

From the acceleration versus time diagrams the acceleration maximums at each of 7 frequencies were measured and recorded.

Twice deriving the acceleration versus time diagrams trunk displacements were obtained for the frequencies examined. Their maximums were also recorded.

**Results**

To be able to draw the amplitude vs. frequency, acceleration vs. frequency and power demand vs. frequency curves for the tree model at 80 cm trunk height, the measured  $M_{def}$ ,  $c'_{def}$  and  $k_{def}$  values, as well as the shaker machine parameters had to be replaced into Eqns (2), (3) and (5).

In Eqn. (2) the total mass of the system  $M_t$  includes the reduced tree mass  $M_{def}$ , the total unbalanced masse  $m$  of the shaker and the mass of the machine frame, attached to the trunk  $M_f$ :

$$M_t = M_{def} + m + M_f = 420 \text{ kg} \quad (16)$$

The eccentricity of unbalanced masses,  $r = 25$  mm.

The accelerations calculated for the model tree (Eqn 3) and measured on a real tree ( $\square$ ) are shown in the frequency range 0–17 Hz on the Figure 5.

Figure 5 includes also the calculated (Eqn. 2) and measured ( $\diamond$ ) amplitudes for the same frequency interval. In both cases there seems to be one peak is the examined frequency range at about 6 Hz.

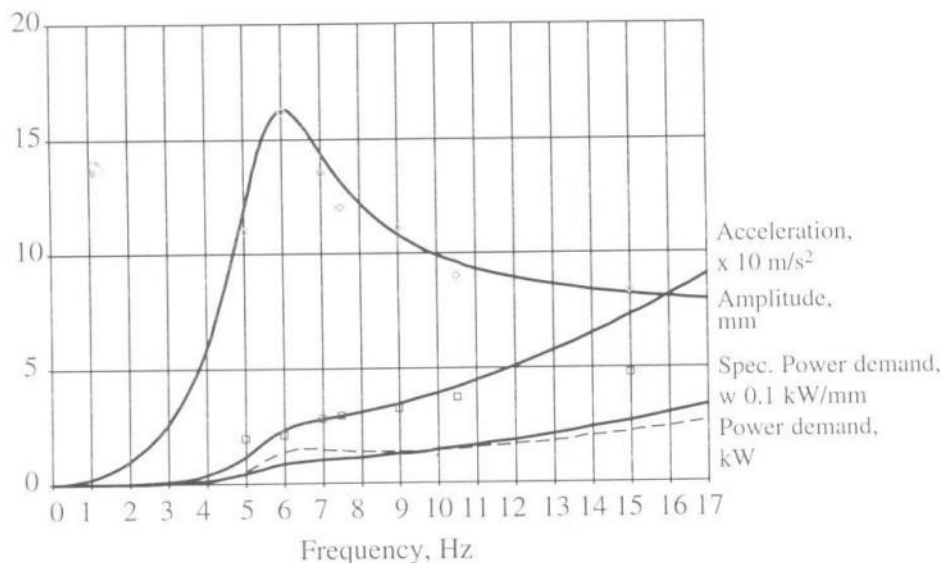


Figure 5. Measured and calculated trunk amplitude, acceleration, power and specific power demand in function of shaking frequency

The calculated average power demand, using Eqn. 5 as well as the specific power demand is also shown on Figure 5. While the power demand has a maximum above 6 Hz, the specific power demand increases continuously.

## Discussion

The replacement of the shaken fruit tree by a three element one degree of freedom model proved to be successful: the acceleration and displacement amplitudes vs. shaking frequency values coincided well for the model and real tree.

The results qualifies also the methods in measuring the model parameters: the definition of spring constant, damping coefficient and reduced mass for a given trunk height.

Drawing the acceleration and amplitude versus frequency curves for 80 cm shaking height it became clear that at the usual

practical frequency range of 12–14 Hz the amplitude is not at maximum, although the acceleration is high enough to an acceptable fruit detachment rate. Further investigations should be made to clear the effect of shaking height in resulting amplitude of the trunk.

## Acknowledgements

The author wishes to thank the valuable advices and discussions of László Csorba, associate professor of Szent István University in Gödöllő. Thanks are also due to the colleagues of the Hungarian Institute of Agricultural Engineering, Gödöllő and of Kevefrukt Co. for their cooperation in field experiments.

## References

- Fridley R B & Adrian P. A. (1966): Mechanical harvesting equipment for deciduous tree fruits. California Agricultural Experiment Station, Bulletin 825: 56.
- Láng, Z. (2003): A fruit tree stability model for static and dynamic loading. *Biosystem Engineering* 85(4): 461–465.
- Láng, Z. (2006): Dynamic Modelling Structure of a Fruit Tree for Inertial Shaker System Design. *Byosystem Engineering* 93(1): 35–44.
- O'Brian, M., Cargill, B.F. & Fridley, R.B. (1983): *Harvesting and Handling Fruits and Nuts*. AVI Publishing Company, Westport, Connecticut, 162–166.
- Whitney, J. D., Smerage, G. H. & Block, W. A. (1990): Dynamic analysis of a trunk shaker-post system. *Trans. ASAE* 33(4):1066–1068.