Mathematical and Statistical Modelling of Impact Symptoms and Application to Tomato

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Summary: Drop tests were performed with different tomato cultivars. „Rupture" and „no rupture" conditions were determined as results. The proportion of „rupture" was shown versus the drop height and the relationships were described by a logistic function. The different cultivars are compared on this basis. The tests and results are suitable for the evaluation of stress sensitivity of tomatoes.

Key words: tomato, dynamic loading, impact, modelling, mathematical statistics

Introduction

Our examination is focusing on the estimation of the tomato characteristics especially considering its physical-mechanical characteristics related to quality. As part of the examination, we are making drop tests. After the tomatoes are dropped on a rigid and heavy plane surface from different distances *(Figure 1)*, they are observed and set in injured and sound classes according cracking because of collision or not. In our opinion, this simple examination could be a practical method to estimate the power of resistance against dynamic effects in other words the sensitiveness to injury caused by impact. That was tried to work out as a demonstration in a few cases, which assist to examine and judge advantages and disadvantages.

Figure 1 The drop tester set at the height of 50 cm. Under the drop tube, the square-shaped wooden colliding surface can be seen, which is mounted to the heavy fixed structure.

Objective

The objective was to develop a method for the possible assessment of the tendency of tomato to mechanic damage concerning dynamic loading and the demonstration of procedures based on measurements and supported by case-studies.

Material and method

We made the drop test with tomato varieties produced under field conditions by the Róna Agricultural Producer's Co-operative and by the Department of Horticulture of the Agricultural University, Gödöllő *(Table 1)*.

It is a general rule – which was firstly submitted with detailed mathematical analysis probably by *Galilei* (1564–1642) – that the physical-mechanical stability of the bodies (also of the fauna and flora) is not constant with the growth of their size (for instance volume, mass ... etc.) but decreases. It is not possible to build a tower with extreme height only by increasing its diameter because it would break down under its own weight sooner or later. Similarly, an elephant is less 'drop resistant' than a cat or a cock-chafer. It is obvious that this aspect can also be examined among 'varieties'. For instance smaller fruits are probably more resistant against mechanical effects than the bigger ones. The size is practically marked with the weight (in the field of gravity) in our experiment, however, refinements are certainly possible.

The initial hypothesis is, when dropping the tomato fruits from the same height it is expected that the bigger ones tolerate the loading worse than the smaller ones with approximately the same maturity. The '*cracked-non-cracked*' indicator function is considered to be the most simple answer function as a possible measure of the performance.

The assumption stating this random phenomenon seems to be obvivus. Since between fruits of same size, only one of them is damaged by cracks: there are bigger fruits which are

Table 1 Circumstances of the drop test with tomato varieties

Site/time	Variety	Drop height [cm]	Sample size [piece]	Injured [piece]	Sound [piece]	Remark
Sz.szállás/31.8.1999.	**UNO**	150	100	57	43	
Gödöllő/24.8.1999.	**Ispán**	50	60	1	59	
		100	60	11	49	
		150	60	43	17	
		200	60	47	13	
	Jubileum	25	60	6	54	
		50	60	30	30	
		75	60	39	21	
		100	60	57	3	
	Pollux	50	60	3	57	
		100	60	16	44	
		150	60	24	36	
		200	60	47	13	
25.8.1999.	**UNO**	50	60	1	59	
		100	60	14	46	
		150	60	31	29	
		200	60	48	12	
26.8.1999.	**Heinz 9706**	50	60	1	59	
		100	60	2	58	
		150	60	19	41	
		200	60	21	39	
2.9.1999.	**Ispán**	50	60	3	57	
		100	60	23	37	
		150	60	39	21	
		200	60	51	9	
	UNO	50	60	0	60	
		100	60	17	43	
		150	60	43	17	
		150	60	41	19	previous repeated

Figure 2 An inhomogeneous sample: tomato fruits at different stage of maturity. Variety: Heinz 9706.

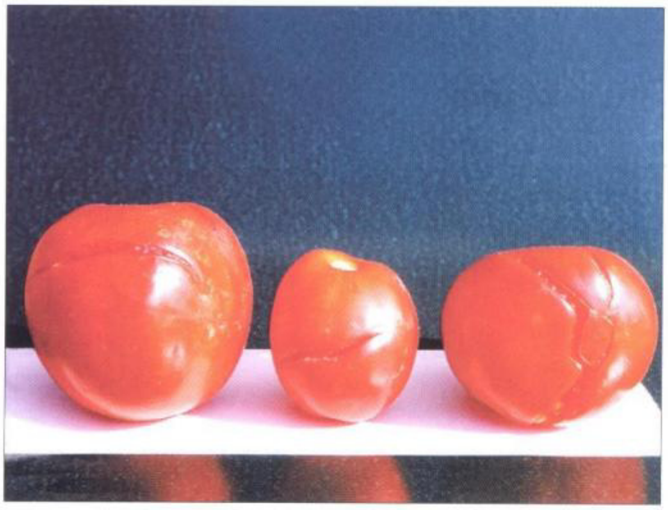

Figure 3 Different varieties of tomato fruits cracked as a result of the drop test

not likely to be cracked and also smaller ones which are usually damaged by cracks. To answer the question, mathematical and statistical methods seem to be suitable.

The followings can be assumed about the number of the tomatoes cracked in the function of the height:

- the shorter the drop distance is, the less fruits will be cracked; this rate must approximate zero: the tomatoes dropped from a distance of 0 cm will not crack;
- the higher is the dropping distance, the more cracked fruits can be expected; in limit value – in other words, in case of an unlimited increase of the drop distance – it must lead to the crack of all elements;
- we are interested in the characteristics of the intermediate range besides the expected asymptotic performance in the extreme range: we assume that the performance of the tomatoes picked from different or the same but of different maturity varieties will be also divergent in this range.

According to the experimental phenomenon outlined above, the symptom is random. Therefore, the – absolute or relative – possible number of the cracked tomatoes dropped from a given distance - that is the conditional possible value – must be practically estimated: for this we will use a regression analysis. To choose the type of regression function it is based on the asymptotic performance. In the medium range, the formation of the curve is determined by the help of measured data when the non-linear regression is accomplished by the method of least squares. One of the functions following the previous argumentation, is called logistic function by the literature [1]:

$$M(h) = \frac{K}{1 + b\exp(-a \cdot h)}$$

where: K, $a>0$, $b>0$ are constants, M is the number of the cracked fruits dropped from h distance. The K is a value to which the function approximates asymptotically with the increase of h that K is the number of tomato fruits which is dropped in a series with the same number of elements (in our case: $K=60$),

that is:

$$\lim_{h\to\infty} M(h) = \lim_{h\to\infty}\left[\frac{K}{1 + b\exp(-a \cdot h)}\right] = K$$

Decreasing the distance, the function approximates the

$$\lim_{h\to 0} M(h) = \lim_{h\to 0}\left[\frac{K}{1 + b\exp(-a \cdot h)}\right] = \frac{K}{1+b}$$

limit value which in contrast with the desirable, is not zero in case of $h=0$, although it will be a small value after the calculation of the regressions. At the level of $K/2$ the function has a point of inflexion. The increase of the growing rate of the strictly monotonously increasing function becomes decreasing and its graph is (centred) symmetrical to this point. The question is whether the cracked tomatoes follow these peculiarities or not?

The function can be linear which is favourable from the point of view of making the regression in a simple way and therefore a and b parameter can be estimated. K is a constant which: the number of tomatoes dropped from the same distance. As an example, we show the results of the *Ispán* variety (24. 8. 1999.) in *Figure 4*.

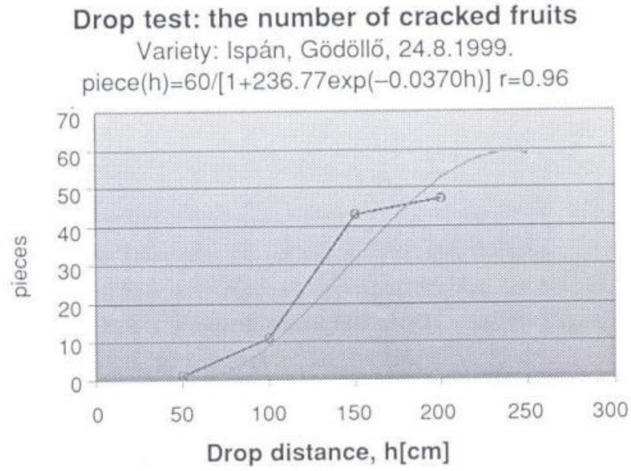

Figure 4 The results of the drop test of Ispán and the fitted logistic curve

The normal distribution function is also similar to the logistic curve, moreover, the arc tangent function too. These are also symmetric to the "intermediate value". Their asymptotic performance is equivalent in both directions with the logistic function, therefore they are suitable for the use as much as the above mentioned logistic function. The result of approximation with the normal distribution function is the probability – and this can also be transformed into the expected number of pieces – which in case of a given height provides the probability (or expected number of pieces) of the crack of the tomatoes dropped from a lower height.

All of the three above mentioned descriptions have the same drawbacks:

- on one hand, they expect the number of injured tomato fruits to be symmetric to the low and high drop distance;
- on the other hand, – even if with small errors but – in case of a given drop distance of zero, the number of cracked fruits do not result in zero, moreover, they are in the negative range of drop distance too.

This research tries to apply only the logistic model worked out in details among the three solutions mentioned.

Results

Completing *Figure 4*, we make another case shown in *Figure 5*. The Heinz-variety is widely known about its loading capacity. In this case, the drop distances planned and sometimes modified, according to the results of the first tests, provided results only at the lower part of the available range.

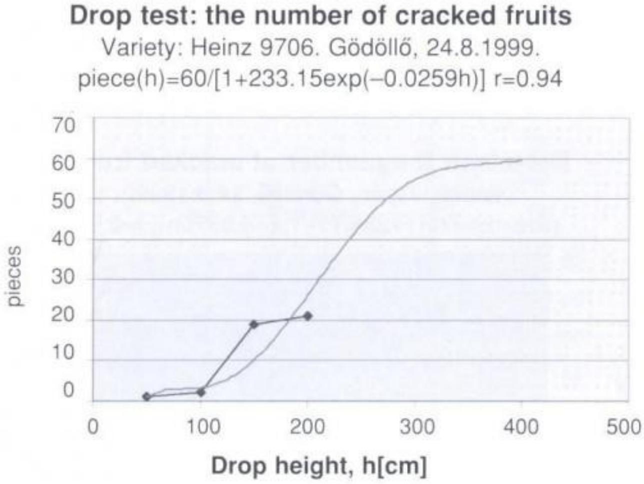

Figure 5 The results of the drop test and its approximation. Variety: Heinz 9706.

Therefore the extrapolation has to be handled carefully: it can be seen very well that the drop distances could have been extended above 300 cm. In the applied measurement range this variety differed from the other tested varieties significantly *(Figure 6)*.

The fit of the logistic function makes the relations more clear and suggests the conclusion that from the point of view of the resistance to impact the *Jubileum* and *Heinz* varieties are the extreme ones and the *Ispán*, the *Pollux* and the *UNO* seem to be similar to each other based on the analysis of the cracks *(Figure 7)*. The two extreme varieties seem to separate from those in middle and this is a considerable difference for the *Heinz* variety.

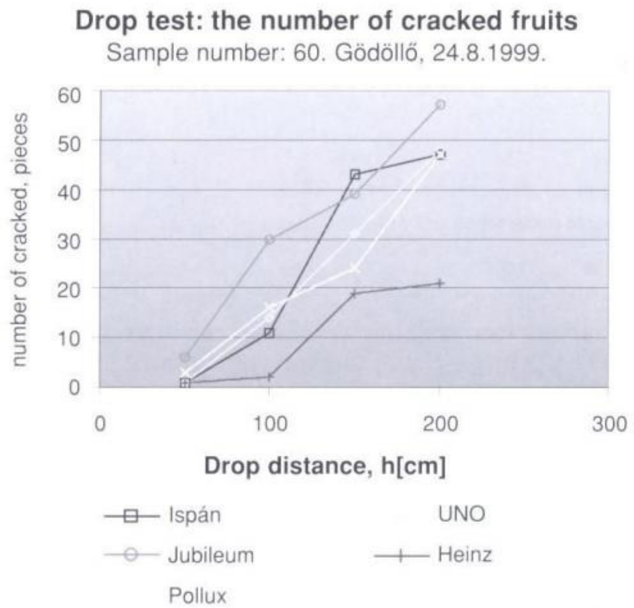

Figure 6 The empirical function of the cracked tomato fruits (24. 8. 1999.).

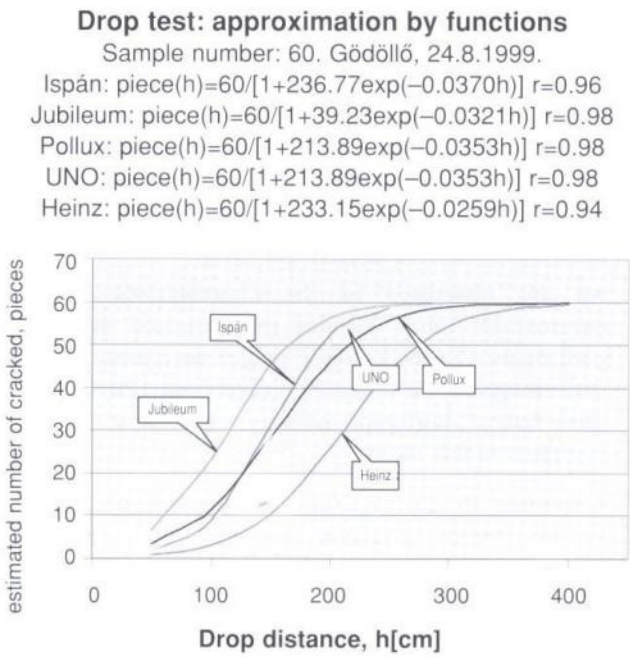

Figure 7 Logistic analytic approximation for the tests on 24.8.1999.

There was possibility to repeat the drop tests only in case of two varieties among the earlier tested ones: the measurement of the *Ispán* and the *UNO* led to the results shown in *Figure 8 and 9* (2.9.1999.). Their performance seems to be quite similar. To make our conclusions more than a hypothesis, it can be stated on the basis of these measurements that the measurements have to be repeated to get known the average number and the empirical standard deviation of the cracked dropped fruits from the same distance. This could make it possible to create the correct

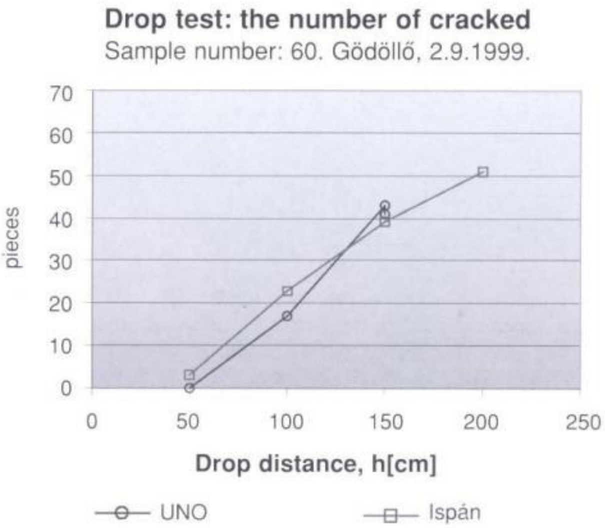

Figure 8 The empirical result of the test on 2.9.1999.

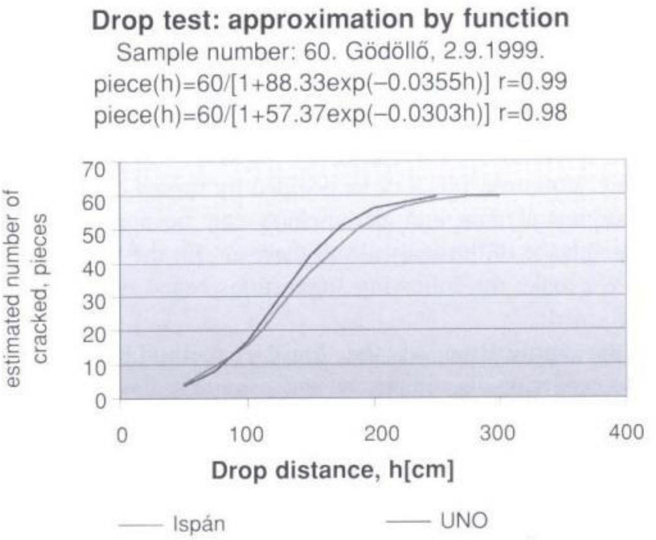

Figure 9 Logistic models fitted to the data of the tests performed on 2. 9. 1999.

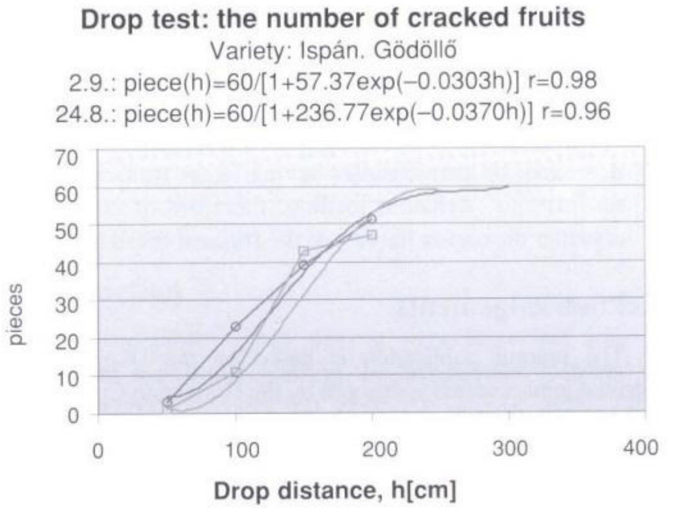

Figure 10 The results of the test made in two different time in case of the variety *Ispán*

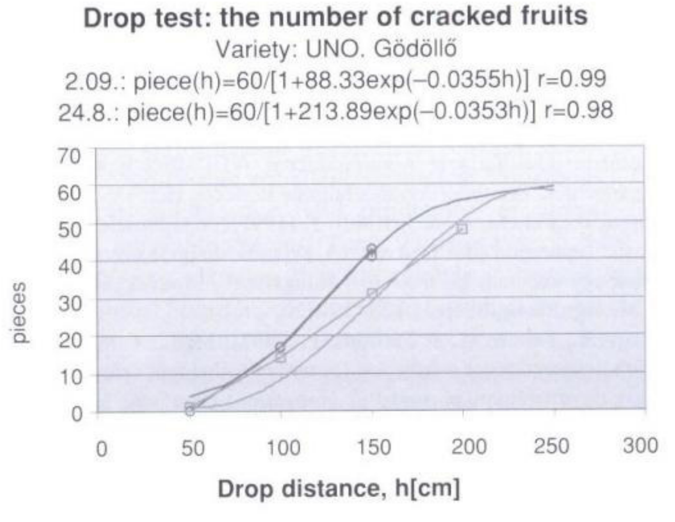

Figure 11 The results of the tests made in two different times, Variety: *UNO*.

conclusions on the basis of the estimation at a given level. The repetition of the drop test at the same distance can be seen here as well: our measurement with *UNO* from 150 cm height has given a 41 and 43-cracked-piece series. It seems that at least three more series should be performed. If we compare the measurements of 24.8.1999. in case of this two varieties, the similar performance seems to be even more established *(Figure 6 and 7)*.

It is obviously an interesting question what is the reaction of the ripe tomato fruits picked in two different time by the drop test in the function of time: this is shown by *Tables 10* and *11* containing the empirical values and their analytical approximations.

It is noticeable beside the correlation of the empirical data and of the fitted logistic model how the latter one helps the qualitative distinction among the data. In both cases, the analytical relations suggest that the higher percent of cracks in ripe tomatoes picked later. Is it possible that the selection in accordance with 'maturity' leads to another result as time is going on? The extrapolation from a drop distance of 250 cm predicts the crack of the whole set practically for both varieties and both periods: so it is not worthwhile increasing the distance above 250 cm in case of these varieties but it is worth applying the 250-cm-distance. This observation seems to be right for all varieties except *Heinz* while in case of *Heinz* it is useful to increase the drop distance with another meter (see *Figure 6.* and *7)*.

The method can be applied inversely: if there is a limit value according to the regulation – for example the 5 percent – for the relative number of injured then this can be looked for by using logistic fit. In this case the measurement in this range with the choose of lower drop distances is obviously practical with that aim that this limit value – $h_{5percent}$ height belonging to the 5 percent – could be estimated with good accuracy. The varieties can be ranked in this way since we can consider those more resistant against the dynamic effects where higher drop height belongs to the 5 percent cracking rate. It can occur that the results at low height can turn at higher height as the case of *Ispán* and *Pollux* shows, although we have to perform more test series on a certain height to get further results about significant level.

Conclusions and recommendations

Although it has some deficiency, the logistic model seems to be useful for the approximation of cracking sensitivity and for the comparison of the varieties at the same time and at different dates by the drop tests of tomato fruits. With the help of the logistic model, the objective estimation of the favourable harvesting time, from the point of view of the tolerance of dynamic – like an impact – mechanical loading, is possible. For this, however, the rational increase of the number of measurements series is necessary because the phenomenon is influenced by random effects. It seems to be practical to increase the drop distance to 350 cm in case of those varieties which are considered to be more resistent to impact by the complete (extended near to 100 percent cracking rate) test (like Heinz 9706) while in case of the rest of the varieties to increase it up to 250 cm (such as *Ispán, Jubileum, Pollux, UNO*). We can be satisfied with much lower drop distances if the aim is to determine a rank based on the drop distances leading to the results of a certain cracking rate (according to agreement). Obviously, it is important for tomatoes picked for the drop test to be representative by size. In the contrary, since the bigger fruits crack sooner, it is presumable that the test of selected fruits based on size leads to false result.

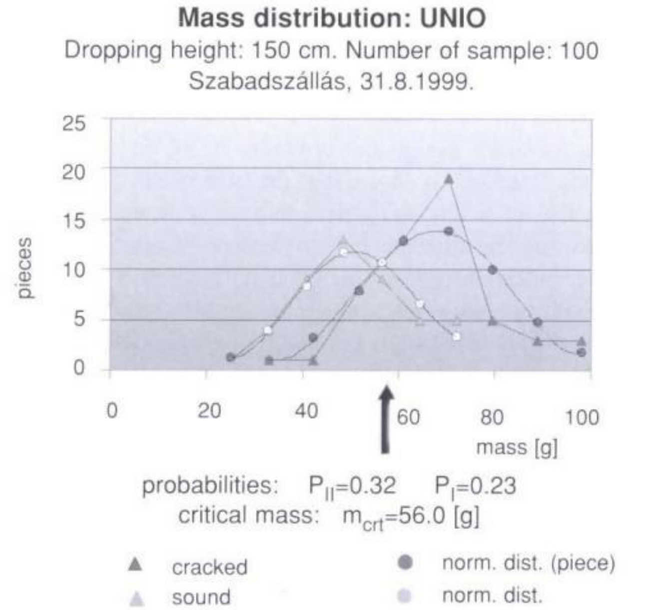

Figure 12 Histogram and the density functions of the normal distributions fitted in case of the set of cracked and non-cracked tomato fruits. The critical mass mcrt, is shown as well

This way of thinking gives possibility for another ranking method [3,4] too: the distribution by mass of the 'cracked-non-cracked' specimens of the representative sample dropped from a given distance can also be considered to be the basis of classification according the mass is the point of intersection of the density functions by mass of cracked and non-cracked tomatoes *(Figure 12)*. If this so-called mcrt critical mass characteristic is larger in case of a certain variety by dropping from a given height, this can be considered to be better than its competitor (in the aspect of the tolerance of dynamic effects). Since the specimens tolerate the loading caused by drop without crack until a larger mass.

The drawback of this method is that it does not make difference between two varieties (dropped from the same height) having the same mcrt critical mass. As for an example 60 percent does not crack in case of one of the varieties while it is only 40 percent in case of the other variety. Obviously, that variety could be considered to be better from which higher rate did not crack during the test. Therefore it seems that this method of judgement of the sensitiveness against dynamic loading requires a bivariate approximation: beside the mcrt, the crack rate is also typical. Generally, different drop distances give different critical mass – in other words a mcrt(h) function can be expected – and the evaluation is extendible. This model of mass distribution fits the random characteristic of the phenomenon, however, this can be efficient if only the mass related density functions of the cracked and non-cracked fruits can be distinguished. *Figure 12*, as one of our first tests made in this field, suggests the existence of such an effect. However, this will be decided by further analysis of experimental data and the method can be applied as a consequence of the analysis: all these are for the future.

We make the following suggestions based on the tests performed:

- the application of the logistic method especially considering its simplicity and accuracy and relatively rapid character;
- as for the next step, we do suggest
  - the development of the model established on lognormal distribution, which is more adaptable to the physical background of the phenomenon;
  - the control of its reliability and fit;
  - its comparison with estimations by the logistic model
  - and in case of the hoped success of the previous, the software development for the automatic data processing for the evaluations;
- the development and control of the (bivariate) method based on mass distribution;
- it is expected that varieties having larger fruits are more sensitive to dynamic loading, therefore, it is wise to consider the earlier harvest of the fruits of these varieties.

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