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An Analytical Solution for the Two-Layered Composite Beam-Column with Interlayer Slip and Constant Axial Load

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Abstract. The authors present an analytical solution for the two-layered composite beams with imperfect shear connections. The considered beam is simply supported at both ends. The beam is subjected to transverse and axial loads. The kinematic assumptions of the Euler-Bernoulli beam theory are used. The connection of the beam components is perfect in normal direction, but the axial displacement field may have jump. The shear axial force derived from the imperfect connection is proportional to the relative slip occurring between the layers. The determination of the analytical solution is based on the Fourier method. Two examples illustrate the application of the presented analytical method.

Keywords: Composite beam-column, Axial load

Introduction

Layered composite structures, especially layered composite beams are widely applied in building and bridge engineering since the advantages of the layers made of different materials can be well married, while their disadvantages can be reduced or eliminated. Therefore, it is very important to understand the mechanical behavior of the layers, composite beams and the influence of the connection between the layers of composite beams which are joined to each other by different shear connections such as nails, screws or rivets. Because of the elastic deformation of those connectors can occur between the connected beam components the appearance of interlayer slip is possible. In this paper it is assumed that the normal direction the connection of layers is perfect. The connection in axial direction may be imperfect, which is described by the concept interlayer slip.

The first theoretical and experimental studies analyzing the behavior of the composite beams with interlayer slip was published in the 1950s [1,2,3]. In paper [2] the Euler-Bernoulli beam theory was utilized to describe the static behavior of composite beams with interlayer slip. Since then, a lot of papers, studies [4,6,7,8,10,11,12,13,14,15,16] have dealt with the problem of layered beams with partial shear interaction based on the results of Newmark et al [2]. Paper by Ecsedi and Baksa [5] presents a slip-cross-sectional rotational formulation to obtain the deflection, slip, cross-sectional rotation and internal forces in shear deformable composite beam with imperfect shear connection. An analytical

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solution is presented for two layered composite beams with interlayer slip based on the assumptions of Timoshenko beam theory by the use of the method of fundamental solution in [17].

Elastic stability of beam-columns with weak shear connection is considered in papers [6] and [9]. Girhammar and Gupta studied the composite beam-columns with interlayer slip which subjected to transverse and axial loads [6]. They gave closed form solutions for simply supported beam, whose transverse load is a uniform distributed force along the whole length of the beam. In [6] the stability problem of the considered beam-column is also investigated. Lengyel and Ecsedi presented a second order analysis of composite beam-columns with interlayer slip. The considered beam carries the transverse load and constant axial load [13].

In the present paper a slip-deflection formulation is presented to solve the statics problem of composite beams with imperfect shear connection and constant axial load.

1. Governing equations

The present paper deals with the solution of a static problem of composite two-layered beam-column with imperfect shear connection. The considered simply supported beam-column and its loads are shown in Figure 1. The beam-column carries the axial and transverse loads.

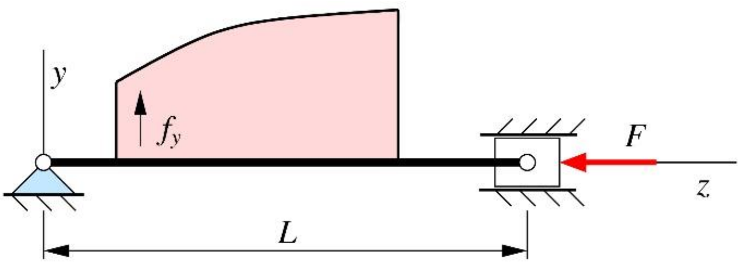

Figure 1 Layered composite beam-column and its applied load.

The cross section of composite beam is given in Figure 2, the shape and data of cross section is borrowed from the paper [6].

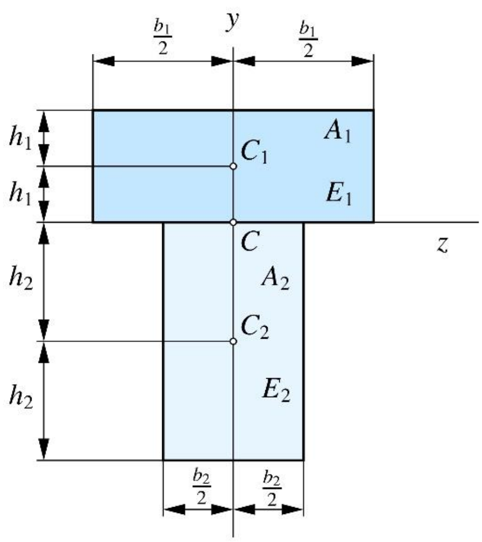

Figure 2 The cross section of the composite beam-column.

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In both examples the following data will be used

$$h_1 = 0.025\ [\mathrm{m}],\ \ b_1 = 0.3\ [\mathrm{m}],\ \ h_2 = 0.075\ [\mathrm{m}],\ \ b_2 = 0.05\ [\mathrm{m}],\ \ L = 4\ [\mathrm{m}]. \tag{1}$$

The modulus of elasticity of the beam component i is E_i $(i = 1{,}2)$ where

$$E_1 = 1.2 \times 10^{10}\ [\mathrm{Pa}], \qquad E_2 = 8 \times 10^{9}\ [\mathrm{Pa}]. \tag{2}$$

The center of the cross section A_i is denoted by C_i $(i = 1{,}2)$. The E-weighted center of composite cross section is C. In present problem the E-weighted center of the composite cross section is on the common boundary curve of cross sections A_1 and A_2 as shown in Figure 2. The length of the beam is L. The origin O of the rectangular Cartesian coordinate system $Oxyz$ is the E-weighted center of the left end cross section $A = A_1 \cup A_2$, so that the axis z is the E-weighted center-line of the considered two-layer composite beam-column with flexible shear connection. Denote the beam-column component i is B_i $(i = 1{,}2)$. A point Q in $B = B_1 \cup B_2$ is illustrated by the position vector $\overrightarrow{OQ} = \boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$, where $\boldsymbol{e}_x$, $\boldsymbol{e}_y$ and $\boldsymbol{e}_z$ are the unit vectors of the coordinate system $Oxyz$. It is known that the position of the E-weighted center C is obtained as (see Figure 2)

$$c_1 = \left|\overrightarrow{OC_1}\right| = \frac{A_2E_2}{A_E}c,\ \ c_2 = \frac{A_1E_1}{A_E}c,\ \ c = h_1 + h_2, \tag{3}$$

$$A_E = A_1E_1 + A_2E_2. \tag{4}$$

The common boundary of the beam component B_1 and B_2 is determined by $y = 0$, $|x| \leq 0.5b_2$ (see Figure 2). The applied axial force acts on the beam-column component B_i is denoted by P_i is the point C_i $(i = 1{,}2)$. The magnitude of P_1 and P_2 are such that the point of application of resultant axial force $P = P_1 + P_2$ is the E-weighted center C of the composite cross section $A = A_1 \cup A_2$. From this fact it follows that (see Figure 3)

$$P_1 = \frac{c_2}{c}P,\ \ P_2 = \frac{c_1}{c}P. \tag{5}$$

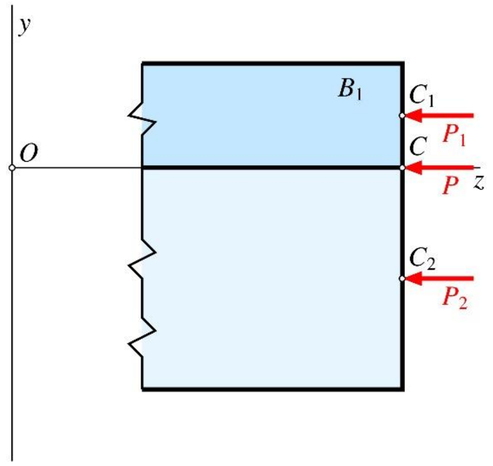

Figure 3 Illustration of the applied axial load.

According to the Euler-Bernoulli beam theory which is valid for each homogenous beam-column component, the deformation of the whole composite beam with constant axial load can be described by the following displacement field

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$$u = 0, \quad v = v(z) \quad (x, y, z) \in B_1 \cup B_2, \tag{6}$$

$$w(y, z) = w_1(z) - y\frac{\mathrm{d}v}{\mathrm{d}z} - \frac{P_1 z}{A_1 E_1} \quad (x, y, z) \in B_1, \tag{7}$$

$$w(y, z) = w_2(z) - y\frac{\mathrm{d}v}{\mathrm{d}z} - \frac{P_2 z}{A_2 E_2} \quad (x, y, z) \in B_2. \tag{8}$$

In equations (6), (7) and (8) u is the displacement in direction of $\boldsymbol{e}_x$, v is the displacement in direction of $\boldsymbol{e}_y$ and w is the displacement in direction of $\boldsymbol{e}_z$, which is the axial displacement (Figure 1). On the common boundary of B_1 and B_2 the axial displacement may have jump which is called the interlayer slip

$$s(z) = w_1(z) - w_2(z) - z\left(\frac{P_1}{A_1 E_1} - \frac{P_2}{A_2 E_2}\right). \tag{9}$$

Form equations (4) and (6) it follows that

$$\frac{P_1}{A_1 E_1} - \frac{P_2}{A_2 E_2} = 0, \tag{10}$$

that is

$$s(z) = w_1(z) - w_2(z). \tag{11}$$

Application of the strain-displacement relationships of the linearized theory of elasticity gives [18,19]

$$\varepsilon_x = \varepsilon_y = \gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0 \quad (x, y, z) \in B, \tag{12}$$

$$\varepsilon_z = \frac{\mathrm{d}w_1}{\mathrm{d}z} - y\frac{\mathrm{d}^2 v}{\mathrm{d}z^2} - \frac{P_1}{A_1 E_1} \quad (x, y, z) \in B_1, \tag{13}$$

$$\varepsilon_z = \frac{\mathrm{d}w_2}{\mathrm{d}z} - y\frac{\mathrm{d}^2 v}{\mathrm{d}z^2} - \frac{P_2}{A_2 E_2} \quad (x, y, z) \in B_2. \tag{14}$$

In equation (12), (13) and (14) ε_x, ε_y and ε_z are the normal strains, γ_{xy}, γ_{yz} and γ_{xz} are the shearing strains. By the use of Hooke's law, the following formulae are derived for the normal stress σ_z

$$\sigma_z = E_1\left(\frac{\mathrm{d}w_1}{\mathrm{d}z} - y\frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right) - \frac{P_1}{A_1} \quad (x, y, z) \in B_1, \tag{15}$$

$$\sigma_z = E_2\left(\frac{\mathrm{d}w_2}{\mathrm{d}z} - y\frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right) - \frac{P_2}{A_2} \quad (x, y, z) \in B_2. \tag{16}$$

are used.

The subsection forces and moments, according to the beam's theory, are defined as

$$N_1 = \int_{A_1} \sigma_z \mathrm{d}A = n_1 - P_1, \qquad N_2 = \int_{A_2} \sigma_z \mathrm{d}A = n_2 - P_2, \tag{17}$$

$$M_1 = \int_{A_1} y\sigma_z \mathrm{d}A = -c_1 E_1 A_1 \frac{\mathrm{d}w_1}{\mathrm{d}z} - E_1 I_1 \frac{\mathrm{d}^2 v}{\mathrm{d}z^2} - c_1 P_1, \tag{18}$$

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$$M_2 = \int_{A_2} y\sigma_z \mathrm{d}A = -c_2 E_2 A_2 \frac{\mathrm{d}w_2}{\mathrm{d}z} - E_2 I_2 \frac{\mathrm{d}^2 v}{\mathrm{d}z^2} + c_2 P_2. \tag{19}$$

In equation (17)

$$n_1 = E_1 A_1 \left(\frac{\mathrm{d}w_1}{\mathrm{d}z} - c_1 \frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right), \qquad n_2 = E_2 A_2 \left(\frac{\mathrm{d}w_2}{\mathrm{d}z} + c_2 \frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right), \tag{20}$$

and in equations (18) and (19)

$$I_i = \int_{A_i} y^2 \mathrm{d}A \qquad (i = 1,2) \tag{21}$$

It is evident, that

$$N_1 + N_2 = -P, \tag{22}$$

that is

$$n_1 + n_2 = 0. \tag{23}$$

By the use of equation (23)

$$(-E_1 A_1 c_1 + E_2 A_2 c_2) \frac{d^2 v}{dz^2} = \left(-\frac{E_1 A_1 E_2 A_2}{A_1 E_1 + A_2 E_2} + \frac{E_2 A_2 E_1 A_1}{A_1 E_1 + A_2 E_2}\right) \frac{d^2 v}{dz^2} = 0. \tag{24}$$

equation (23) can be written in the form

$$E_1 A_1 \frac{\mathrm{d}w_1}{\mathrm{d}z} - E_2 A_2 \frac{\mathrm{d}w_2}{\mathrm{d}z} = 0. \quad 0 \le z \le L. \tag{25}$$

From equation (9) it follows that

$$\frac{\mathrm{d}s}{\mathrm{d}z} = \frac{\mathrm{d}w_1}{\mathrm{d}z} - \frac{\mathrm{d}w_2}{\mathrm{d}z}. \tag{26}$$

The combination of equation (25) with equation (26) gives

$$\frac{\mathrm{d}w_1}{\mathrm{d}z} = \frac{E_2 A_2}{A_1 E_1 + A_2 E_2} \frac{\mathrm{d}s}{\mathrm{d}z} = \frac{c_1}{c} \frac{\mathrm{d}s}{\mathrm{d}z}, \tag{27}$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}z} = -\frac{E_1 A_1}{A_1 E_1 + A_2 E_2} \frac{\mathrm{d}s}{\mathrm{d}z} = -\frac{c_2}{c} \frac{\mathrm{d}s}{\mathrm{d}z}. \tag{28}$$

Inserting this result into the expression of N_1, N_2 and M_1, M_2 gives

$$N_1 = B\left(\frac{\mathrm{d}s}{\mathrm{d}z} - c \frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right) - P_1 = n_1(z) - P_1, \tag{29}$$

$$N_2 = -B\left(\frac{\mathrm{d}s}{\mathrm{d}z} - c \frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right) - P_2 = n_2(z) - P_2, \tag{30}$$

$$M_1 = c_1 B \frac{\mathrm{d}s}{\mathrm{d}z} - E_1 I_1 \frac{\mathrm{d}^2 v}{\mathrm{d}z^2} - c_1 P_1, \tag{31}$$

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$$M_2 = c_2 B \frac{\mathrm{d}s}{\mathrm{d}z} - E_2 I_2 \frac{\mathrm{d}^2 v}{\mathrm{d}z^2} + c_2 P_2. \tag{32}$$

From equations (29) – (32) it follows that

$$n_1(z) = -n_2(z) = B\left(\frac{\mathrm{d}s}{\mathrm{d}z} - c\frac{\mathrm{d}^2 v}{\mathrm{d}z^2}\right) \tag{33}$$

and

$$M(z) = M_1(z) + M_2(z) = cB\frac{\mathrm{d}s}{\mathrm{d}z} - j\frac{\mathrm{d}^2 v}{\mathrm{d}z^2}, \tag{34}$$

where $M = M(z)$ is the bending moment on the whole cross section, and $j = E_1 I_1 + E_2 I_2$.

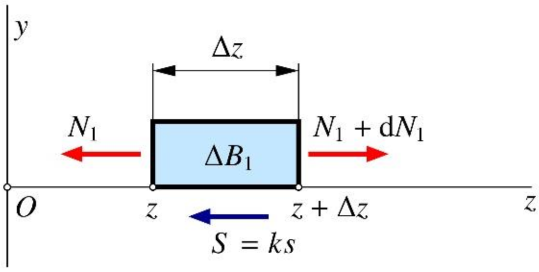

Figure 4 Free body diagram of beam component ΔB_1.

Application of the condition of equilibrium for the forces acting in axial direction of the beam-column component B_1 (see Figure 4) the following equation can be derived

$$\frac{\mathrm{d}N_1}{\mathrm{d}z} - k(w_1 - w_2) = 0. \tag{35}$$

In equation (35) $S = ks$ is the interlayer shear force and k is the slip modulus [2,6,8]. The detailed form of equation (35) is

$$B\frac{\mathrm{d}^2 s}{\mathrm{d}z^2} - ks - cB\frac{\mathrm{d}^3 v}{\mathrm{d}z^3} = 0. \tag{36}$$

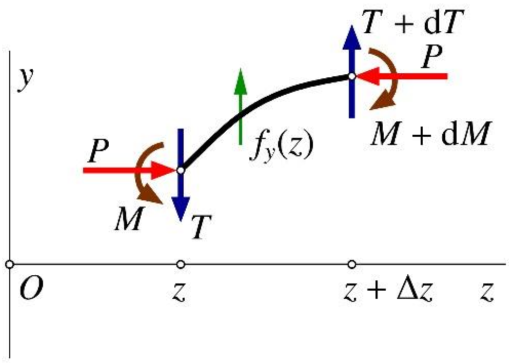

Figure 5 Beam element with its load.

Figure 5 shows a beam element and its load. According to Figure 5 the following equilibrium equations are valid

$$\frac{\mathrm{d}T}{\mathrm{d}z} + f_y = 0 \qquad 0 < z < L, \tag{37}$$

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$$\frac{\mathrm{d}M}{\mathrm{d}z} - T - P\frac{\mathrm{d}v}{\mathrm{d}z} = 0 \qquad 0 < z < L. \tag{38}$$

In equations (37) and (38) $f_y = f_y(z)$ is the applied distributed force in direction of axis y and $T = T(z)$ is the cross-sectional shear force. Combination of equation (37) with equation (38) gives

$$\frac{\mathrm{d}^2 M}{\mathrm{d}z^2} - P\frac{\mathrm{d}^2 v}{\mathrm{d}z^2} + f_y = 0 \qquad 0 < z < L. \tag{39}$$

Substitution of expression $M = M(z)$ into equation (39) provides a fundamental equation of slip-deflection formulation

$$cB\frac{\mathrm{d}^3 s}{\mathrm{d}z^3} - j\frac{\mathrm{d}^4 v}{\mathrm{d}z^4} + P\frac{\mathrm{d}^2 v}{\mathrm{d}z^2} + f_y = 0 \qquad 0 < z < L. \tag{40}$$

The determination of deflection and of slip function is based on the system of equations (36), (39). In the case of first order analysis the following equilibrium equation

$$\frac{\mathrm{d}M}{\mathrm{d}z} - T = 0 \qquad 0 < z < L. \tag{41}$$

is used instead of equation (38).

2. Simply supported beam-columns with constant axial load

Figure 1 shows the simply supported beam with its load. The boundary conditions for simply supported beam are formulated as [4,6]

$$v(0) = 0, \qquad M(0) = 0, \qquad n(0) = 0, \tag{42}$$

$$v(L) = 0, \qquad M(L) = 0, \qquad n(L) = 0. \tag{43}$$

The Fourier series representation of the applied transverse load is used

$$f_y(z) = \sum_{l=1}^{\infty} f_l \sin\frac{l\pi}{L}z \qquad 0 \le z \le L, \tag{44}$$

where

$$f_l = \frac{2}{L}\int_0^L f_y(z)\sin\frac{l\pi}{L}z\,\mathrm{d}z \qquad (l = 1,2,\dots). \tag{45}$$

The solution of the system of differential equations (36), (39) is looked as

$$v(z) = \sum_{l=1}^{\infty} V_l \sin\frac{l\pi}{L}z, \quad s(z) = \sum_{l=1}^{\infty} S_l \sin\frac{l\pi}{L}z. \tag{46}$$

A simple computation gives the following result

$$M(z) = \sum_{l=1}^{\infty}\left[-c\frac{l\pi}{L}BS_l + \left(\frac{l\pi}{L}\right)^2 jV_l\right]\sin\frac{l\pi}{L}z, \tag{47}$$

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$$n(z) = \sum_{l=1}^{\infty} -\left[B\frac{l\pi}{L}S_l + cB\left(\frac{l\pi}{L}\right)^2 V_l\right]\sin\frac{l\pi}{L}z\ , \tag{48}$$

$$T(z) = \sum_{l=1}^{\infty}\left[-cB\left(\frac{l\pi}{L}\right)^2 S_l + \left(\left(j\frac{l\pi}{L}\right)^3 - P\frac{l\pi}{L}\right)V_l\right]\sin\frac{l\pi}{L}z\ . \tag{49}$$

It is evident that the functions given by equation (46), (47) and (48) satisfy the prescribed boundary conditions for arbitrary values of V_l, S_l $(l = 1, 2, 3, \ldots)$. From the governing differential equation (36) and (39) it follows that

$$V_l = \frac{f_l}{-j\left(\frac{l\pi}{L}\right)^4 + P\left(\frac{l\pi}{L}\right)^2 + \frac{c^2B^2\left(\frac{l\pi}{L}\right)^6}{k + B\left(\frac{l\pi}{L}\right)^2}} \qquad (l = 1,2,3, \ldots) \tag{50}$$

$$S_l = \frac{cB\left(\frac{l\pi}{L}\right)^3}{k + B\left(\frac{l\pi}{L}\right)^2}V_l \qquad (l = 1,2,3, \ldots). \tag{51}$$

The expression of normal stress σ_z in terms of $s = s(z)$ and $v = v(z)$ can be derived by the use of equations (15), (16), (26) and (27)

$$\sigma_z(y,z) = -E_1\left(\frac{c_1}{c}\frac{\mathrm{d}s}{\mathrm{d}z} - y\frac{\mathrm{d}^2v}{\mathrm{d}z^2}\right) - \frac{P_1}{A_1} \qquad 0 \le y \le h_1, \tag{52}$$

$$\sigma_z(y,z) = -E_2\left(\frac{c_2}{c}\frac{\mathrm{d}s}{\mathrm{d}z} + y\frac{\mathrm{d}^2v}{\mathrm{d}z^2}\right) - \frac{P_2}{A_2} \qquad -h_1 \le y \le 0. \tag{53}$$

The shear stress resultant N_{yz} is introduced as

$$N_{yz} = \tau_{yz}b_1 \quad \text{for} \quad 0 \le y \le 2h_1, \tag{54}$$

$$N_{yz} = \tau_{yz}b_2 \quad \text{for} \quad -2h_1 \le y \le 0. \tag{55}$$

Integration of the equilibrium equation

$$\frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0, \tag{56}$$

gives

$$\tau_{yz}(y,z) - \tau_{yz}(0,z) = -\int_0^z \frac{\partial\sigma_z}{\partial z}\mathrm{d}z. \tag{57}$$

The shearing stress $\tau_{yz} = \tau_{yz}(y,z)$ satisfies the boundary conditions

$$\tau_{yz}(2h_1,z) = 0, \quad \tau_{yz}(-2h_2,z) = 0. \tag{58}$$

The continuity condition of shearing stress resultant N_{yz} in terms of shearing stress τ_{yz} can be formulated as

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$$\lim_{\varepsilon\to 0}\left(b_1\tau_{yz}(\varepsilon^2,z)-b_2\tau_{yz}(-\varepsilon^2,z)\right)=0. \tag{59}$$

Knowing the normal stress σ_z and τ_{yz} it is possible to obtain from equilibrium equation the normal stress σ_y

$$\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}=0. \tag{60}$$

Integration of equation (60) gives

$$\sigma_y(y,z)-\sigma_y(0,z)=-\int_0^y\frac{\partial\tau_{yz}}{\partial z}\,\mathrm{d}y. \tag{61}$$

The normal stress $\sigma_y=\sigma_y(y,z)$ satisfies the following stress boundary conditions

$$\sigma_y(2h_1,z)=\frac{f_y(z)}{b_1},\quad \sigma_y(-2h_2,z)=0. \tag{62}$$

Subservient to define the normal stress resultant N_y as

$$N_y=\sigma_y b_1 \quad \text{for } 0\le y\le 2h_1, \tag{63}$$

$$N_y=\sigma_y b_2 \quad \text{for} -2h_2\le y\le 0. \tag{64}$$

The continuity condition of N_y at $y=0$ gives

$$\lim_{\varepsilon\to 0}\left(b_1\sigma_y(\varepsilon^2,z)-b_2\sigma_y(-\varepsilon^2,z)\right)=0. \tag{65}$$

In the numerical examples the graphs of $\sigma_z=\sigma_z(y,z)$, $N_{yz}=N_{yz}(y,z)$ and $N_y=N_y(y,z)$ for some values of axial coordinate z are presented.

3. Examples

3.1. Example 1

Figure 6 shows the beam-column and its applied load.

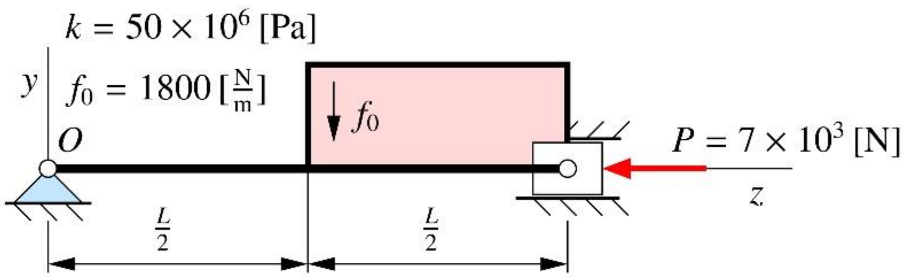

Figure 6 Partially loaded beam-column with uniform load.

The plots of function $v=v(z)$ and $\phi=\phi(z)$ are shown in Figure 7 and 8.

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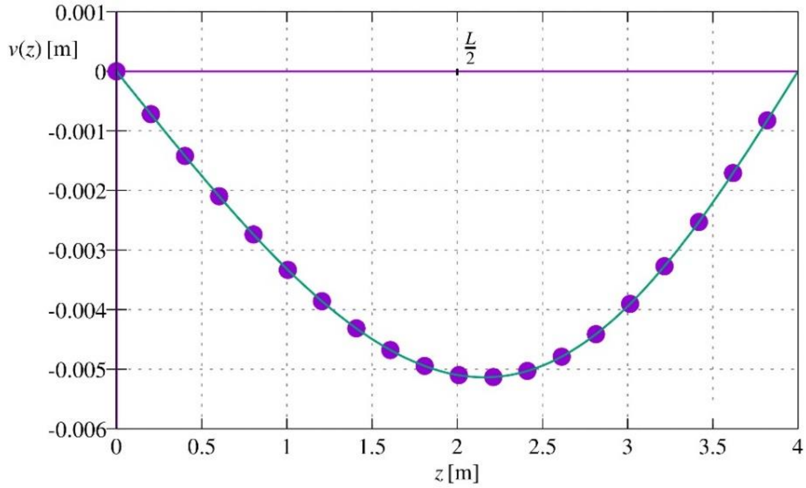

Figure 7 The graph of the deflection function $v = v(z)$.

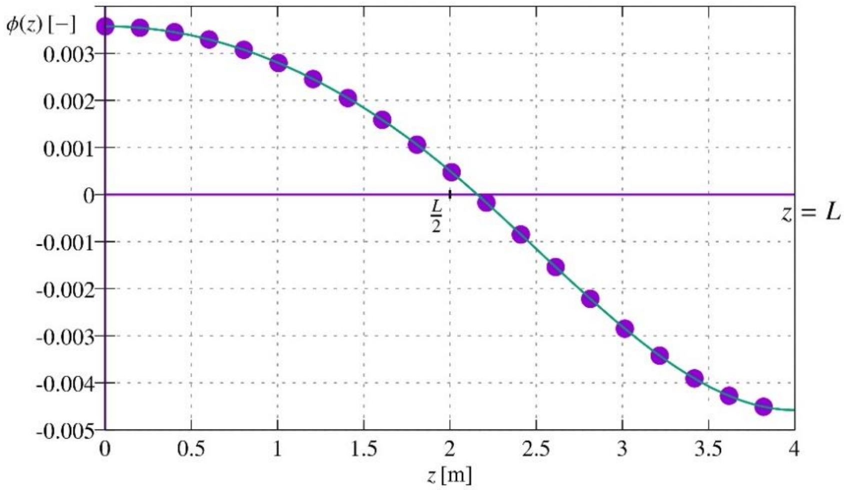

Figure 8 The graph of the cross-sectional rotation function $\phi = \phi(z)$.

The graph of the slip function $s = s(z)$ is illustrated in Figure 9.

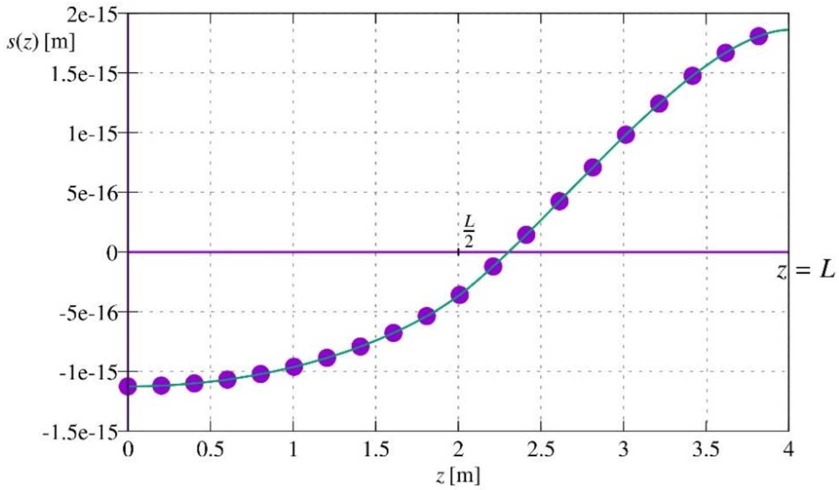

Figure 9 The graph of slip function $s = s(z)$.

The graphs of $M = M(z)$, $n = n(z)$ and $T = T(z)$ are shown in Figures 10, 11 and 12. Since the composite beam-columns with simply supported boundary conditions is statically determinate, the

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bending moment and shear force diagrams can be obtained immediately from the applied load. This fact gives an opportunity to check the validity of results obtained by the applications of formulae (47) and (49).

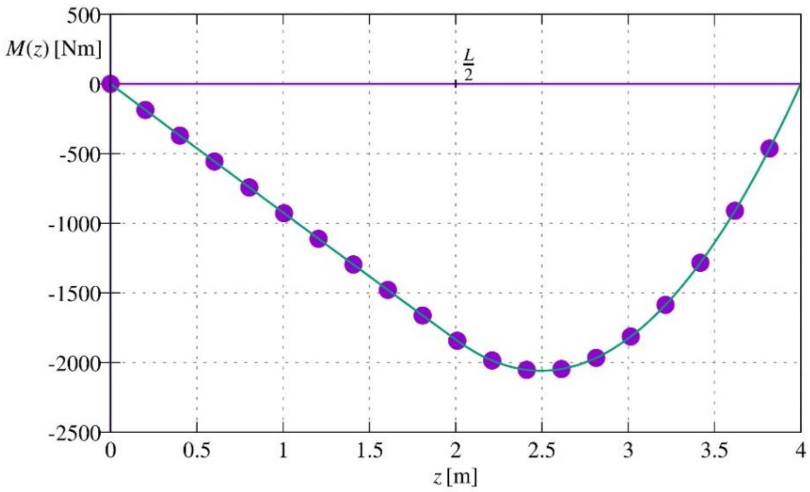

Figure 10 The graph of the bending moment function $M = M(z)$.

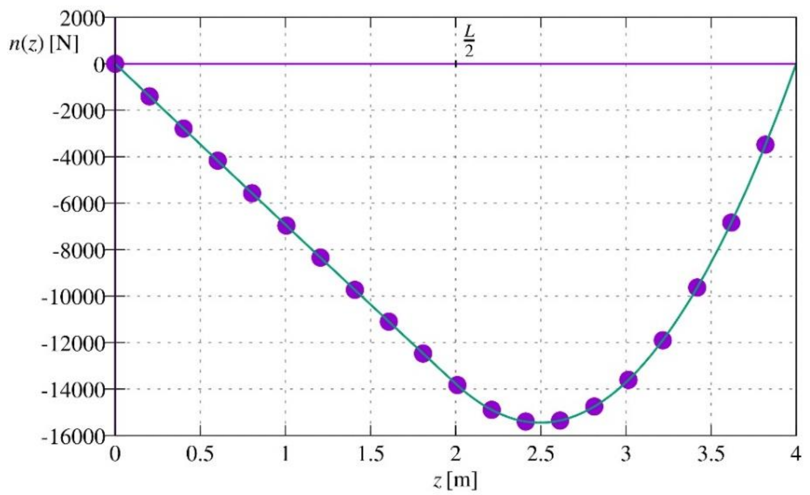

Figure 11 The graph of the function $n = n(z)$.

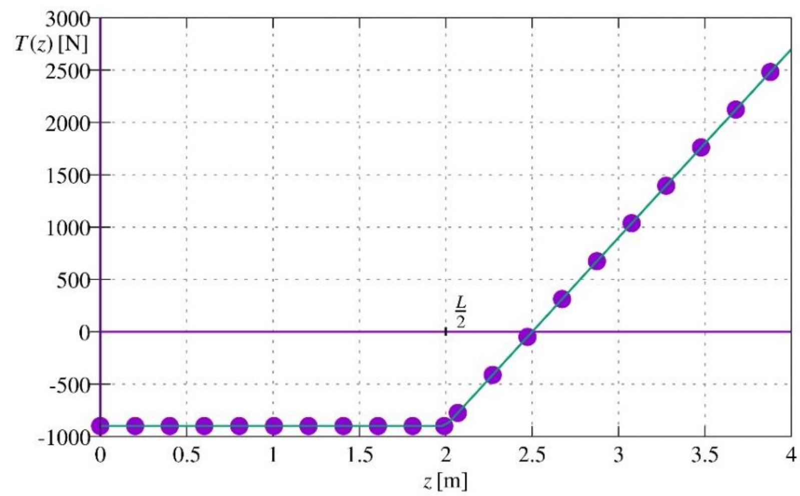

Figure 12 The graph of the shear force function $T = T(z)$.

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The stress field is obtained by the application of equations (52), (53) and equations (54), (55), (57), (58), (61), (63) and (64).

The plots of normal stress $\sigma_z(y,z)$ as a function of y for $z = L/6$, $z = L/4$, $z = 3L/4$ and $z = L$ are shown in Figure 13.

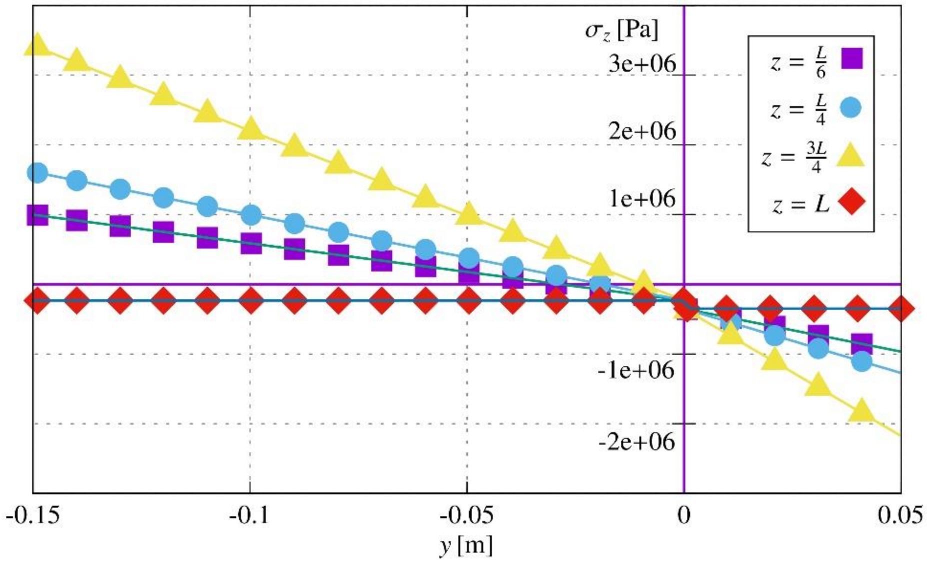
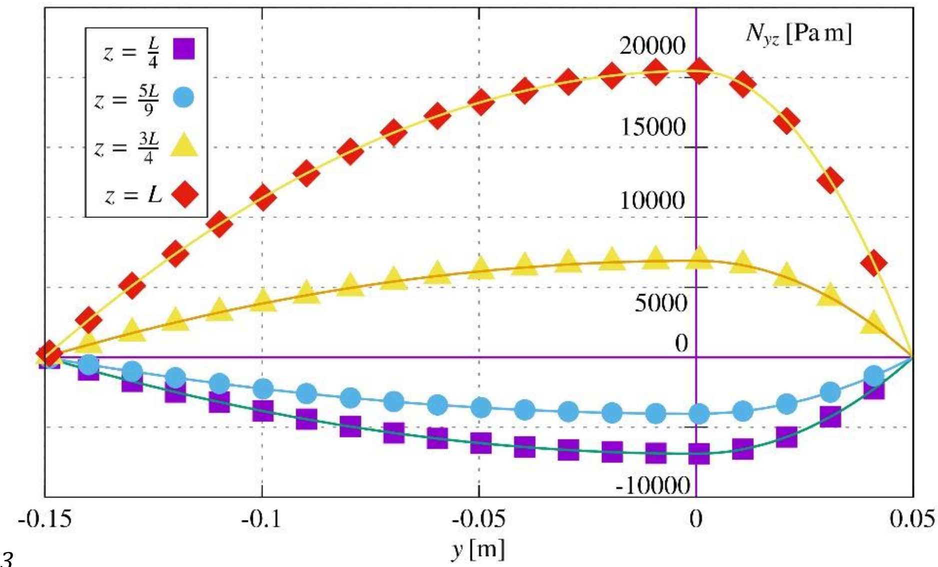


Figure 13 Plots of the normal stress $\sigma_z = \sigma_z(y,z)$ for four different values of axial coordinate.

The graphs of the shearing stress resultant $N_{yz}(y,z)$ as a function of y are presented in Figure 14.



3

Figure 14 The graphs of the shearing stress resultant as a function of y for four different values of z.

The plots of the normal stress resultants $N_y(y,z)$ as a function of y for four different values of the z coordinate are given in Figure 15.

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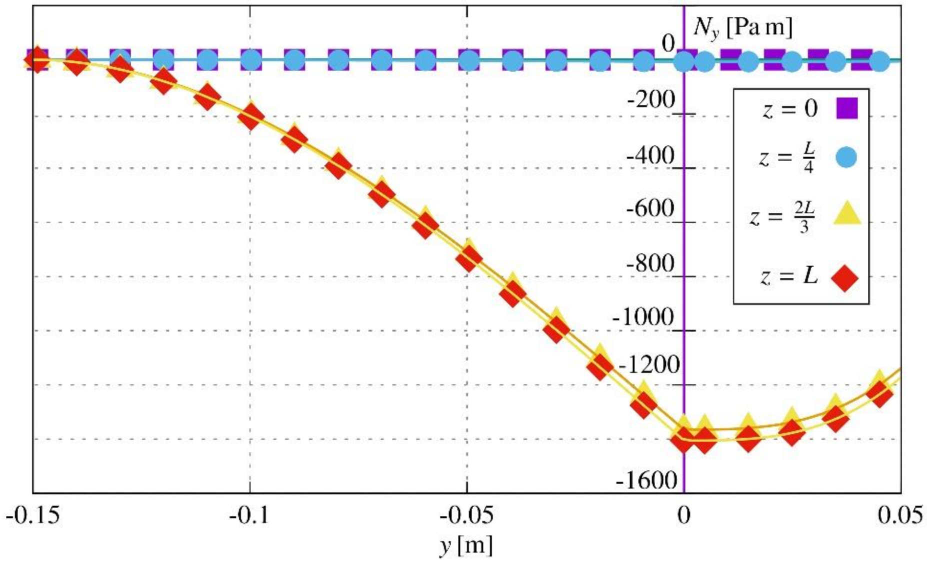

Figure 15 The plots of the function $N_y = N(y, z)$ as a function of y for $z = 0$, $z = 0.25L$, $z = 0.66L$ and $z = 0.99L$.

3.2. Example 2

Figure 16 shows the beam-column and its applied load. The expression of the applied transverse load is

$$f_y(z) = \frac{f_0}{L} z \qquad \text{for } 0 \le z \le L. \tag{66}$$

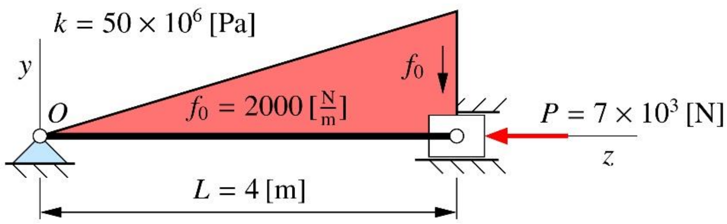

Figure 16 The beam-column with a linearly varying load and constant axial load.

The graphs of the deflection function $v = v(z)$ and the cross-sectional rotation function $\phi = \phi(z)$ are presented in Figures 17 and 18.

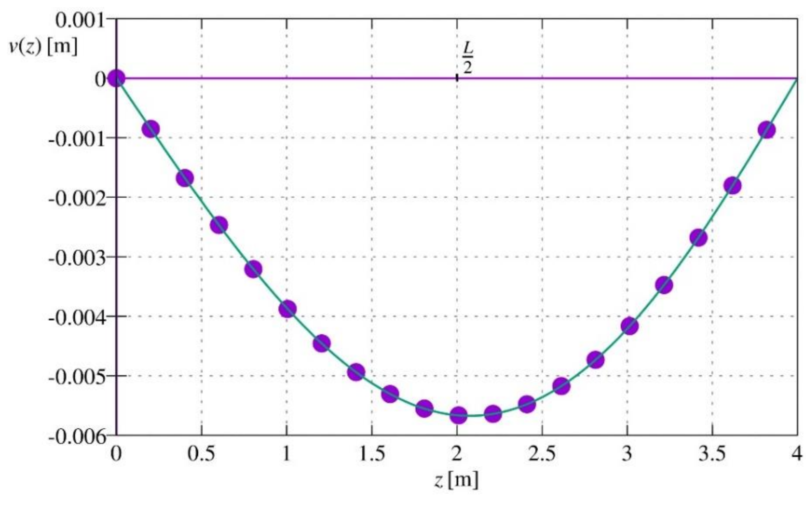

Figure 17 The plot of the deflection function $v = v(z)$.

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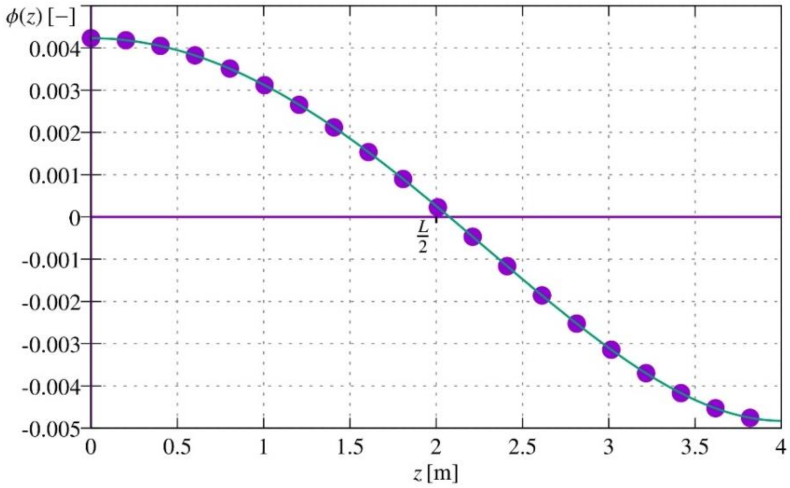

Figure 18 The graph of the cross-sectional rotation function $\phi = \phi(z)$.

The plot of slip function $s = s(z)$ is shown in Figure 19.

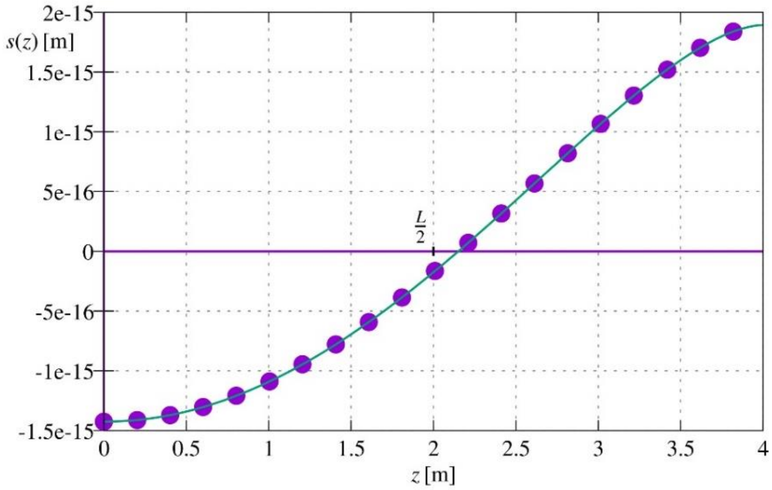

Figure 19 The plot of the slip function $s = s(z)$.

The graphs of the bending moment $M = M(z)$ and shear force $T = T(z)$ are given by in Figures 20 and 21.

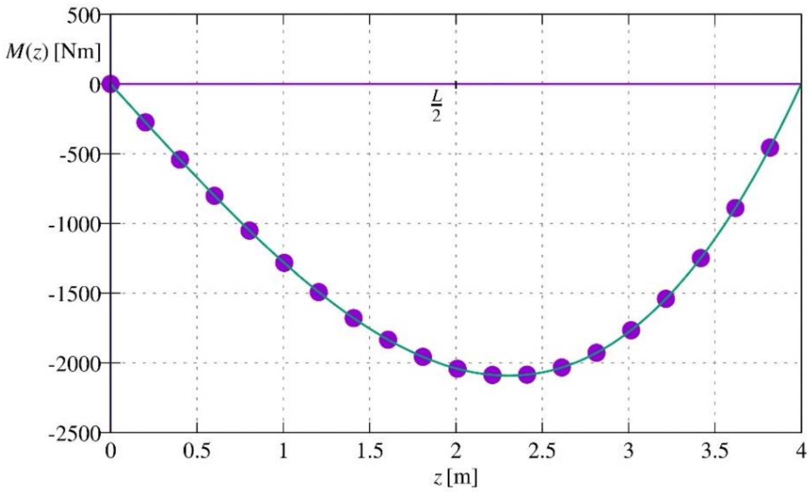

Figure 20 The graph of the bending moment function $M = M(z)$.

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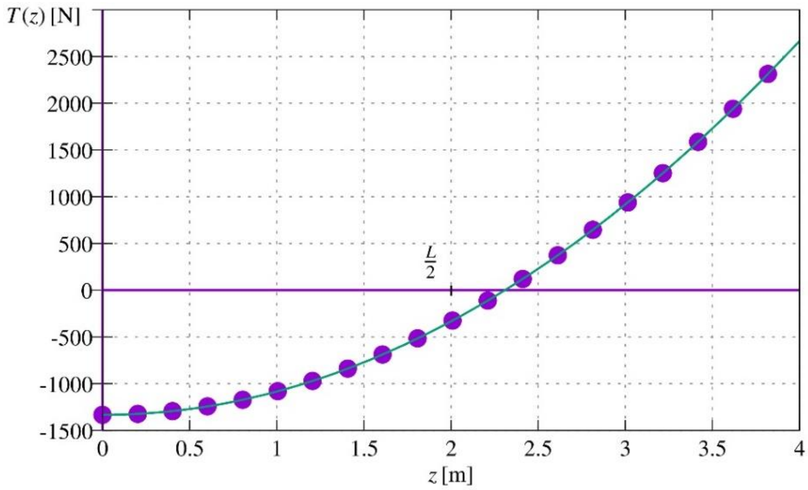

Figure 21 The plot of the cross-sectional shear force function $T = T(z)$.

The plot of the function $n = n(z)$ is presented in Figure 22.

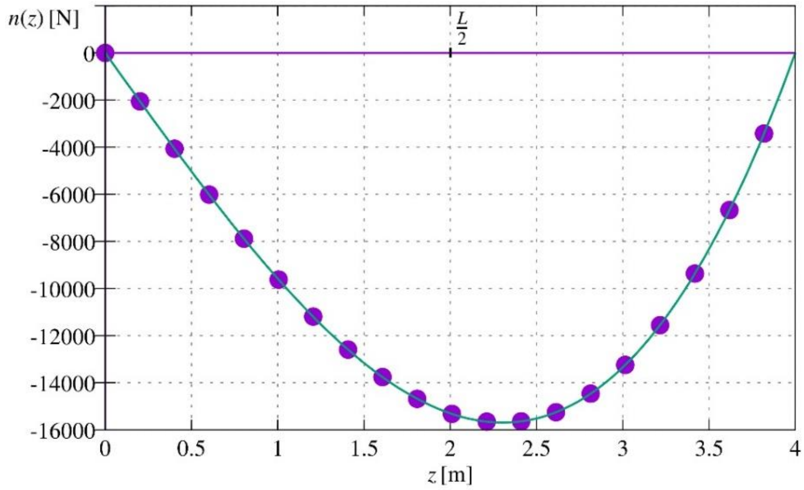

Figure 22 The graph of the function $n = n(z)$.

The plots of normal stress $\sigma_z = \sigma_z(y, z)$ and the stress resultant $N_{yz} = N_{yz}(y, z)$ are shown in Figures 23 and 24 as a function of y for four different values of he axial coordinate z.

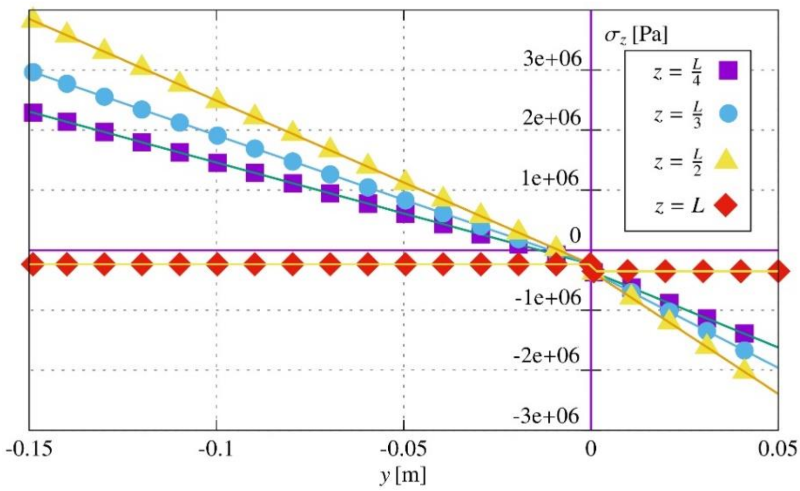

Figure 23 The graphs of the normal stresses as a function of y for four different values of axial coordinate.

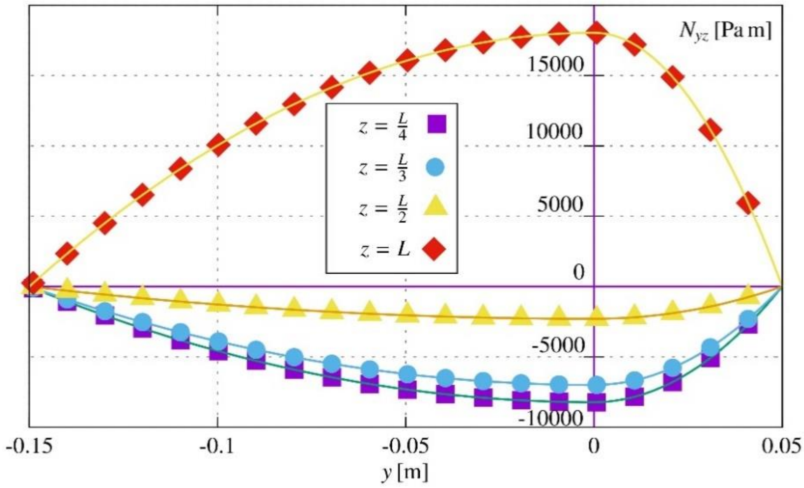

Figure 24 The graphs of the shearing stress resultant $N_{yz} = N_{yz}(y,z)$ *as a function of* y *for* $z = \frac{L}{4}, \frac{L}{3}, \frac{L}{2}, L$.

The plots of the normal stress resultant $N_y = N_y(y,z)$ as a function of y are presented in Figure 25 for $z = \frac{L}{4}, \frac{L}{3}, \frac{L}{2}, L$.

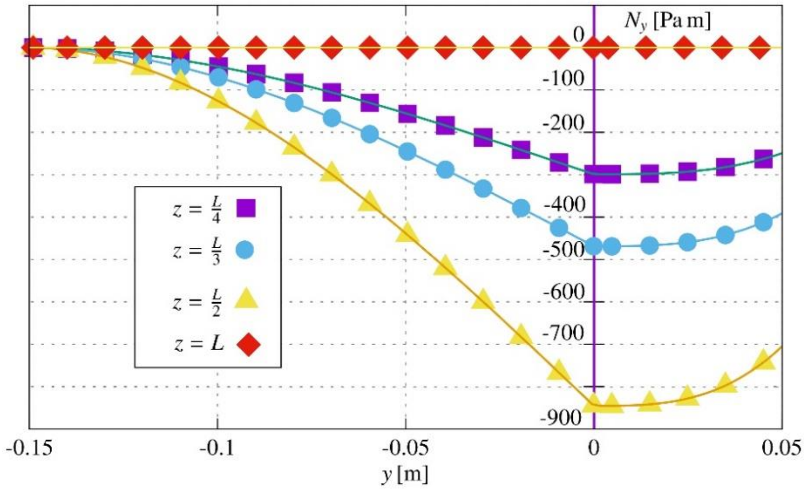

Figure 25 The graphs of the shearing stress resultant $N_y = N_y(y,z)$ *as a function of* y *for* $z = \frac{L}{4}, \frac{L}{3}, \frac{L}{2}, L$.

4. Conclusions

In this paper a two-layered composite beam-column with imperfect shear connection is considered. The beam-column at both end is simply supported and its applied load is transverse load which depends on the axial coordinate and constant axial load. The presented closed form solution is obtained by the Fourier series representation of the displacement and slip functions. Two examples illustrate the application of the developed analytical method.

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